18/11/2025

Linear Algebra , AI46 , Mansoura

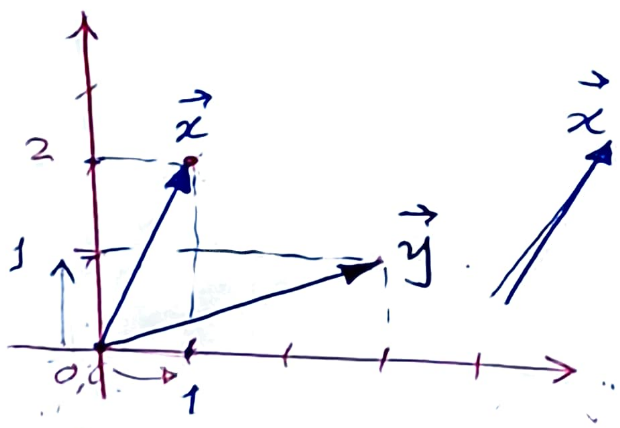
scalar ; quantity that has amagnitude.
(no direction) ; $\underline{5}$

Vector ; ⇒ ordered set of scalar values

⇒ quantity with ~~magnitda~~ magnitude
& direction

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

column vector representation



$$\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\vec{y} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$\vec{x} = (x_1, x_2) = \nless \quad \ngtr$

ordered set of numbers

dimensions

$$\vec{x} \in \mathbb{R}^2$$

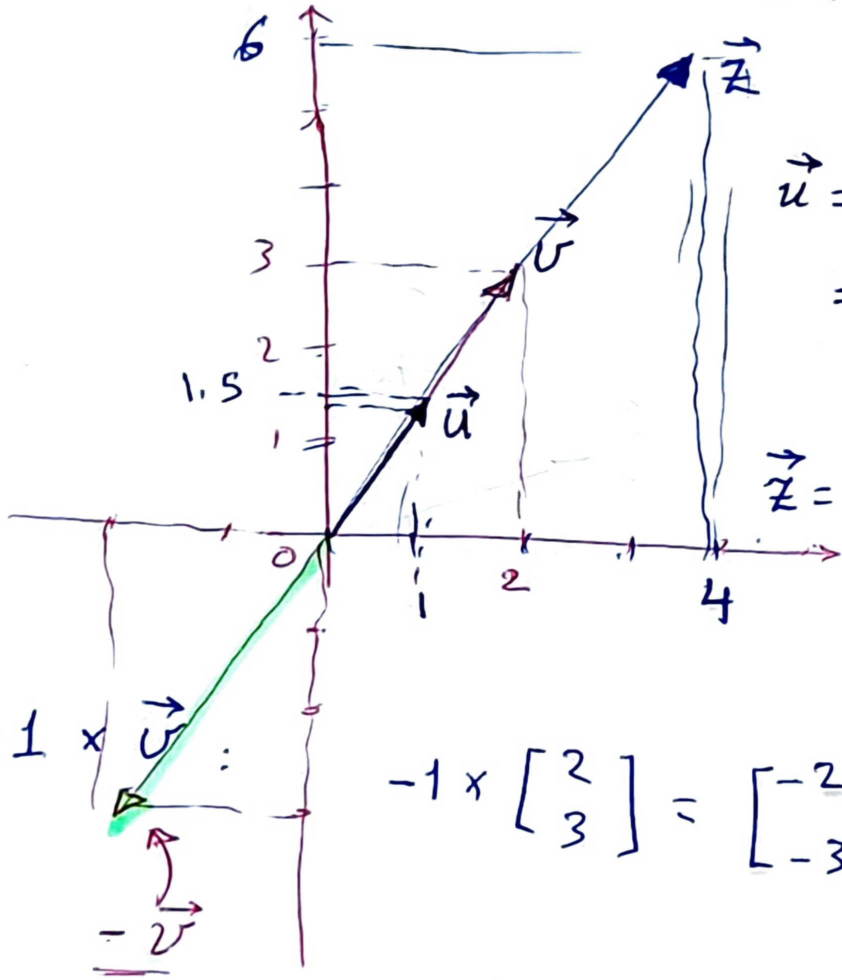
$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \Leftrightarrow \vec{v} \in \mathbb{R}^n$$

→ Vector scaling ; scalar multiplication

$$a\vec{v} = a\begin{bmatrix} 2 \\ 3 \end{bmatrix} = a(2, 3) \equiv a[2 \quad 3]^T$$

a ↑ scalar value

$$a\vec{v} = a\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} av_1 \\ av_2 \end{bmatrix}$$



$$\vec{u} = 0.5\vec{v} = 0.5\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.5\times 2 \\ 0.5\times 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$$

$$\vec{z} = 2\vec{v} = \begin{bmatrix} 2\times 2 \\ 2\times 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

→ $-1 \times \vec{v}$:

$$-1 \times \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

→ Vector addition

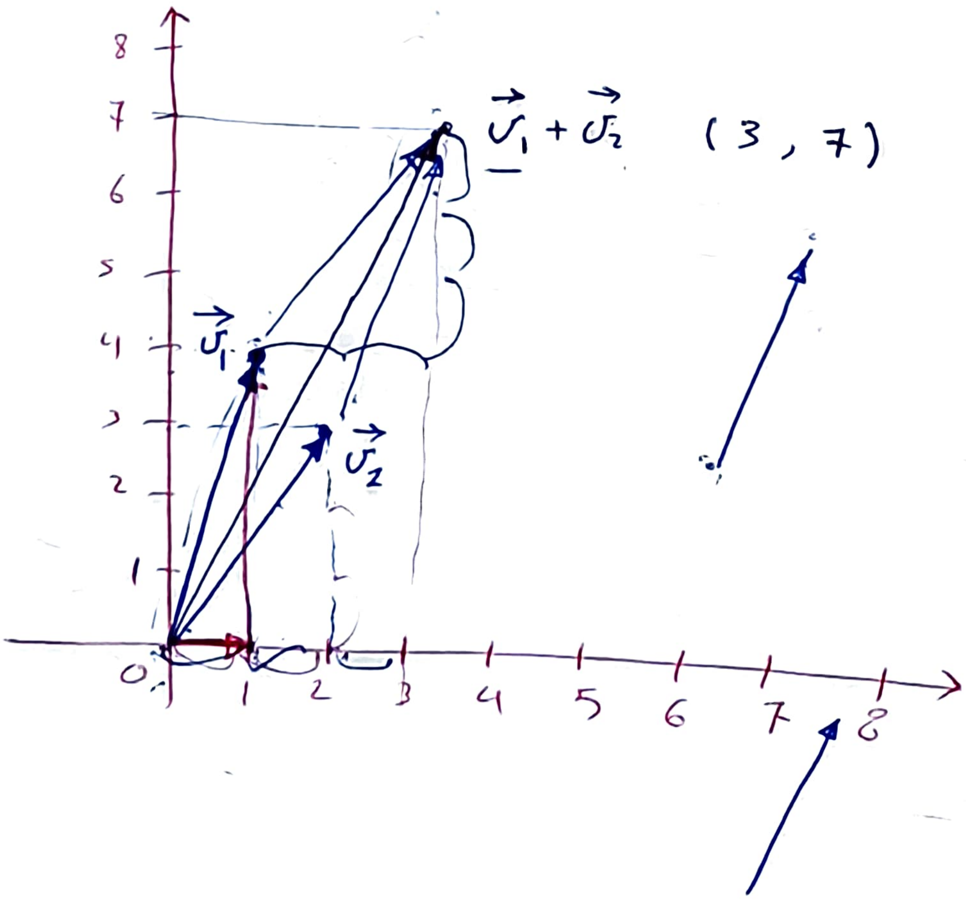
$$\vec{v}_1 = \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$\vec{v}_2 = \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\vec{v}_1 + \vec{v}_2 = \begin{bmatrix} \overset{1}{v_{11}} + \overset{2}{v_{21}} \\ \overset{4}{v_{12}} + \overset{3}{v_{22}} \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$



→ Vector subtraction

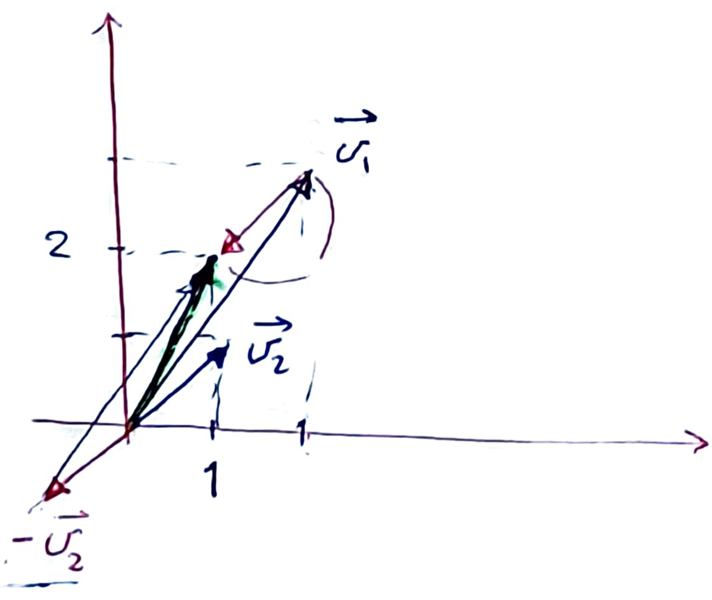
$$\vec{v_1} = \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} \qquad \vec{v_2} = \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix}$$

$$\vec{v_1} - \vec{v_2} = \begin{bmatrix} v_{11} - v_{21} \\ v_{12} - v_{22} \end{bmatrix}$$

$$\vec{v_1} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \vec{v_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\vec{v_1} - \vec{v_2} = \begin{bmatrix} 2-1 \\ 3-1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\vec{v_1} + (-\vec{v_2})$$



dot (Inner) product)

$\vec{u} \cdot \vec{v} \quad \in \mathbb{R}^2$

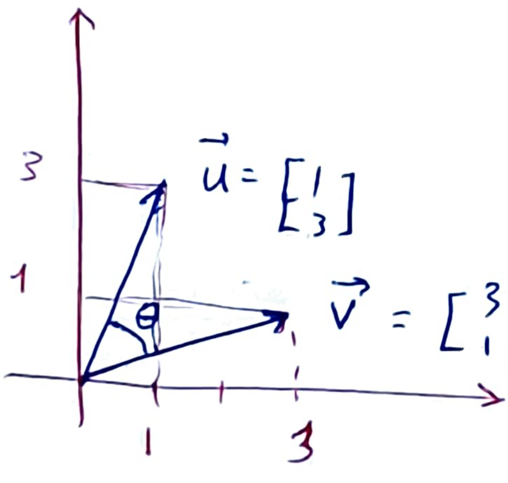
$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = u_1 \times v_1 + u_2 \times v_2 =$$

$$= \|\vec{u}\| \, \|\vec{v}\| \cos\theta$$

$\vec{u}, \vec{v} \in \mathbb{R}^n$

$$\boxed{\cos\theta = \frac{\vec{v} \cdot \vec{u}}{\|\vec{u}\| \, \|\vec{v}\|}}$$

$$\vec{u} \cdot \vec{v} = \sum_{i=1}^{n} u_i \times v_i$$



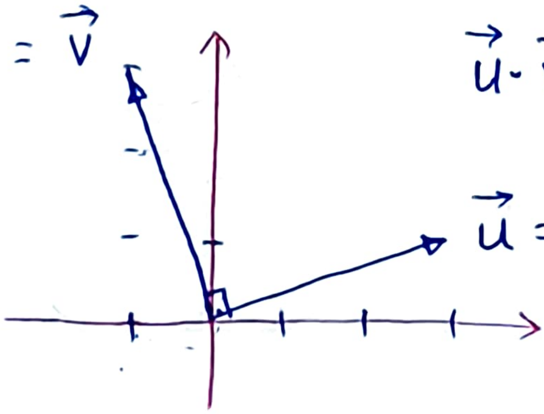
$\vec{u} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\vec{u} \cdot \vec{v} = 1 \times 3 + 3 \times 1 = 6$

$=$

$\begin{bmatrix} -1 \\ 3 \end{bmatrix} = \vec{v}$



$\vec{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

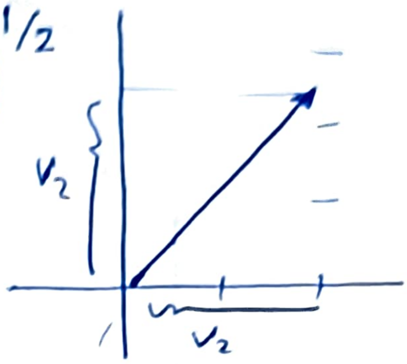
$$\vec{u} \cdot \vec{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = -3 + 3$$

$$= 0$$

$=$

"Cosine similarity"

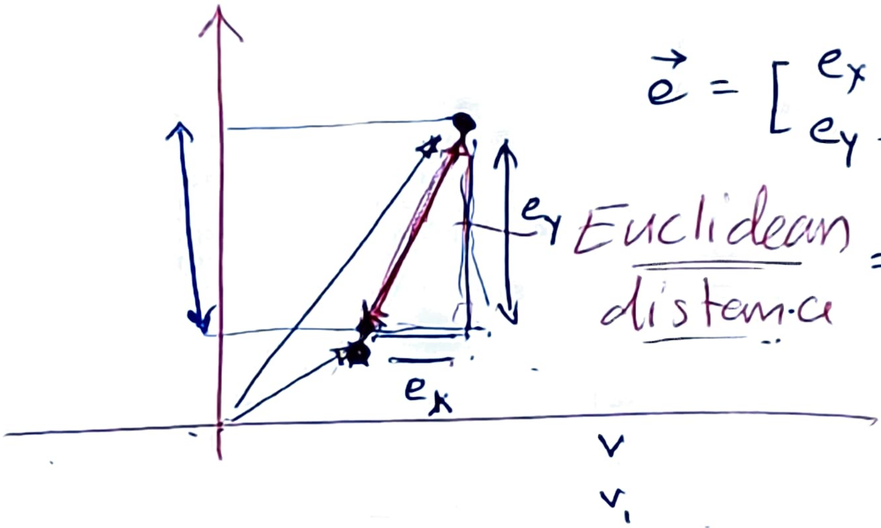
$$\|\vec{V}\|_2 = \left(\sum_i (V_i)^2\right)^{1/2}$$



l_2 - norm

Euclidean distance

length $= \sqrt{v_1^2 + v_2^2}$



$$\vec{e} = \begin{bmatrix} e_x \\ e_y \end{bmatrix}$$

Euclidean distance $= \sqrt{e_1^2 + e_2^2}$

$$\vec{e} \in \mathbb{R}^m \Rightarrow \|\vec{e}\|_2 = \sqrt{e_1^2 + e_2^2 + e_3^2 + \cdots + e_m^2}$$

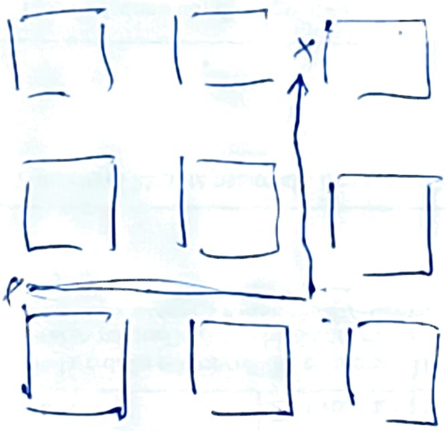
$$= \left(\sum_{i=1}^{m} e_i^2\right)^{1/2}$$

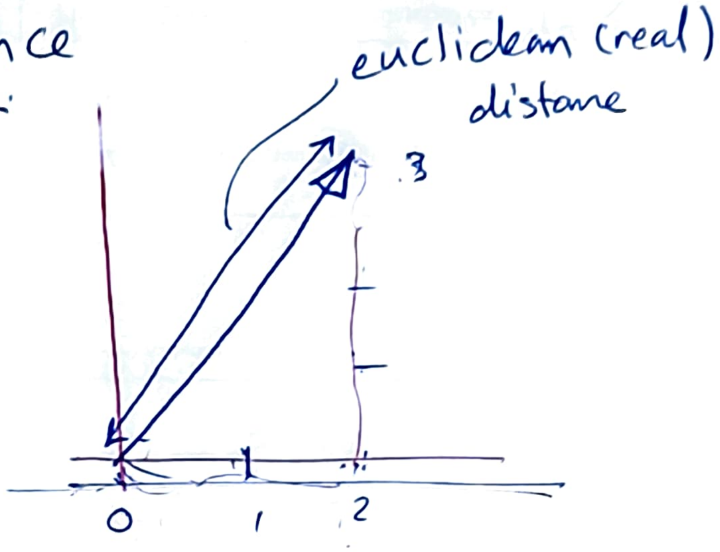
$\Rightarrow$ "n" norm of a vector

$$= \|\vec{e}\|_n = \left(\sum_{i=1}^{m} e_i^n\right)^{1/n}$$

l_1-norm $= \sum_{i=1}^{m} |(e_i)|$

Manhattan distance





cosine similarity

Matrices

$$A_{m\times n} \in R^{m\times n}$$

$$A_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & & \vdots \\ \vdots & & & \vdots \\ a_{m\times 1} & & & a_{mn} \end{bmatrix}$$

$$\vec{u} \to \boxed{T} \to \vec{v}$$

$$\vec{u} \longrightarrow \boxed{A\vec{u}} \longrightarrow \vec{v} \quad \Leftrightarrow \quad x \to \boxed{f(\,)} \to y$$

$$A_{m\times n} \times \vec{u}_{n\times 1} = \vec{v}_{m\times 1}$$

$$\underline{\vec{u} \in R^n} \xrightarrow{A \in R^{m\times n}} \underline{\vec{V} \in R^m}$$

$$A_{2\times 3} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}}_{\vec{v}} = \begin{bmatrix} 1v_1 + 2v_2 + 3v_3 \\ 4v_1 + 5v_2 + 6v_3 \end{bmatrix}$$

Row picture

$$\begin{bmatrix} \overbrace{(1, 2, 3)}^{\vec{r}_{A_1}} \\ \underbrace{(4, 5, 6)}_{\vec{r}_{A_2}} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \vec{r}_{A_1} \cdot \vec{v} \\ \vec{r}_{A_2} \cdot \vec{v} \end{bmatrix}$$

Column Picture

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = v_1 \vec{C}_{A_1} + v_2 \vec{C}_{A_2} + v_3 \vec{C}_{A_3}$$

$$\vec{C}_{A_1} \quad \vec{C}_{A_2} \quad \vec{C}_{A_2}$$

$$= v_1 \begin{bmatrix} 1 \\ 4 \end{bmatrix} + v_2 \begin{bmatrix} 2 \\ 5 \end{bmatrix} + v_3 \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} v_1 + 2v_2 + 3v_3 \\ 4v_2 + 5v_2 + 6v_3 \end{bmatrix}$$

Matrix - Matrix Multiplication

$$A_{m\times K} \times B_{K\times n} = C_{m\times n}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & B_{32} \end{bmatrix} = \begin{bmatrix} \; & \; \end{bmatrix}$$

Row Echelon form

$$\begin{bmatrix} 2 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} \text{row } 1 \\ \text{row}2 - 2\times\text{row}1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 3 \\ 4-2\times 2 & 5-2\times 2 & 6-2\times 3 \end{bmatrix}$$

$$2 \quad \begin{bmatrix} 2 & 2 & 3 \\ 0 & 1 & 0 \end{bmatrix}$$

Reduced Row Echelon form

$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{row 1} \div 2$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1.5 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{array}{l} row_1 \rightarrow row_1 - row_2 \\ row_2 \rightarrow row_2 \end{array}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1.5 \\ 0 & 1 & 0 \end{bmatrix}$$

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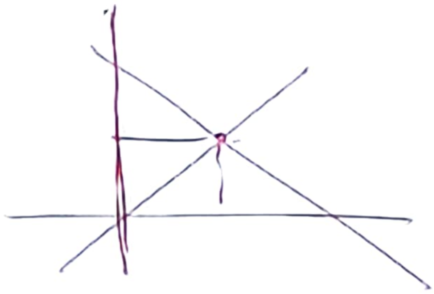
sys. of linear eqns.

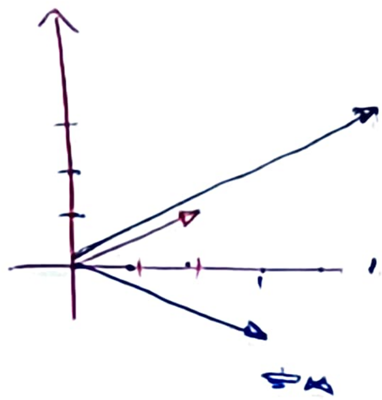
$$2x + 3y = 5$$

$$x - y = 3$$



RRREF $\longrightarrow$ $\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ — solution.

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} x + \begin{bmatrix} 3 \\ -1 \end{bmatrix} y = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$



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