

# Exercise 3.1-3.3

## 3.1

$$\begin{array}{c} \text{length}(S; 1) \quad \text{length}(K; 1) \\ \hline a_1 \text{ comb } a_2 \text{ comb } \text{length}(a_1; l_1) \text{ length}(a_2; l_2) \text{ sum}(l_1; l_2; l_{\text{tot}}) \\ \hline \text{length}(ap(a_1; a_2); l_{\text{tot}}) \end{array}$$

## 3.2

$$x \mid x \text{ comb} \vdash_C a_2 \text{ comb}$$

$a_1 \text{ comb}$

then  $[a_1/x] a_2 \text{ comb}$

Structure Ind on  $a_2$ :

1)  $a_2$  is  $S$   $[a_1/x] a_2 \stackrel{\Delta}{=} S \text{ comb}$

2)  $a_2$  is  $K$   $[a_1/x] a_2 \stackrel{\Delta}{=} K \text{ comb}$

3)  $a_2$  is  $ap(a_1'; a_2')$  and

$$a_1'' \stackrel{\Delta}{=} [a_1/x] a_1' \text{ comb}$$

$$a_2'' = [a_1/x] a_2' \text{ comb}$$

$$[a_1/x] a_2 \stackrel{\Delta}{=} ap([a_1/x] a_1'; [a_1/x] a_2')$$

$$\stackrel{\Delta}{=} ap(a_1''; a_2'') \text{ comb}$$

4)  $a_2$  is  $x$   $[a_1/x] a_2 \stackrel{\Delta}{=} a_1 \text{ comb}$

### 3.3

$$x \mid x \text{ comb} \vdash_{\text{CUE}} s \text{ K K } x$$

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$$3.16f$$

$$x \mid x \text{ comb} \vdash_{\text{CUE}} (\text{K } x)(\text{K } x)$$

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$$3.16e$$

$$x \mid x \text{ comb} \vdash_{\text{CUE}} x$$

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$$3.16a$$

$$x \text{ comb}$$