

1. Exponential Function

x	-7	-4	-1	2	5	8	11
$H(x)$	-125	-13	1	2.75	2.969	2.996	2.999

a) $\lim_{x \rightarrow -\infty} H(x) =$

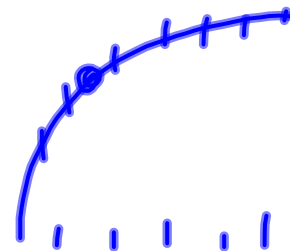
$-\infty$

b) $\lim_{x \rightarrow -1} H(x) =$

1

c) $\lim_{x \rightarrow \infty} H(x) =$

3



2. Rational Function

x	-1000	-2.001	-2	-1.999	0.999	1	1.001	1000
$G(x)$	0.998	0.333	Undefined	0.333	-1999	Undefined	2001	1.002

a) $\lim_{x \rightarrow -\infty} G(x) = 1$

b) $\lim_{x \rightarrow -2^-} G(x) = \frac{1}{3}$

c) $\lim_{x \rightarrow -2^+} G(x) = \frac{1}{3}$

d) $\lim_{x \rightarrow -2} G(x) = \frac{1}{3}$

e) $\lim_{x \rightarrow 1^-} G(x) = -\infty$

f) $\lim_{x \rightarrow 1^+} G(x) = +\infty$

g) $\lim_{x \rightarrow 1} G(x) = \text{DNE}$

h) $\lim_{x \rightarrow \infty} G(x) = 1$
(HA?)

3. Rational Function

x	-10000	0.999	1	1.001	3.999	4	4.001	10000
$H(x)$	1.9999	-2.331	Undefined	-2.335	-12998	Undefined	13002	2.001

a) $\lim_{x \rightarrow \infty} H(x) = 2$

b) $\lim_{x \rightarrow 1} H(x) = -2.3 \approx -\frac{7}{3}$

c) $\lim_{x \rightarrow 4^+} H(x) = +\infty$

d) $\lim_{x \rightarrow 4^-} H(x) = -\infty$

e) $\lim_{x \rightarrow 4} H(x) = \text{DNE}$

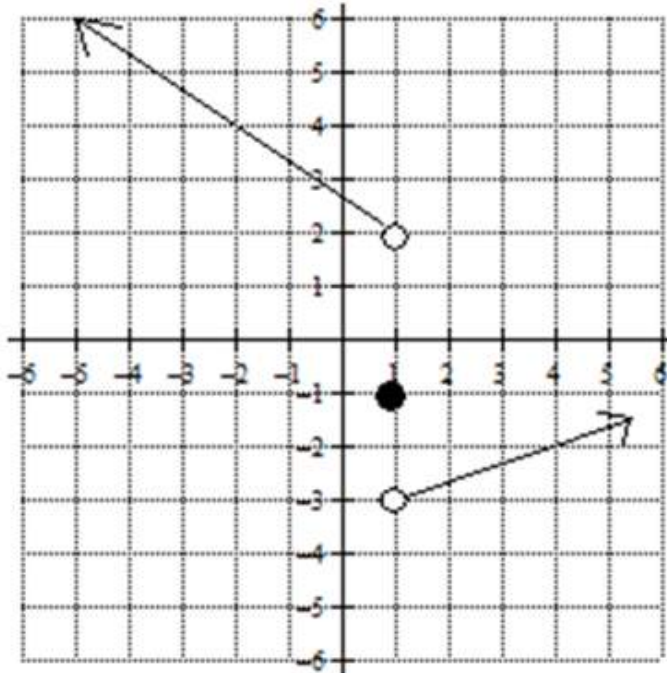
Characteristics

At $x=1$ hole

At $x=4$ VA

*NOTE - Correct 4w

4. The graph of $h(x)$ is given.

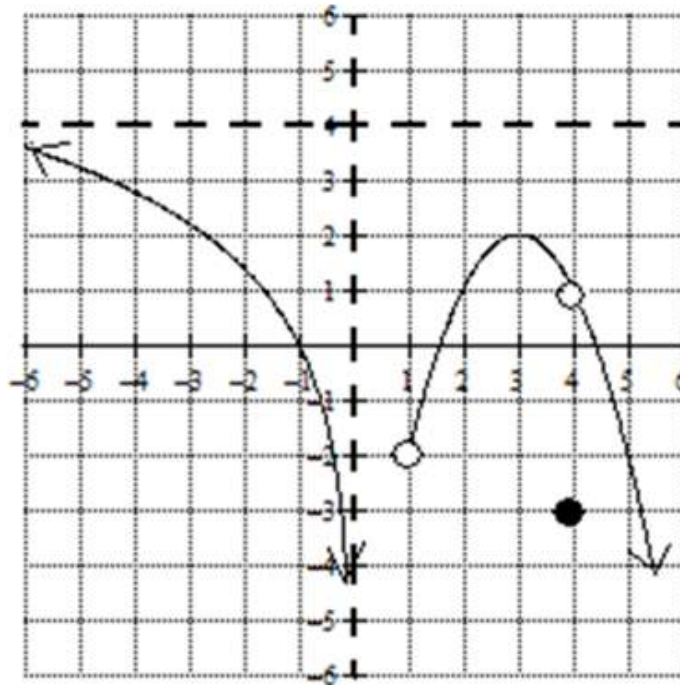


a) $\lim_{x \rightarrow 1^-} h(x) = 2$ b) $\lim_{x \rightarrow 1^+} h(x) = -3$

c) $\lim_{x \rightarrow 1} h(x) = \text{DNE}$ d) $h(1) = -1$

e) $h(-2) = 4$ f) $\lim_{x \rightarrow -2} h(x) = 4$

5. The graph of $g(x)$ is given.

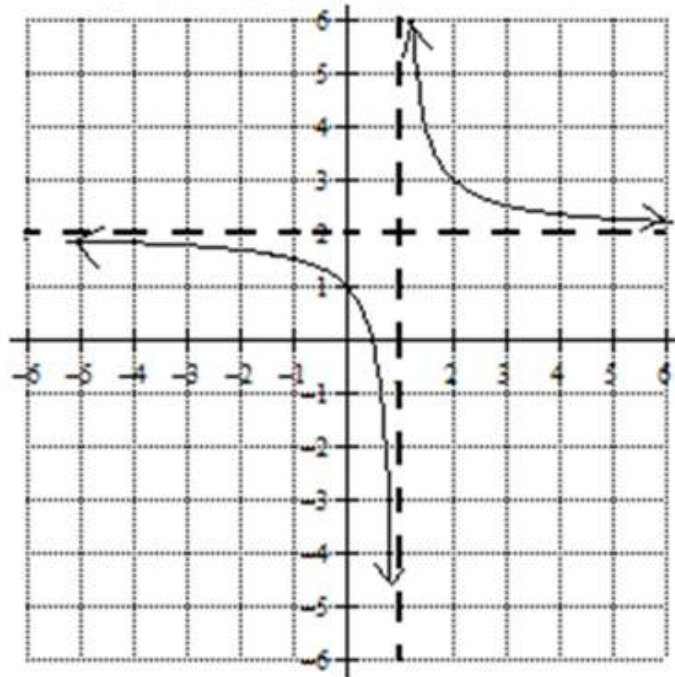


a) $\lim_{x \rightarrow 0^-} g(x) = -\infty$ b) $\lim_{x \rightarrow 1^+} g(x) = -2$

c) $\lim_{x \rightarrow -\infty} g(x) = 4$ d) $\lim_{x \rightarrow 4} g(x) = 1$

e) $g(4) = -3$ f) $\lim_{x \rightarrow 3} g(x) = 2$

6. The graph of $f(x)$ is given.



a) $\lim_{x \rightarrow 0} f(x) = 1$

b) $\lim_{x \rightarrow -\infty} f(x) = 2$

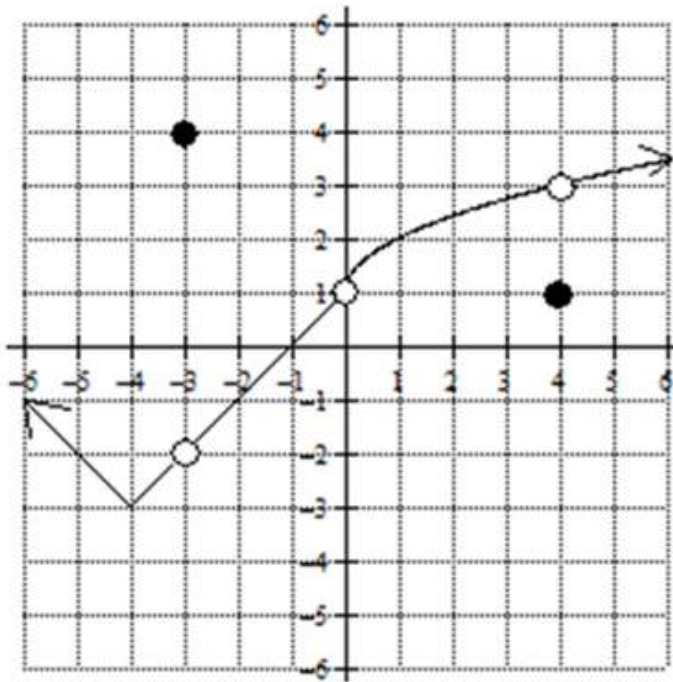
c) $\lim_{x \rightarrow \infty} f(x) = 2$

d) $\lim_{x \rightarrow 1^+} f(x) = +\infty$

e) $\lim_{x \rightarrow 1^-} f(x) = -\infty$

f) $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

7. The graph of $q(x)$ is given.

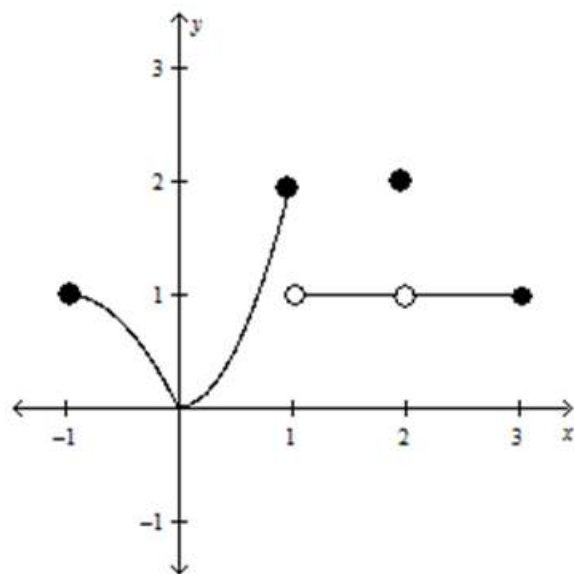


a) $\lim_{x \rightarrow 0} q(x) = 1$ b) $\lim_{x \rightarrow -3} q(x) = -2$

c) $\lim_{x \rightarrow 4} q(x) = 3$ d) $\lim_{x \rightarrow -4} q(x) = -3$

e) $q(-3) = 4$ f) $q(4) = 1$

Given the graph of the function, $g(x)$, below, determine if the statements are true or false. For statements that are false, explain why.



8. $\lim_{x \rightarrow 1} g(x) = 2$

False
 $g(1) = 2 \rightarrow \lim_{x \rightarrow 2} g(x) \text{ DNE}$ (left & right limits don't equal)

9. $\lim_{x \rightarrow c} g(x)$ exists for every value of c on the interval $(-1, 1)$.

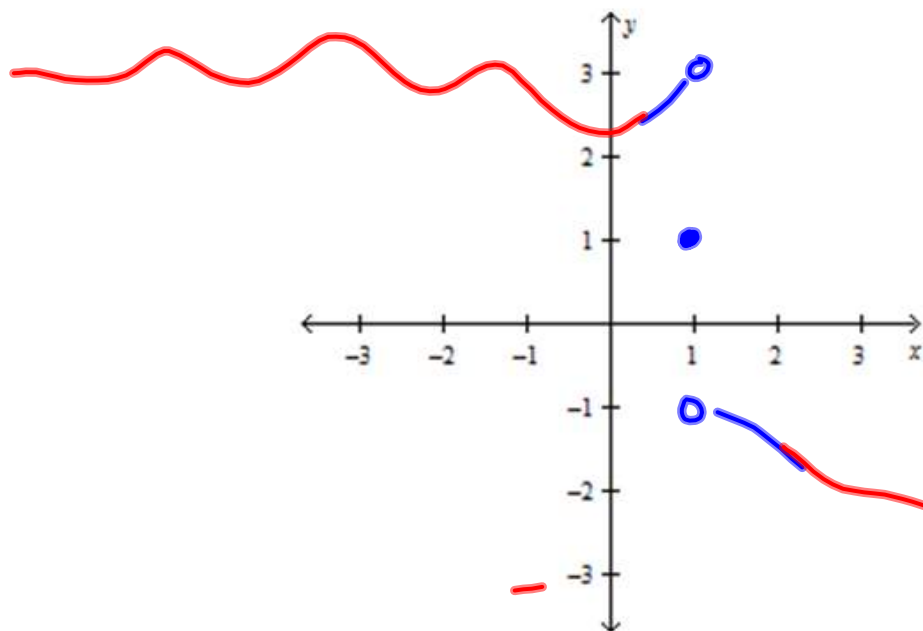
True

open \checkmark

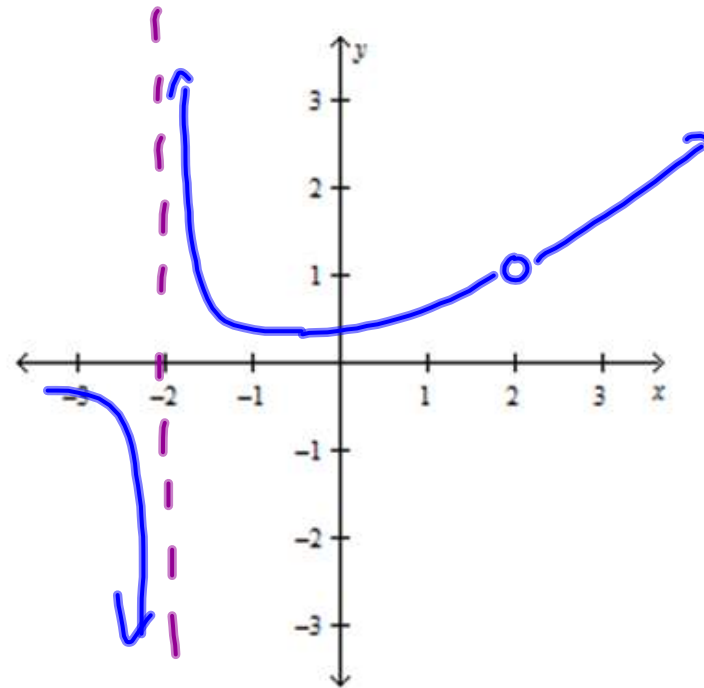
10. $\lim_{x \rightarrow 2} g(x)$ does not exist.

False $\lim_{x \rightarrow 2^-} g(x) = 1 > \therefore \lim_{x \rightarrow 2} g(x) = 1$
 $\lim_{x \rightarrow 2^+} g(x) = 1$

11. $\lim_{x \rightarrow 1^-} f(x) = 3$ $\lim_{x \rightarrow 1^+} f(x) = -1$ $f(1) = 1$



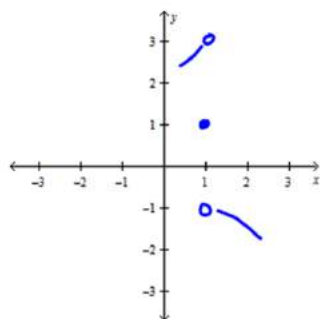
12. $\lim_{x \rightarrow -2^-} f(x) = -\infty$ $\lim_{x \rightarrow 2^+} f(x) = \infty$ \leftarrow +1P.
 $f(2)$ is undefined but $\lim_{x \rightarrow 2} f(x)$ exists.



✓ ,

13. In exercise 11, does $\lim_{x \rightarrow 1} f(x)$ exist? Explain why or why not.

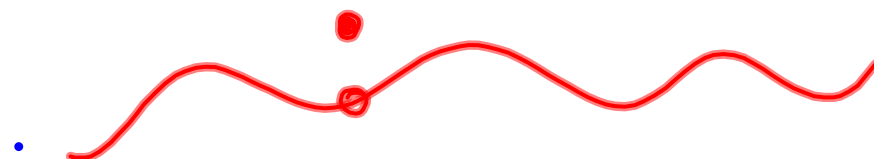
11. $\lim_{x \rightarrow 1^-} f(x) = 3$ $\lim_{x \rightarrow 1^+} f(x) = -1$ $f(1) =$



No the $\lim_{x \rightarrow 1} f(x)$ DNE

b/c the left-hand limit and
right-hand limit do not match

At $x=1$



$$14. \lim_{x \rightarrow -\frac{1}{2}} 3x^2(2x-1)$$

$$3\left(-\frac{1}{2}\right)^2(2\left(-\frac{1}{2}\right)-1)$$

$$3\left(\frac{1}{4}\right)(-1-1)$$

$$3\left(\frac{1}{4}\right)(-2)$$

$$\frac{3}{4}(-2)$$

$$\frac{-6}{4} = \left\{ \frac{-3}{2} \right\}$$

$$15. \lim_{x \rightarrow -1} x^3 + 2x^2 - 3x + 3$$

$$(-1)^3 + 2(-1)^2 - 3(-1) + 3$$

$$-1 + 2(1) + 3 + 3$$

$$-1 + 2 + 3 + 3$$

$$\lim_{x \rightarrow -1} = \textcircled{7}$$

$$16. \lim_{x \rightarrow -2} (x-6)^{2/3}$$

$$(-2-6)^{2/3}$$

$$(-8)^{2/3}$$

rewrite

$$\left(\sqrt[3]{-8}\right)^2$$

$$(-2)^2 = 4$$

$$17. \lim_{x \rightarrow 2} \frac{x^2 + 5x + 6}{x+2}$$

plug in first

$$\frac{20}{4} = 5$$

don't ask here :)

$$\frac{(x+3)(x+2)}{(x+2)}$$

$$18. \lim_{\theta \rightarrow \frac{\pi}{6}} \theta^2 \tan \theta$$

$$\left(\frac{\pi}{6}\right)^2 \tan\left(\frac{\pi}{6}\right)$$

$$\left(\frac{\pi^2}{36}\right) \left(\frac{\sqrt{3}}{3}\right) =$$

$$\frac{\pi^2 \sqrt{3}}{108}$$



$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \quad \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$19. \lim_{x \rightarrow 0} \frac{(x+4)^2 - 16}{x}$$

$$\frac{(0+4)^2 - 16}{(0)}$$

$\frac{0}{0}$ Some work needs to be done

Expand
 $\lim_{x \rightarrow 0}$

$$\frac{x^2 + 8x + 16 - 16}{x}$$

$$\lim_{x \rightarrow 0}$$

$$\frac{x(x+8)}{x}$$

$$\lim_{x \rightarrow 0}$$

$$(x+8) = 8$$

$$20. \lim_{x \rightarrow 1} \frac{x-1}{x^2-1} = \frac{0}{0}$$

$$\lim_{x \rightarrow 1} \frac{(x-1)}{(x-1)(x+1)}$$

$$\lim_{x \rightarrow 1} \frac{1}{x+1} = \frac{1}{2}$$

$$21. \lim_{x \rightarrow 2} \frac{x^2-3x+2}{x^2-4} = \frac{0}{0}$$

$$\lim_{x \rightarrow 2} \frac{(x-2)(x-1)}{(x-2)(x+2)}$$

$$\lim_{x \rightarrow 2} \frac{x-1}{x+2} = \frac{1}{4}$$

$$22. \lim_{x \rightarrow 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2} \quad \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{x^2(5x+8)}{x^2(3x^2-16)}$$

$$\lim_{x \rightarrow 0} \frac{5x+8}{3x^2-16} = \frac{8}{-16}$$

$$= -\frac{1}{2}$$

$$23. \lim_{x \rightarrow 0} \frac{\frac{1}{x+2} - \frac{1}{2}}{x} \quad \frac{0}{0}$$

Be careful!

$$\lim_{x \rightarrow 0} \frac{2 - (x+2)}{2(x+2)x}$$

$$\lim_{x \rightarrow 0} \left(\frac{-x}{2(x+2)} \right) \left(\frac{1}{x} \right)$$

$$\lim_{x \rightarrow 0} \frac{-1}{2(x+2)} = \frac{-1}{4}$$

$$24. \lim_{x \rightarrow 0} \frac{(2+x)^3 - 8}{x}$$

$$\lim_{x \rightarrow 0} \frac{x^3 + 8x^2 + 12x + 8 - 8}{x}$$

$$\lim_{x \rightarrow 0} \frac{x(x^2 + 8x + 12)}{x}$$

$$\lim_{x \rightarrow 0} x^2 + 8x + 12 = 12$$

Scratch work

$$(2+x)^2 = (4 + 4x + x^2)(2+x)$$

$$\begin{array}{r} 8 + 8x + 2x^2 \\ 4x + 4x^2 + x^3 \end{array}$$

$$8 + 12x + 8x^2 + x^3$$

$$25. \lim_{h \rightarrow 0} \frac{(x+h)^2 + 2(x+h) - 3 - (x^2 + 2x - 3)}{h}$$

$$\lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2x + 2h - 3 - x^2 - 2x + 3}{h}$$

$$\lim_{h \rightarrow 0} \frac{2xh + h^2 + 2h}{h} \quad \frac{0}{0}$$

$$\lim_{h \rightarrow 0} \frac{h(2x + h + 2)}{h}$$

$$\lim_{h \rightarrow 0} 2x + h + 2 = 2x + 2$$

careful

26. $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ if $f(x) = 3x^2 - 2x$

$$\lim_{h \rightarrow 0} \frac{3(x+h)^2 - 2(x+h) - (3x^2 - 2x)}{h} \leftarrow \text{must protect the group}$$

$$\lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) - 2x - 2h - 3x^2 + 2x}{h} = 3x^2 + 2x$$

$$\lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 2x - 2h - 3x^2 + 2x}{h}$$

$$\lim_{h \rightarrow 0} \frac{6xh + 3h^2 - 2h}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(6x + 3h - 2)}{h}$$

$$\lim_{h \rightarrow 0} 6x + 3h - 2 = 6x - 2$$

typo on homework should be "h" going to zero not x

$$27. \lim_{x \rightarrow 2} f(x) \text{ if } f(x) = \begin{cases} 2x^2 - 4x, & x < 2 \\ 4 \sin\left(\frac{\pi x}{4}\right), & x > 2 \end{cases}$$

$$\lim_{x \rightarrow 2^-} f(x)$$

$$\lim_{x \rightarrow 2^-} 2x^2 - 4x$$

$$= 0$$

$$\lim_{x \rightarrow 2^+} f(x)$$

$$\lim_{x \rightarrow 2^+} 4 \sin\left(\frac{\pi x}{4}\right)$$

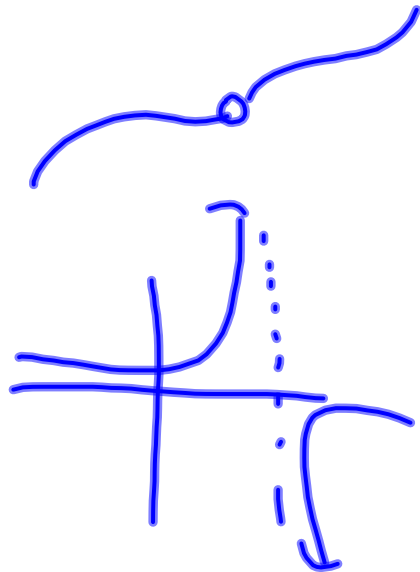
$$\lim_{x \rightarrow 2^+} 4 \sin\left(\frac{\pi}{2}\right)$$

$$4 \cdot 1$$

$$4$$

$$\neq$$

DNE



$$28. \lim_{x \rightarrow 3} e^x \cos\left(\frac{\pi x}{3}\right)$$

$$e^3 \cos\left(\frac{3\pi}{3}\right)$$

$$e^3 (\cos \pi)$$

$$-1e^3$$

$$29. \lim_{x \rightarrow 1} \frac{\sqrt{x+3}-2}{x-1} \quad \frac{0}{0}$$

$$\lim_{x \rightarrow 1} \frac{\sqrt{x+3}-2}{(x-1)} \left(\frac{\sqrt{x+3}+2}{\sqrt{x+3}+2} \right)$$

$$\lim_{x \rightarrow 1} \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)}$$

$$\lim_{x \rightarrow 1} \frac{(x-1)}{(x-1)(\sqrt{x+3}+2)}$$

$$= \frac{1}{2+2}$$

$$\frac{1}{4}$$

$$30. \lim_{x \rightarrow 3^+} \frac{x+3}{x-3}$$

VA at 3 DNE

$+\infty$

for more info

Approach from #'s
on the right

reason



like 4

$$\frac{x+3}{x-3} \text{ at } x=4$$

$\frac{+}{+}$ has positive values
So $+\infty$

$$31. \lim_{x \rightarrow -3^+} \frac{(2x-3)(x-3)}{x^2-9}$$

VA at $x=-3$

DNE

$-\infty$

for more info:

-3 from right so I choose $x=0$

$$\frac{2x-3}{x+3} \text{ at } x=0 = \frac{-}{+}$$

neg values

So $-\infty$

$$32. \lim_{x \rightarrow 0} \frac{\frac{1}{x-2} + \frac{1}{2}}{x} \quad \frac{0}{0}$$

$$\cancel{cd} \lim_{x \rightarrow 0} \frac{\frac{2 + (x-2)}{2(x-2)}}{x}$$

$$\lim_{x \rightarrow 0} \frac{x}{2(x-2) \cdot x}$$

$$\lim_{x \rightarrow 0} \frac{1}{2(x-2)} \quad \left(\div -\frac{1}{4} \right)$$

$$33. \lim_{x \rightarrow -2} \begin{cases} 2-x, & x < -2 \\ x^2 - 2x, & x > -2 \end{cases}$$

Piecewise

Look at $x = -2$ from both sides

$$\lim_{x \rightarrow -2^-} 2-x = 4$$

$$\lim_{x \rightarrow -2^+} x^2 - 2x = 8$$

$$4 - 2(-2) \\ 4 + 4$$

don't match
limit
DNE

34. If $\lim_{x \rightarrow 3} f(x) = 2$ and $\lim_{x \rightarrow 3} g(x) = -4$, find each of the following limits. Show your analysis applying the properties of limits.

a. $\lim_{x \rightarrow 3} \left[\frac{5f(x)}{g(x)} \right]$

$$\frac{5 \lim_{x \rightarrow 3} f(x)}{\lim_{x \rightarrow 3} g(x)} = \frac{5 \cdot (2)}{-4}$$

$$= \frac{10}{-4}$$

$$= -\frac{5}{2}$$

b. $\lim_{x \rightarrow 3} [f(x) + 2g(x)]$

$$\lim_{x \rightarrow 3} f(x) + 2 \lim_{x \rightarrow 3} g(x)$$

$$2 + 2(-4)$$

$$2 - 8$$

$$-6$$

c. $\lim_{x \rightarrow 3} \sqrt{4f(x)}$

$$2 \sqrt{\lim_{x \rightarrow 3} f(x)}$$

$$2(\sqrt{2})$$

$$2\sqrt{2}$$

34. If $\lim_{x \rightarrow 3} f(x) = 2$ and $\lim_{x \rightarrow 3} g(x) = -4$, find each of the following limits. Show your analysis applying the properties of limits.

d. $\lim_{x \rightarrow 3} \frac{g(x)}{8}$

$$\frac{1}{8} \lim_{x \rightarrow 3} g(x)$$

$$\frac{1}{8} \cdot (-4)$$

$$\boxed{-\frac{1}{2}}$$

e. $\lim_{x \rightarrow 3} [3f(x) - g(x)]$

$$3 \lim_{x \rightarrow 3} f(x) - \lim_{x \rightarrow 3} g(x)$$

$$3(2) - (-4)$$
$$6 + 4$$

$$\boxed{10}$$

f. $\lim_{x \rightarrow 3} \left[\frac{f(x)g(x)}{12} \right]$

$$\frac{1}{12} \left[\lim_{x \rightarrow 3} f(x) \cdot \lim_{x \rightarrow 3} g(x) \right]$$

$$\frac{1}{12} [2 \cdot (-4)]$$

$$\frac{1}{12} [-8]$$

$$\boxed{-\frac{2}{3}}$$

35. If $\lim_{x \rightarrow 4} f(x) = 0$ and $\lim_{x \rightarrow 4} g(x) = 3$, find each of the following limits. Show your analysis applying the properties of limits.

a. $\lim_{x \rightarrow 4} \left[\frac{g(x)}{f(x)-1} \right]$

$$\frac{\lim_{x \rightarrow 4} g(x)}{\lim_{x \rightarrow 4} f(x) - 1}$$

$$\frac{3}{0-1} = \frac{3}{-1} = -3$$

b. $\lim_{x \rightarrow 4} xf(x)$

$$\lim_{x \rightarrow 4} x \cdot \lim_{x \rightarrow 4} f(x)$$

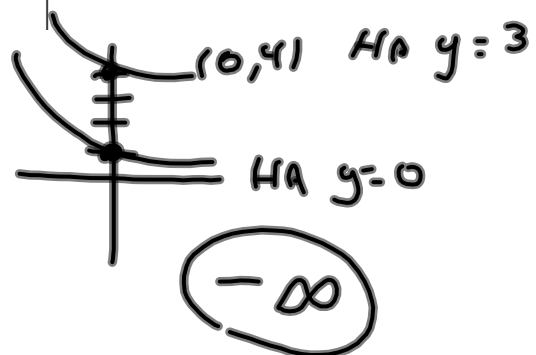
$$4 \cdot 0$$

$$0$$

Find the limit of each of the following exponential functions. Sketch a graph of each function to aid in your determination of the limit.

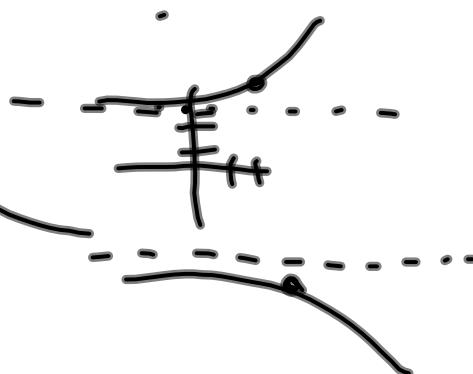
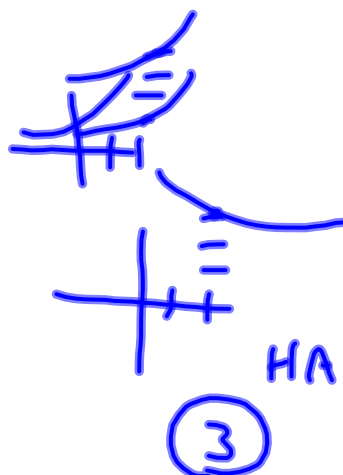


36. $\lim_{x \rightarrow \infty} -(0.5)^{-x-2} + 3$



Pt	HA
(0, 1)	$y=0$
(0, 4)	$y=3$
(2, 1)	$y=3$
(2, 4)	

37. $\lim_{x \rightarrow \infty} 2^{-x-2} + 3$



38. $\lim_{x \rightarrow -\infty} -\left(\frac{1}{4}\right)^{-x-2} + 3$



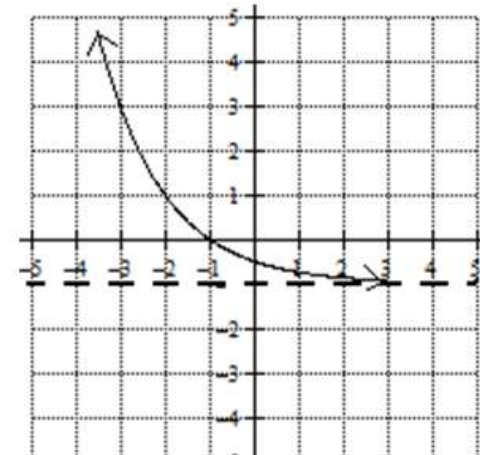
39. Using the graph of $g(x)$ pictured to the right, find each of the following limits.

a. $\lim_{x \rightarrow \infty} g(x) = \underline{-1}$

b. $\lim_{x \rightarrow -\infty} g(x) = \underline{+\infty \text{ (DNE)}}$

c. $\lim_{x \rightarrow -1} g(x) = \underline{0}$

d. $\lim_{x \rightarrow -3} g(x) = \underline{3}$





$$\lim_{x \rightarrow \infty} \left(\frac{3}{2} \right)^{-x} + 3$$

Handwritten: 3



$$\lim_{x \rightarrow -\infty} -(3)^{x-2} + 3$$

Handwritten: 3

Warm-ups

$$\lim_{x \rightarrow 0} \frac{1 - \cos(5x)}{5x}$$

$$\lim_{x \rightarrow 0} \frac{\sin(5x)}{5x}$$

$$\lim_{x \rightarrow 0} \frac{2 \sin(2x)}{5x}$$

$$\frac{2}{5} \lim_{x \rightarrow 0} \frac{\sin(2x)}{2x}$$

$$\frac{4}{5}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{5x}$$

$$\frac{1}{5} \lim_{x \rightarrow 0} \frac{1 - \cos(3x)}{3x}$$

$$\frac{1}{5} (3) \cdot 0$$

$$0$$

$$\lim_{x \rightarrow 0} \frac{2x}{5 \sin(2x)}$$

$$\frac{1}{5} (1) = \frac{1}{5}$$

Find the value of each limit. For a limit that does not exist, state why.

40.

$$\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{\cos^2 \theta}{1 - \sin \theta}$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{1 - \sin^2 \theta}{1 - \sin \theta}$$

$$\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{(1 - \sin \theta)(1 + \sin \theta)}{(1 - \sin \theta)}$$

$$\lim_{\theta \rightarrow \frac{\pi}{2}} 1 + \sin \theta$$

$$1 + 1 = 2$$

$$41. \lim_{x \rightarrow 0} \frac{x + \sin x}{x}$$

$$\frac{0}{0}$$

split up

$$\lim_{x \rightarrow 0} \left(\frac{x}{x} + \frac{\sin x}{x} \right)$$

$$\lim_{x \rightarrow 0} \frac{x}{x} + \lim_{x \rightarrow 0} \frac{\sin x}{x}$$

$$\lim_{x \rightarrow 0} 1 + \lim_{x \rightarrow 0} \frac{\sin x}{x}$$

$$1 + 1 = 2$$

$$42. \lim_{x \rightarrow 3} \begin{cases} 2x^2 - 3x, & x < 3 \\ 8 - \cos\left(\frac{\pi x}{3}\right), & x > 3 \end{cases}$$

$$\lim_{x \rightarrow 3^-} 2x^2 - 3x = 9$$

18 - 9

$$\lim_{x \rightarrow 3^+} 8 - \cos \frac{\pi x}{3} = 9$$

8 - -1

$$\lim_{x \rightarrow 3} \begin{cases} 2x^2 - 3x & x < 3 \\ 8 - \cos \frac{\pi x}{3} & x > 3 \end{cases} = 9$$

$$43. \lim_{\theta \rightarrow 0} \frac{2 \sin 3\theta}{\theta}$$

$$\lim_{\theta \rightarrow 0} \frac{2 \cdot 2 \sin 3\theta}{3\theta} \quad 2 \lim_{\theta \rightarrow 0} \frac{3 \sin 3\theta}{3\theta}$$

$$\lim_{\theta \rightarrow 0} \frac{6 \cdot \sin 3\theta}{1 \cdot 3\theta} \quad 2 \cdot 3$$

6

$$6 \lim_{\theta \rightarrow 0} \frac{\sin 3\theta}{3\theta}$$

$$6 \cdot 1$$

6

$$\lim_{x \rightarrow 0} \frac{\sin x}{x}$$

$$44. \lim_{x \rightarrow 0} \frac{\sin x}{2x^2 - x}$$

factor

$$\lim_{x \rightarrow 0} \frac{\sin x}{x(2x-1)}$$

rewrite

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1}{2x-1}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{1}{2x-1}$$

$$1 \cdot (-1) = -1$$

$$45. \lim_{x \rightarrow 0} \frac{5x + \sin 3x}{x}$$

rewrite

$$\lim_{x \rightarrow 0} \frac{5x}{x} + \lim_{x \rightarrow 0} \frac{\sin 3x}{x}$$

$$\lim_{x \rightarrow 0} 5 + \lim_{x \rightarrow 0} \frac{3 \sin 3x}{3 \cdot x}$$

$$5 + 3 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x}$$

$$5 + 3 \cdot (1)$$

$$8$$

$$46. \lim_{x \rightarrow 0} \frac{\sin 2x}{6x}$$

rewrite

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{3 \cdot 2x}$$

$$\frac{1}{3} \lim_{x \rightarrow 0} \frac{\sin 2x}{2x}$$

$$\frac{1}{3} \cdot 1$$
$$\frac{1}{3}$$

$$47. \lim_{x \rightarrow 0} \frac{2 \sin 4x}{3x}$$

$$\frac{2}{3} \lim_{x \rightarrow 0} \frac{4 \sin 4x}{4x}$$

$$\frac{8}{3} \lim_{x \rightarrow 0} \frac{\sin 4x}{4x}$$

$$\frac{8}{3} \cdot 1$$
$$\frac{8}{3}$$

$$48. \quad \lim_{\theta \rightarrow 0} \frac{\cos \theta \tan \theta}{3\theta}$$

$$\lim_{\theta \rightarrow 0} \frac{\cos \theta \cdot \frac{\sin \theta}{\cos \theta}}{3\theta}$$

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{3\theta}$$

$$\lim_{\theta \rightarrow 0} \frac{1}{3} \cdot \frac{\sin \theta}{\theta}$$

$$\frac{1}{3} \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$$

$$\frac{1}{3} \cdot 1 = \left\{ \frac{1}{3} \right\}$$

$$49. \quad \lim_{\theta \rightarrow 0} \frac{3 - 3 \cos \theta}{\theta}$$

$$\lim_{\theta \rightarrow 0} \frac{3(1 - \cos \theta)}{\theta}$$

$$3 \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$$

$$3 \cdot 0$$



$$50. \lim_{\theta \rightarrow \frac{\pi}{2}} \frac{\cos \theta}{\cot \theta} \quad \frac{0}{0}$$

$$\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{\cos \theta}{\frac{\cos \theta}{\sin \theta}}$$

$$\lim_{\theta \rightarrow \frac{\pi}{2}} \cos \theta \cdot \frac{\sin \theta}{\cos \theta}$$

$$\lim_{\theta \rightarrow \frac{\pi}{2}} \sin \theta$$

$$\sin\left(\frac{\pi}{2}\right) = 1$$

$$51. \lim_{\theta \rightarrow 0} \frac{1 - \tan \theta}{\sin \theta - \cos \theta}$$

plug in
:-)

$$\frac{1 - 0}{0 - 1} = -1$$

$$-1$$

$$52. \lim_{c \rightarrow 3} \frac{c^3 - 27}{c - 3}$$

$$\lim_{c \rightarrow 3} \frac{(c-3)(c^2 + 3c + 9)}{(c-3)}$$

$$\lim_{c \rightarrow 3} c^2 + 3c + 9$$

$$9 + 9 + 9$$

27

1
1 2 1
1 3 3 1

53

$$\lim_{x \rightarrow -1} \frac{(x+3)^3 - 8}{x+1}$$

$$\lim_{x \rightarrow -1} \frac{x^3 + 3x^2 \cdot 3 + 3x \cdot 3^2 + 1 \cdot 3^3 - 8}{(x+1)}$$

$$\lim_{x \rightarrow -1} \frac{x^3 + 9x^2 + 27x + 27 - 8}{(x+1)}$$

$$\lim_{x \rightarrow -1}$$

$$\frac{x^3 + 9x^2 + 27x + 19}{(x+1)}$$

synthetic
division

$$\lim_{x \rightarrow -1}$$

$$\frac{(x+1)(x^2 + 8x + 19)}{(x+1)}$$

next page

$$1 - 8 + 19 = 12$$

$$\frac{x^3 + 9x^2 + 27x + 19}{(x+1)}$$

-1		1	9	27	19
		↓	-1	-8	-19
		1	8	19	0

so $(x+1)(x^2 + 8x + 19)$

$$\begin{array}{ccc} a^3 \pm b^3 & & \text{SOAP} \\ \downarrow \text{SOAP} & & \downarrow \\ (a \pm b)^3 & & \\ \downarrow & & \\ (a \pm b)(a^2 \mp ab + b^2) & & \end{array}$$

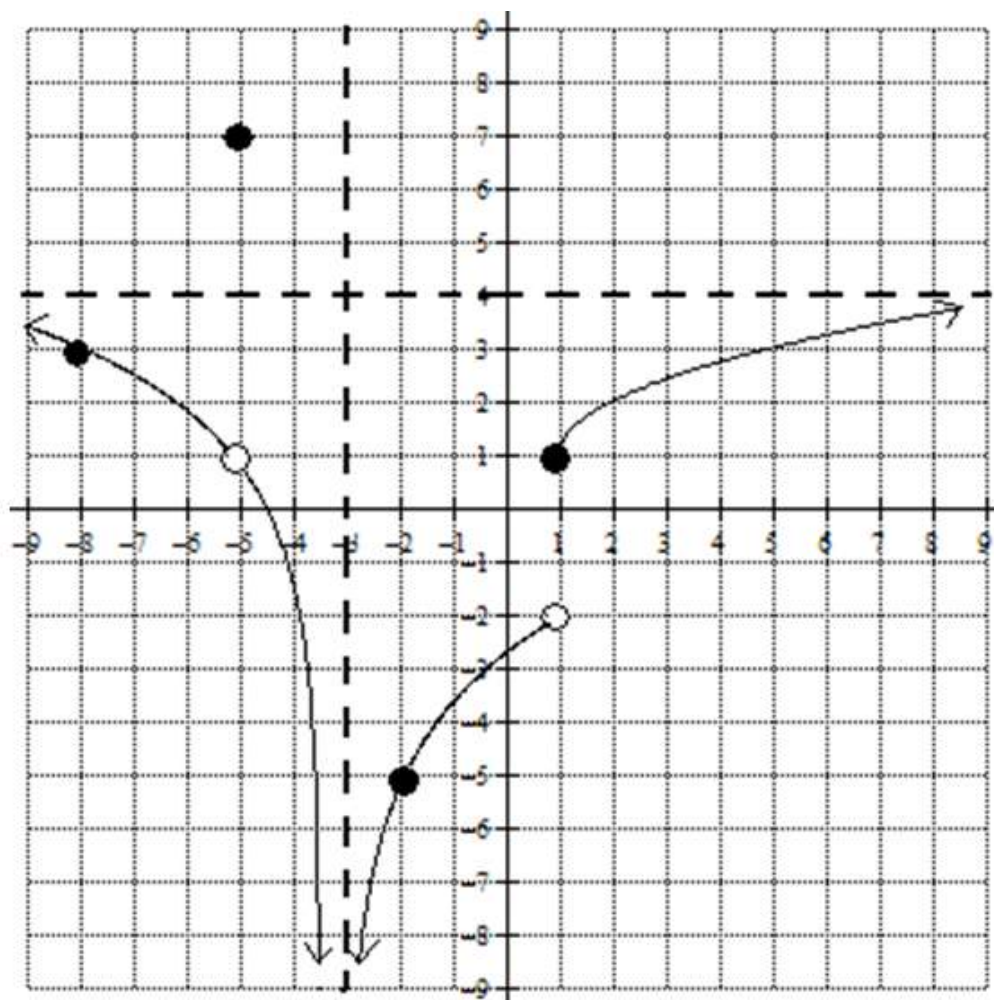
$$a^3 - 8$$

$$\frac{8x^3 - 27y^3}{(2x)^3 - (3y)^3}$$

$$\frac{(2x - 3y)(4x^2 + 6xy + 9y^2)}{(2x - 3y)(4x^2 + 6xy + 9y^2)}$$

$$\frac{(x+3)^3}{(x+3)(x+3)(x+3)}$$

$$\frac{(x^2 + 6x + 9)(x+3)}{(x^2 + 6x + 9)(x+3)}$$



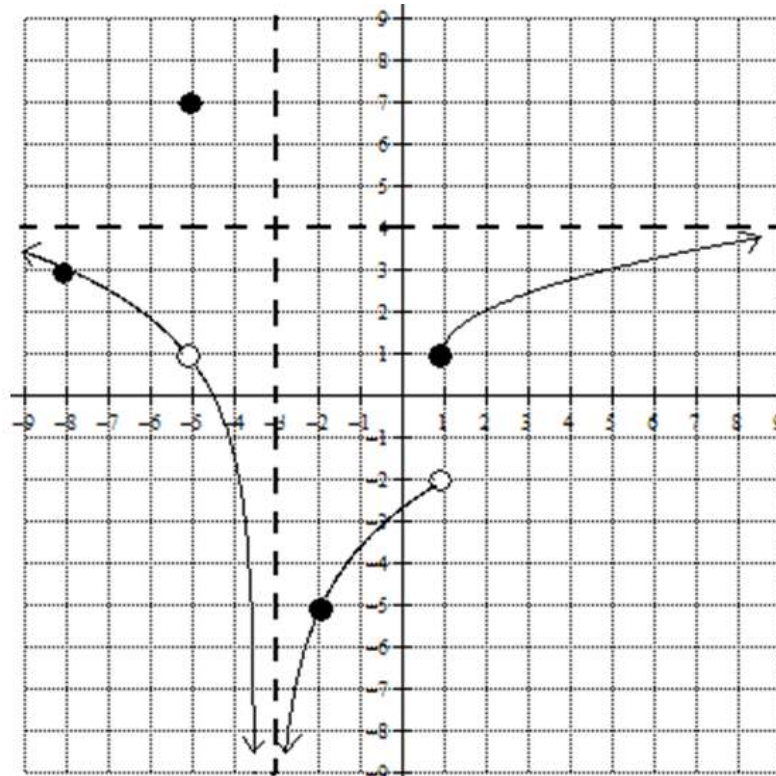
$$54. \lim_{x \rightarrow \infty} F(x) = \underline{4}$$

$$55. \lim_{x \rightarrow 2} F(x) = \underline{2}$$

$$56. \lim_{x \rightarrow -2^-} F(x) = \underline{-5}$$

$$57. \lim_{x \rightarrow -2^+} F(x) = \underline{-5}$$

$$58. \lim_{x \rightarrow -2} F(x) = \underline{-5}$$



59. $F(-4.5) =$ 0

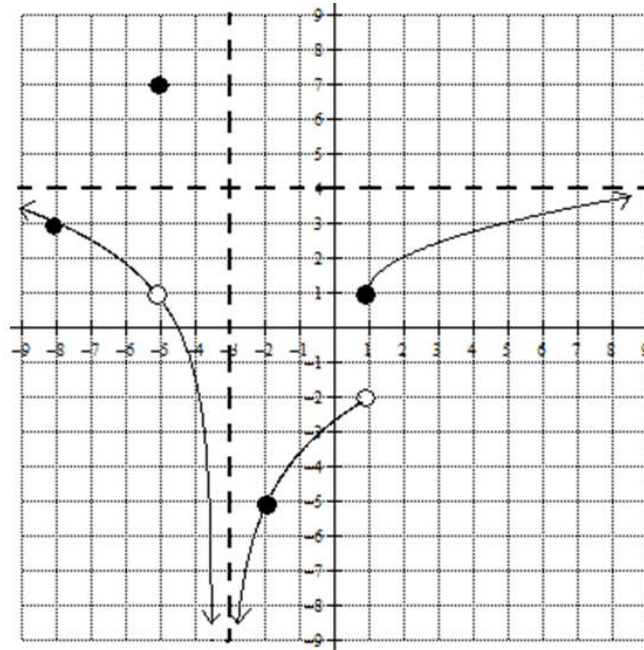
60. $\lim_{x \rightarrow -5} F(x) =$ 1

61. $F(-5) =$ 7

62. $F(1) =$ _____

63. $\lim_{x \rightarrow 1^+} F(x) =$ _____

64. $\lim_{x \rightarrow 1^-} F(x) =$ _____



65. $\lim_{x \rightarrow 1} F(x) = \underline{1}$

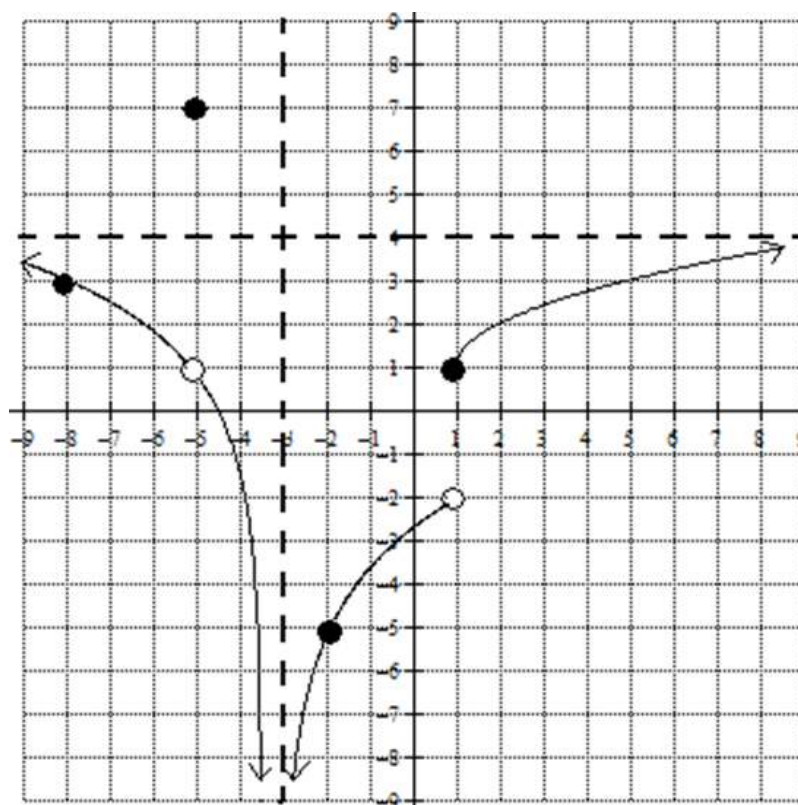
66. $\lim_{x \rightarrow -3^+} F(x) = \underline{1}$

67. $\lim_{x \rightarrow -3^-} F(x) = \underline{-2}$

68. $F(-2) = \underline{-5}$

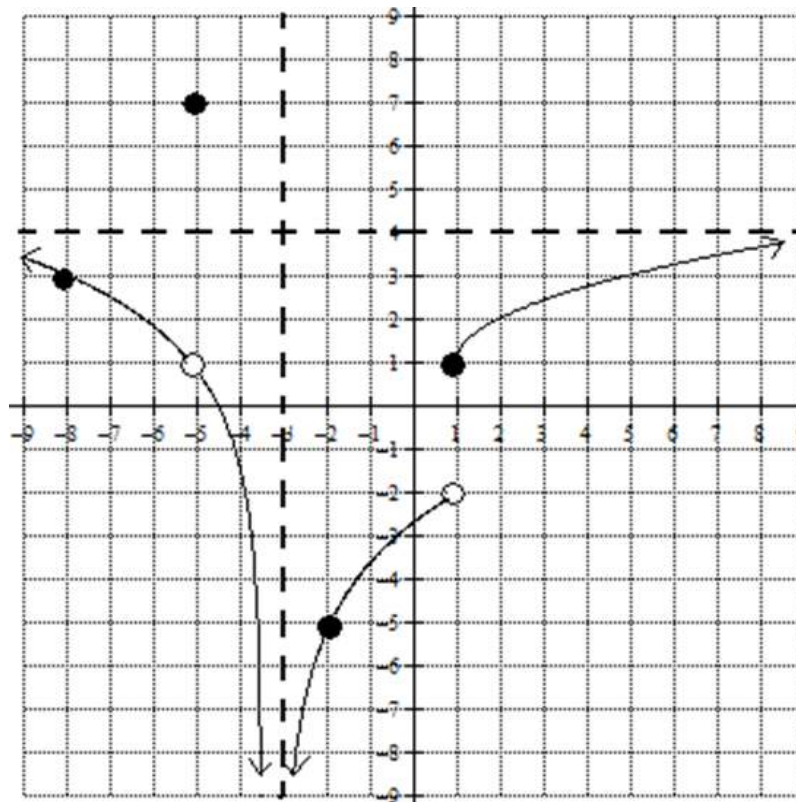
69. $F(-3) = \underline{\text{DNE}}$ VA

Determine if the function is continuous at each of the indicated values below. Use the three part definition of continuity to perform your analysis.



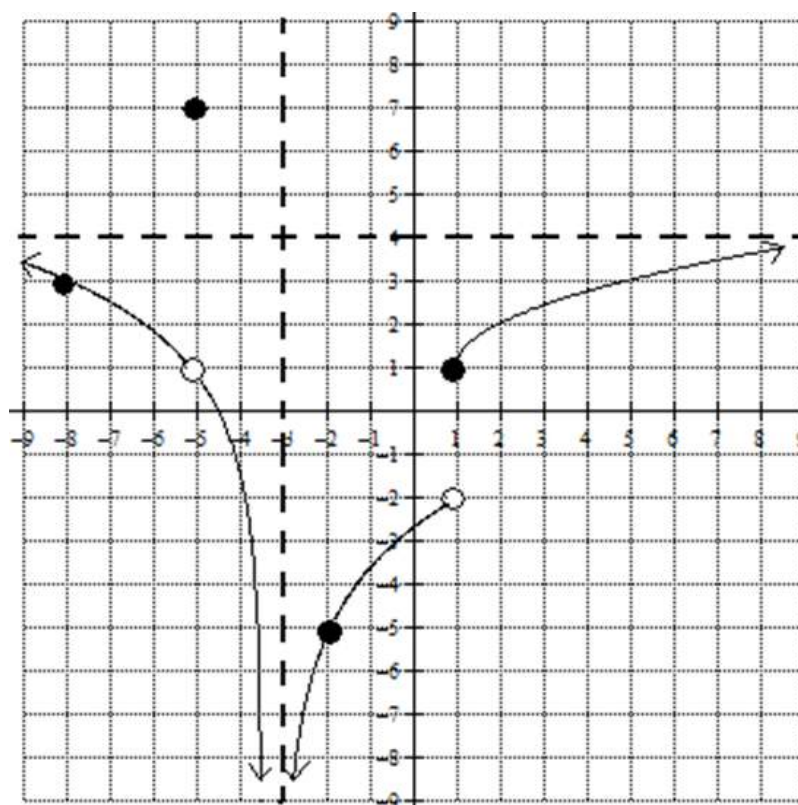
70. x = -5

Determine if the function is continuous at each of the indicated values below. Use the three part definition of continuity to perform your analysis.



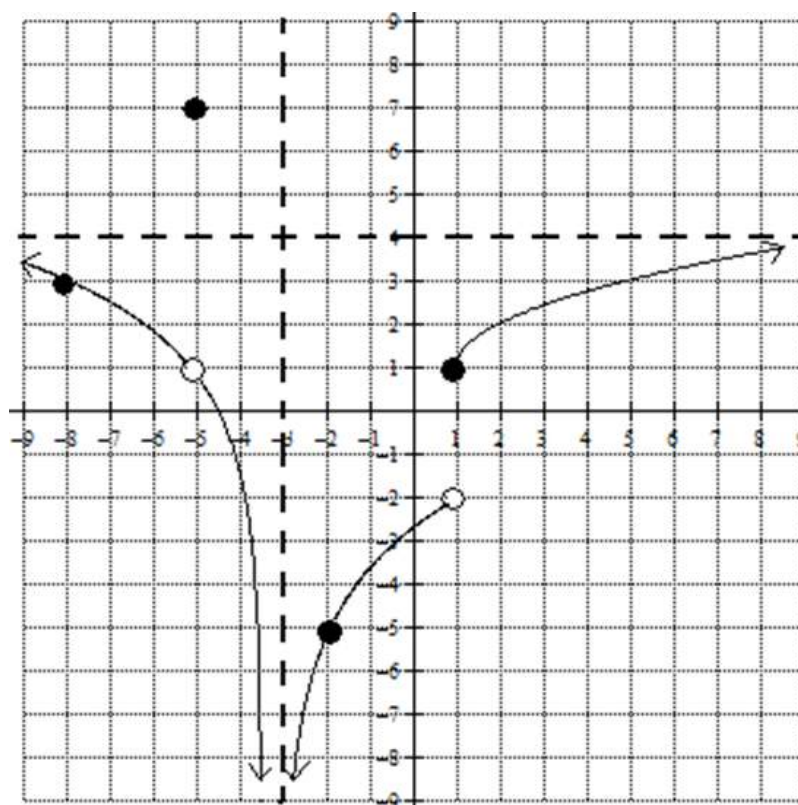
71. x = 1

Determine if the function is continuous at each of the indicated values below. Use the three part definition of continuity to perform your analysis.



72. $x = -3$

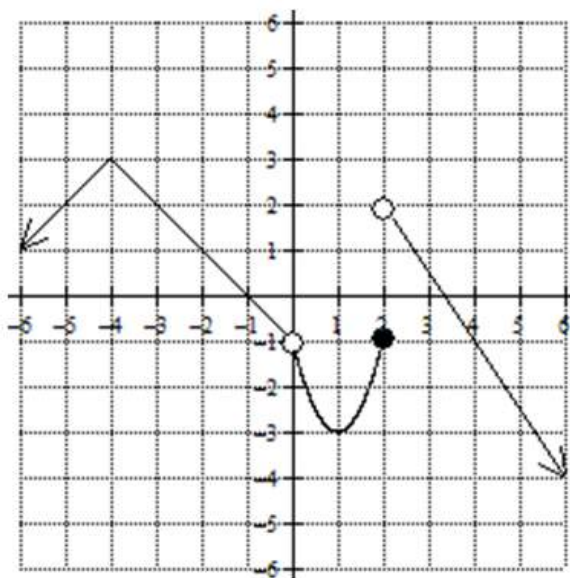
Determine if the function is continuous at each of the indicated values below. Use the three part definition of continuity to perform your analysis.



73. $\underline{x} = -2$

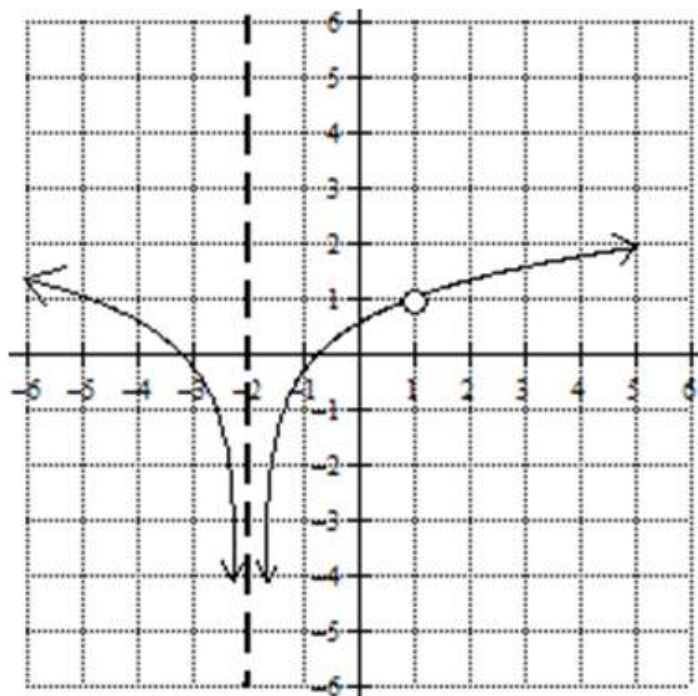
Write a discussion of continuity for each of the functions below. Be sure to include in your discussion where the function is continuous, and, for the values where the function is not continuous, use the three part definition to establish discontinuity.

74. Graph of $g(x)$



include

- removable discontinuity
- jump disc limit from left \neq limit from right
- Continuous everywhere except $x=0$ and $x=2$
then show why not

75. Graph of $h(x)$ 

Don't say VA w/out proving it exists
by limits

- don't say removable disc without
proving it exist by limits

$$\lim_{x \rightarrow -2^-} h(x) = -\infty \therefore \text{DNE}$$

Find the value of a that makes each of the functions below everywhere continuous. Write the two limits that must be equal in order for the function to be continuous.

$$76. f(x) = \begin{cases} 4 - x^2, & x < -1 \\ ax^2 - 1, & x \geq -1 \end{cases}$$

$$\lim_{x \rightarrow -1^-} f(x) \text{ must equal } \lim_{x \rightarrow -1^+} f(x)$$

$$\lim_{x \rightarrow -1^-} 4 - x^2 \stackrel{\text{Must}}{=} \lim_{x \rightarrow -1^+} ax^2 - 1 \text{ for } f(x) \text{ to be continuous}$$

$$\begin{aligned} 3 &= a - 1 \\ 4 &= a \end{aligned}$$

check
when
 $x = -1$

$$\begin{aligned} 4 - x^2 &\stackrel{?}{=} 4(-1)^2 - 1 \\ 4 - (-1)^2 &\stackrel{?}{=} 4(1) - 1 \\ 4 - 1 &\stackrel{?}{=} 4 - 1 \\ 3 &= 3 \checkmark \end{aligned}$$