LAFDS Session 4 Homework

Full Name: _____

Group No.: ____

Please write down all the steps not the final answer only

Questions:

1. (5 points)

Find the eigenvalues and eigenvectors of A and A^2 and A^{-1} and $A + 4I$:

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad A^2 = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}.$$

Check the trace $= \lambda_1 + \lambda_2$ and the determinant $= \lambda_1 \lambda_2$ for A and also A^2.

2. (5 points)

Let Π be the plane in R^3 spanned by vectors $x_1 = (2, 4, 4)$ and $x_2 = (-3, 0, 6)$.

(i) Find an orthonormal basis for Π.

(ii) Extend it to an orthonormal basis for $\mathbb{R}^3$

3. (5 points)

Let $V = \mathbb{R}^2$, $B = \{(1, 2), (3, 4)\}$, $C = \{(7, 3), (4, 2)\}$

the set ***B*** forms a basis for $\mathbb{R}^2$. Also, the set ***C*** forms a basis for $\mathbb{R}^2$.

(Note that the vectors are not unit vectors, we have to normalize by the determinant)

a. Verify that B and C are bases for $\mathbb{R}^2$.
b. Find the change of basis matrix from ***B*** to ***C***.
c. Find the change of basis matrix from ***C*** to ***B***

Hint; the change of basis matrices are given below, verify them.

$$P_{C \leftarrow B} = \begin{bmatrix} -3 & -5 \\ 5.5 & 9.5 \end{bmatrix}.$$

$$P_{B \leftarrow C} = \begin{bmatrix} -9.5 & -5 \\ 5.5 & 3 \end{bmatrix}.$$