Operations Management and Inventory Control

Supplement 6

Statistical Process Control

Process Capability

- ☑ Is the ability of the process to meet the design specifications (engineering design , customer requirements).
- ☑ The natural variation of a process should be small enough to produce products that meet the standards required
- ☑ A process in statistical control does not necessarily meet the design specifications
- ☑ Process capability is a measure of the relationship between the natural variation of the process and the design specifications

Process Capability

- ***there are two popular measures for the quantitatively measure if the process in capable ?***

Process Capability Ration

Process Capability Index

Process Capability Ratio

$$C_p = \frac{\textit{Upper Specification - Lower Specification}}{6\sigma}$$

- ☑ *is a ratio of the specification to the process variation*
- ☑ *A capable process must have a* C_p *of at least* 1.0 (if C_p less that 1.0, so the good/service is outside the allowable tolerance
- ☑ *Often a target value of* C_p = 1.33 *is used to allow for off-center processes*
- ☑ *Six Sigma quality requires a* C_p = 2.0

Process Capability Ratio

Insurance claims process

Process mean $\bar{x} = 210.0$ *minutes*
Process standard deviation $\sigma = .516$ *minutes*
Design specification $= 210 \pm 3$ *minutes*

$$C_p = \frac{\text{Upper Specification - Lower Specification}}{6\sigma}$$

Process Capability Ratio

Insurance claims process

Process mean $\bar{x}$ = 210.0 *minutes*
Process standard deviation σ = .516 *minutes*
Design specification = 210 ± 3 *minutes*

$$C_p = \frac{\text{Upper Specification - Lower Specification}}{6\sigma}$$

$$= \frac{213 - 207}{6(.516)} = 1.938$$

Process Capability Ratio

Example

Insurance claims process

Process mean $\bar{x} = 210.0$ minutes
Process standard deviation $\sigma = .516$ minutes
Design specification $= 210 \pm 3$ minutes

$$C_p = \frac{\text{Upper Specification - Lower Specification}}{6\sigma}$$

$$= \frac{213 - 207}{6(.516)} = 1.938$$

Process is capable

Process Capability Index

$$C_{pk} = \text{minimum of } \left[\frac{\text{Upper Specification Limit} - \bar{x}}{3\sigma}, \frac{\bar{x} - \text{Lower Specification Limit}}{3\sigma} \right]$$

- ☑ *Measures the desired and actual dimensions of goods / services produced.*
- ☑ *A capable process must have a* C_{pk} *of at least* 1.0
- ☑ *A capable process is not necessarily in the center of the specification, but it falls within the specification limit at both extremes*

Process Capability Index

Example:

New Cutting Machine

New process mean $\bar{x}$ = .250 inches, tolerance is ± 0.001

Process standard deviation σ = .0005 inches

Upper Specification Limit = .251 inches

Lower Specification Limit = .249 inches

Process Capability Index

New Cutting Machine

New process mean $\bar{x}$ = .250 inches
Process standard deviation σ = .0005 inches
Upper Specification Limit = .251 inches
Lower Specification Limit = .249 inches

$$C_{pk} = \textit{minimum of } \left[\frac{(.251) - .250}{(3).0005}, \right]$$

Process Capability Index

New Cutting Machine

New process mean $\bar{x}$ = .250 inches
Process standard deviation σ = .0005 inches
Upper Specification Limit = .251 inches
Lower Specification Limit = .249 inches

$$C_{pk} = \textit{minimum of} \left[\frac{(.251) - .250}{(3).0005} \right], \left[\frac{.250 - (.249)}{(3).0005} \right]$$

Both calculations result in

$$C_{pk} = \frac{.001}{.0015} = 0.67$$

New machine is NOT capable

Interpreting C_{pk}

C_{pk} = ***negative number*** — Process does not meet specifications

C_{pk} = ***zero*** — Process does not meet specifications

C_{pk} = ***between* 0 *and* 1** — Process does not meet specifications

$C_{pk} = 1$ — Process meets specifications

$C_{pk} > 1$ — Process is better than the specifications requires

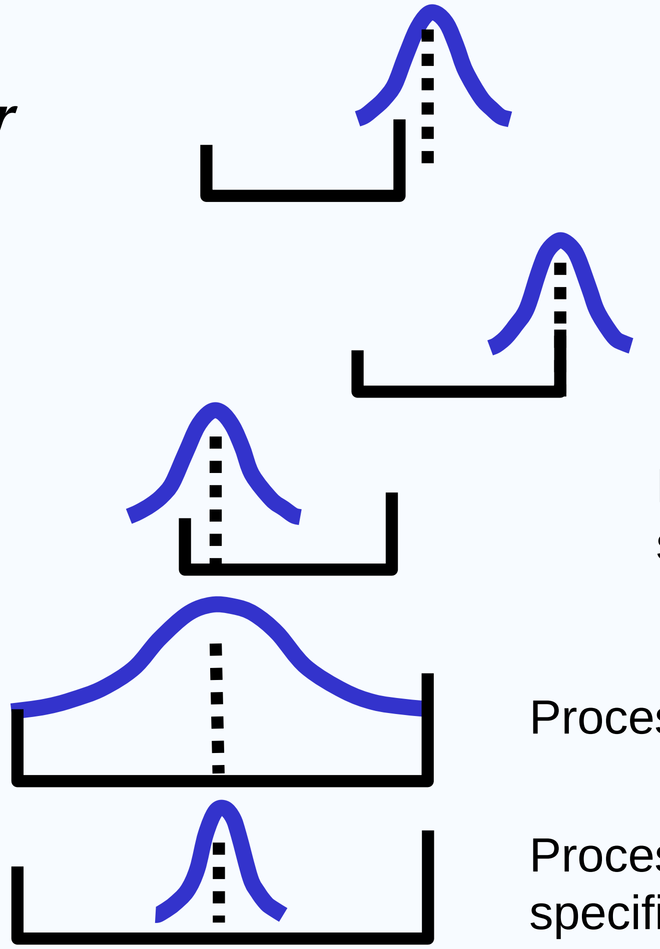


Figure S6.8

Acceptance Sampling

- ☑ *A method of measuring random samples of lots or batches of products against predefined standards*
- ☑ *Form of quality testing used for incoming materials or finished goods*
  - ☑ *Take samples at random from a lot (shipment) of items*
  - ☑ *Inspect each of the items in the sample*
  - ☑ *Decide whether to reject the whole lot based on the inspection results*
- ☑ *Attribute inspection is more commonly used by acceptance sampling.*

Operating Characteristic Curve (OC Curve)

- ☑ ***A graph*** *shows (describes) how well a sampling plan discriminates between good and bad lots (shipments)*
- ☑ *Shows the relationship between the probability of accepting a lot and its quality level*

The "Perfect" OC Curve

AQL and LTPD

- ☑ ***Acceptable Quality Level (AQL): is the quality level of a lot considered good***
  - ☑ ***Poorest level of quality we are willing to accept (i.e. we wish to accept lots that have this or a better level of quality)***
- ☑ ***Lot Tolerance Percent Defective (LTPD):***

is the quality level of a lot that we considered bad.

  - ☑ ***Consumer (buyer) does not want to accept lots with more defects than LTPD***
  - ❑ **To derive a sampling plan, producer and consumer must define not only "good lots" ," bad lots" But they also must specify risk levels.**

Producer’s and Consumer’s Risks

- ☑ ***Producer's risk*** (α)
  - ☑ *Probability of rejecting a good lot*
  - ☑ *Probability of rejecting a lot when the fraction defective is at or above the AQL*
  - ☑ *a lot with an acceptable quality level of AQL still has* $\propto$ *chance of being rejected*
  - ☑ *sampling plans designed often to set* $\alpha = 0.05$ *or 5%.*
- ☑ ***Consumer's risk*** (β)
  - ☑ *Probability of accepting a bad lot*
  - ☑ *Probability of accepting a lot when fraction defective is below the LTPD*
  - ☑ *a common value of* $\beta = 0.10$ *or* 10%

An OC Curve

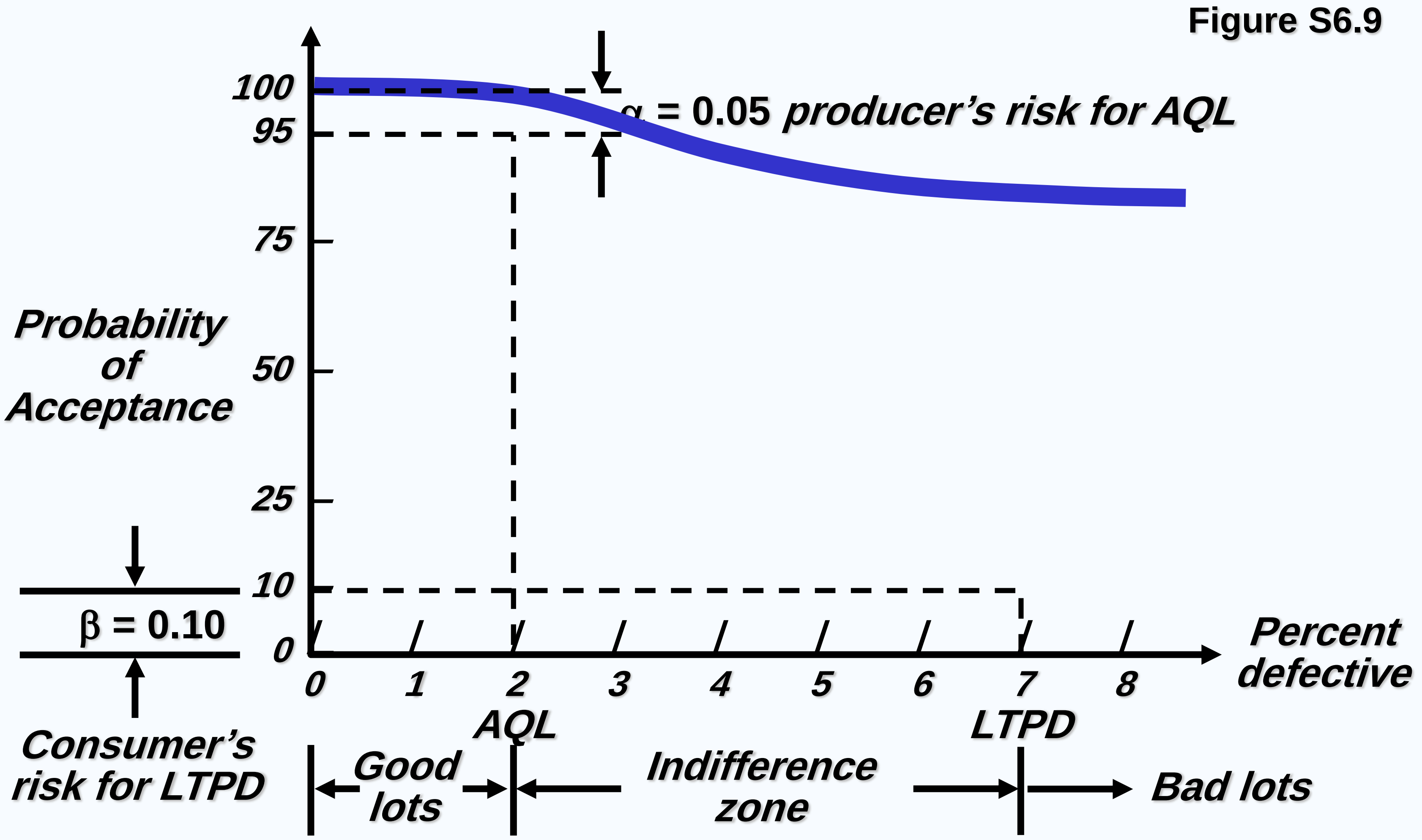


Figure S6.9
100
95
75
50
25
10
0
Probability of Acceptance
α = 0.05 producer's risk for AQL
β = 0.10
Consumer's risk for LTPD
0
1
2
3
4
5
6
7
8
Percent defective
AQL
LTPD
Good lots
Indifference zone
Bad lots

Average Outgoing Quality

- Is the percentage defective in an average lot of goods inspected through acceptance sampling.
- In most of sampling plans when a lot is rejected, the entire lot is inspected and all defective items replaced; using this replacement technique improves the AOQ in terms of percent defective.

$$\text{AOQ} = \frac{(P_d)(P_a)(N - n)}{N}$$

where

P_d = *true percent defective of the lot*

P_a = *probability of accepting the lot*

N = *number of items in the lot*

n = *number of items in the sample*

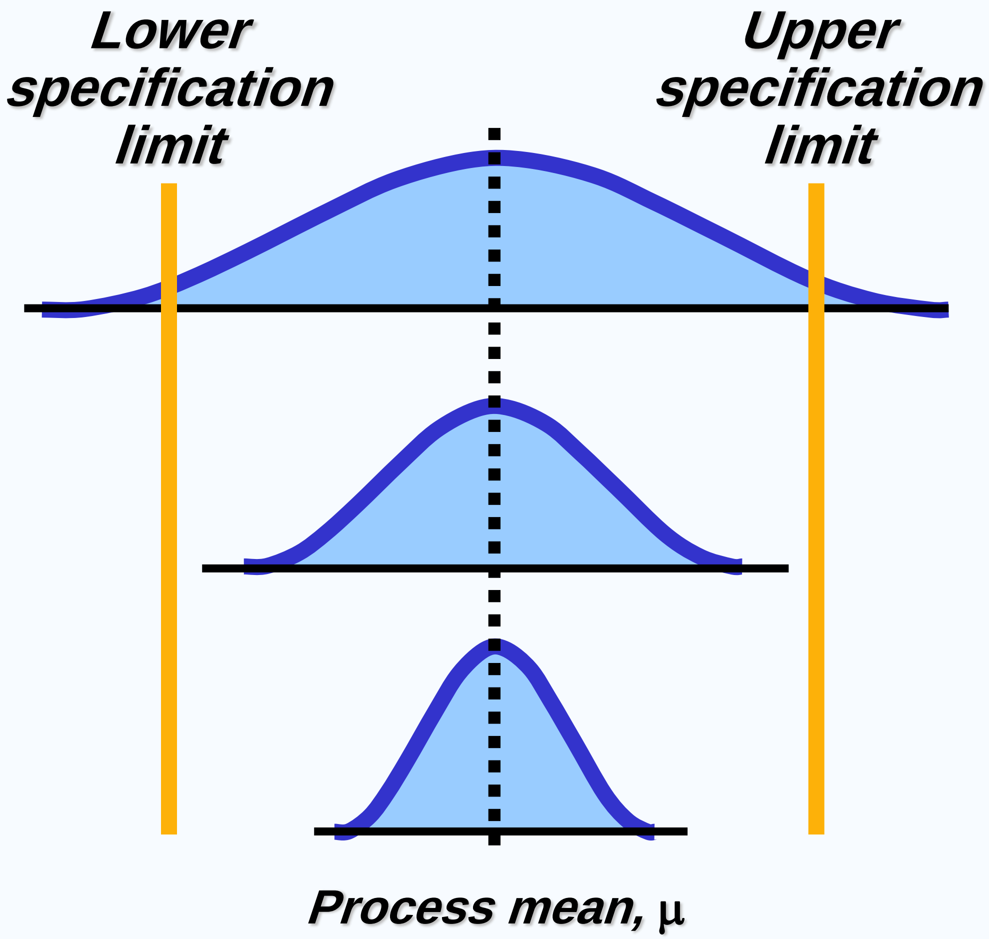
Average Outgoing Quality

1. *If a sampling plan replaces all defectives*
2. *If we know the incoming percent defective for the lot*

We can compute the average outgoing quality (AOQ) in percent defective

The maximum AOQ is the highest percent defective or the lowest average quality and is called the average outgoing quality level (AOQL)

Statistical Process Control (SPC) & Process Variability



(a) Acceptance sampling (Some bad units accepted)

(b) Statistical process control (Keep the process in control)

(c) C_{pk} >1 (Design a process that is in control)

Figure S6.10

Thank You