1. Matrix decompositions

Problem 1. Cholesky decomposition

Find Cholesky decomposition matrix of the matrix $A = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$.

Solution: We need to find following decomposition

$$A = LL^*, \tag{1}$$

where L is lower triangular matrix.

We solve the problem using method of undetermined coefficients. Let $L = \begin{bmatrix} x & 0 \\ y & z \end{bmatrix}$, then L has to satisfy (1)

$$\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \equiv \begin{bmatrix} x & 0 \\ y & z \end{bmatrix} \begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$$
$$\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \equiv \begin{bmatrix} x^2 & xy \\ xy & y^2 + z^2 \end{bmatrix},$$

meaning

$$\begin{cases} x^2 = 2 \\ xy = -3 \\ y^2 + z^2 = 5 \end{cases} \Rightarrow \begin{cases} x = \sqrt{2} \\ y = -\dfrac{3}{\sqrt{2}} \\ z = \dfrac{1}{\sqrt{2}} \end{cases}. \tag{2}$$

Using (2), we get Cholesky deconposition

$$L = \frac{1}{\sqrt{2}} \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} \Longrightarrow A = \frac{1}{\sqrt{2}} \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}^*$$

■

Problem 2. QR decomposition

Find QR decomposition matrix of the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$.

Solution: We need to find following decomposition

$$A = QR,$$

where Q is unitary (orthogonal) and R is upper triangular matrix. Since A is already upper triangular matrix, QR decomposition is trivial. Let

$$Q = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad R = A.$$

■