Homework #2

Question 1

```
procedure MyProc(A, k)
    if k < A.length then
        x = ∞
        i = 0
        for j = k to A.length do
            if A[j] < x then
                x = A[j]
                i = j
        A[i] = A[k]
        A[k] = x
        MyProc(A, k + 1)
```

$$T(n) = \begin{cases} c_1, & n = 1 \\ T(n-1) + c_2(n-1), & n > 1 \end{cases}$$

The time it takes to sort the remaining unsorted elements after the minimum element has been swapped into its proper position is represented by the first term in the recurrence, T(n-1), and the time it takes to scan the unsorted elements to find the next minimum element is represented by the second term, n-1. c_1 is the runtime during the base case.

Question 2

$MergeSort(A, 1, 4)$

$MergeSort(A, 1, 2)$

$MergeSort(A, 1, 1)$

$MergeSort(A, 2, 2)$

$MergeSort(A, 3, 4)$

$MergeSort(A, 3, 3)$

$MergeSort(A, 4, 4)$

Question 3

$$\Omega(g(n)) = \{f(n): there\ exist\ positive\ constants\ c\ and\ n_0\ such\ that$$
$$0 \leq c_1 g(n) \leq f(n)$$
$$for\ all\ n > n_1\}$$

$$O\big(g(n)\big) = \{f(n): there\ exist\ positive\ constants\ c\ and\ n_0\ such\ that$$
$$0 \leq f(n) \leq c_2 g(n)$$
$$for\ all\ n > n_2$$

Combining both definitions, we get

$$0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)\ \ for\ n > \max{(n1, n2)}$$

Since $f(n)$ is bounded between $c_1 g(n)$ -best case- and $c_2 g(n)$ -worst case- and the above is the definition of $\Theta - notation$

$$\therefore \text{ the running time of an algorithm is } \Theta(\text{g(n)}) \text{ if and only if its worst}$$
$$- \text{ case running time is } O(\text{g(n)}) \text{ and its best} - \text{case running time is } \Omega(\text{g(n)}).$$

Question 4

Definition

$$O\big(g(n)\big) = \{f(n): there\ exist\ positive\ constants\ c\ and\ n_0\ such\ that$$
$$0 \leq f(n) \leq cg(n)$$
$$for\ all\ n > n_0$$

Solution

$$3nlg(n) \leq cnln(n)$$

Let $c = 3$ and $n_0 = 2$ and we observe that $0 \leq 3nlg(n) \leq cnln(n)$ holds for all $n \geq n_0$

Question 5

Definition

$$o\big(g(n)\big) = \{f(n): there\ exist\ positive\ constants\ c > 0, there\ exist\ a\ constant\ n_0 > 0\ such\ that$$
$$0 \leq f(n) < cg(n)$$
$$for\ all\ n > n_0$$

Solution

$$100n^2 < cn^3$$

$for\ all\ c > 0, let\ n_0 = \frac{100}{c} + 1$ and we observe that $0 \leq 100n^2 < cn^3$ holds for all $n \geq n_0$