CSC-654 Algorithms Design & Analysis

Homework #4

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Question 1

1. $T(n) = T(\lceil n/2 \rceil) + 1$ is $O(\lg n)$

Assume $T(n) \leqslant c \lg n \quad \forall\ n \geqslant n_0,\ c > 0,\ n_0 > 0$

$$T(n) \leqslant c \lg \lceil \tfrac{n}{2} \rceil + 1$$

We can get rid of the ceiling by adding 1

$$T(n) \leqslant c \lg \left(\tfrac{n}{2} + 1\right) + 1$$

$$T(n) < c \lg \left(\frac{n+2}{2}\right) + 1$$

$$= c \lg (n+2) - c \lg 2 + 1$$

$$= c \lg (n+2) - c + 1$$

We can't conclude from the algebra our assumption

So, we will subtract a lower-order term from the guess

Assuming $T(n) \leq c\lg(n-d)$

$$T(n) \leq c\lg\lceil \tfrac{n}{2} - d \rceil + 1$$

$$< c\lg\left(\tfrac{n}{2} - d + 1\right) + 1$$

$$< c\lg\left(\frac{n+2d-2}{2}\right) + 1$$

$$= c\lg(n+2d-2) - c\lg 2 + 1$$

$$= c\lg(n-2+2d) - (c-1)$$

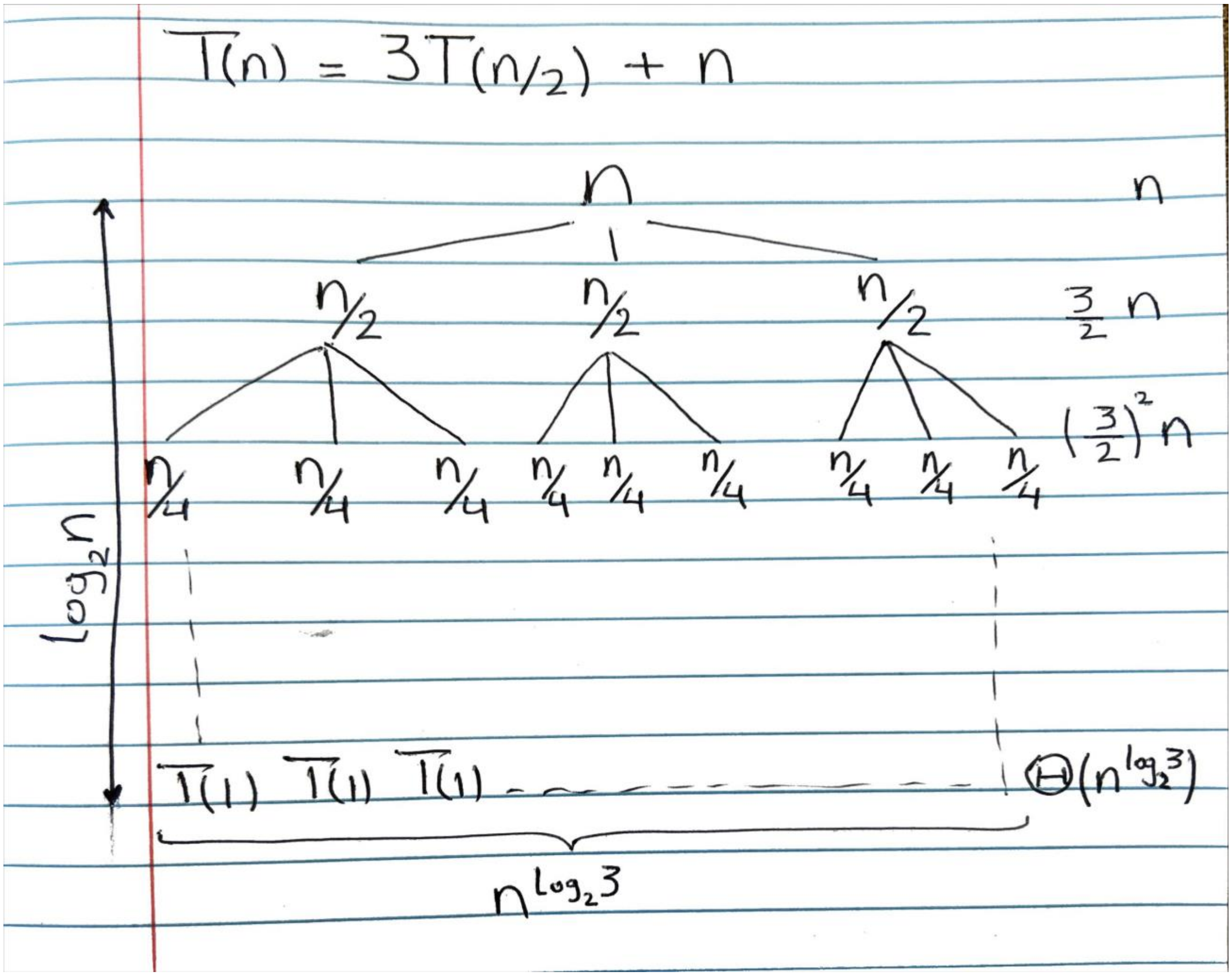
~~Assuming~~

$T(n) = c\lg(n-2+2d) - (c-1)$ we can use $d = 2$

to get $T(n) \leq c\lg(n-2)$

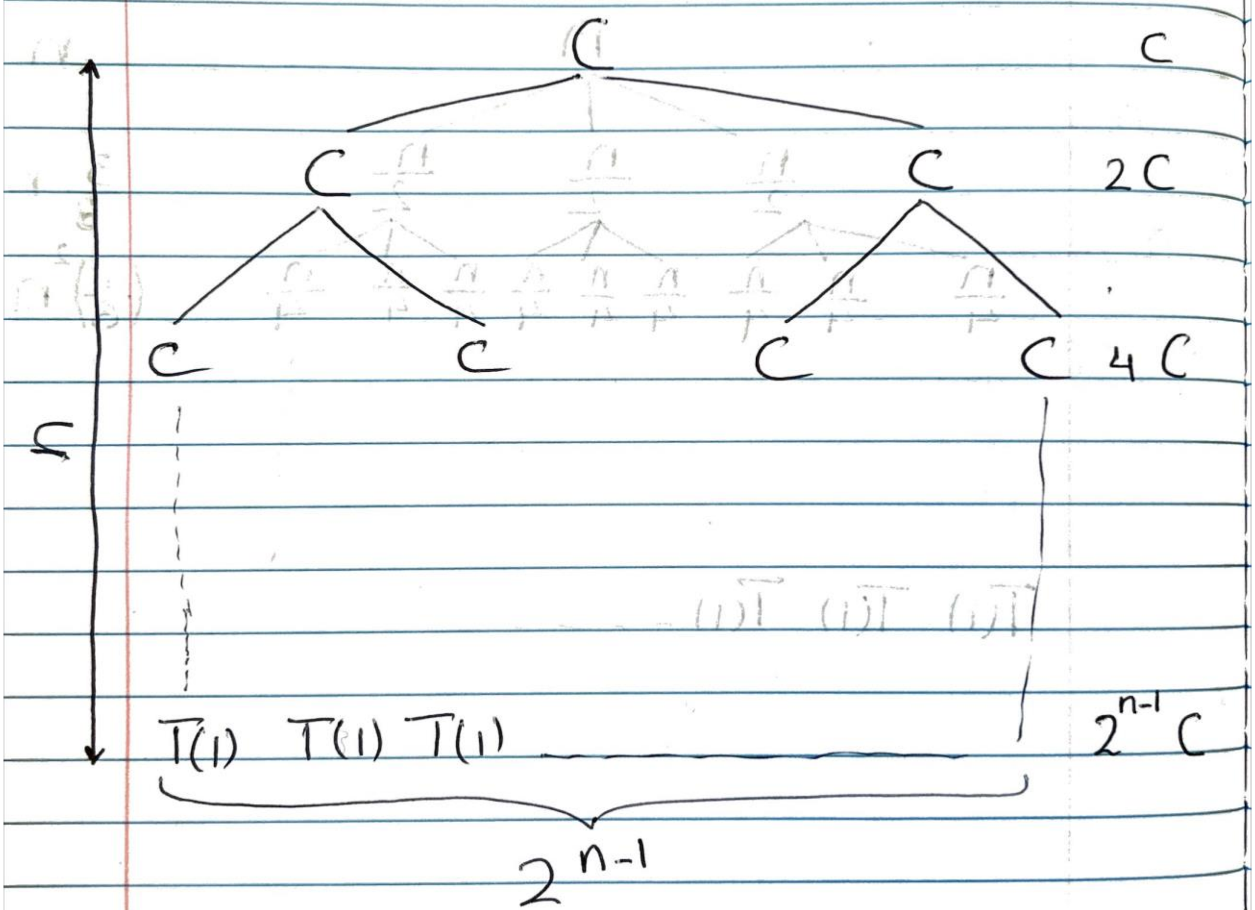
for all $c \geq 1$

Question 3


T(n) = 3T(n/2) + n
n
n
n/2
n/2
n/2
3/2 n
(3/2)^2 n
n/4
n/4
n/4
n/4
n/4
n/4
n/4
n/4
n/4
log_2 n
T(1)
T(1)
T(1)
Θ(n^{log_2 3})
n^{log_2 3}

Question 4

$T(n) = 2T(n-1) + 1$



Question 5

A. $T(n) = 2T(n/4) + 1$

$a = 2 \quad b = 4 \quad f(n) = 1$

$n^{\log_b a} = n^{\log_4 2} = \sqrt{n} = \Theta(\sqrt{n}) = \Theta(n^{0.5})$

$\therefore f(n) = O(1) = O(n^{0.5-0.5})$

by Assuming $\varepsilon = 0.5$

$\therefore$ Case 1 applies

$T(n) = \Theta(\sqrt{n})$

B. $T(n) = 2T(n/2) + \sqrt{n}$

$a = 2 \qquad b = 2 \qquad f(n) = \sqrt{n} = n^{0.5}$

$n^{\log_b a} = n^1 = \Theta(n)$

~~$\therefore f(n) = \Theta(n$~~

~~$\therefore f(n) = \Omega(n$~~

$\therefore f(n) = O(n^{1-0.5})$ where $\varepsilon = 0.5$

$\therefore$ Case 1 applies

$\therefore T(n) = \Theta(n)$

C. $T(n) = 2T(n/2) + n$

$a = 2 \qquad b = 2 \qquad f(n) = n$

$n^{\log_b a} = n^1 = \Theta(n)$

$\therefore \; f(n) = \Theta(n^{\log_b a}) = \Theta(n)$

$\therefore$ Case 2 applies

$T(n) = \Theta(n \lg n)$

Question 6

6. We know that Strassen's $\boxed{\Theta(n^{\lg 7})}$

$T(n) = aT(n/4) + \Theta(n^2)$

$b = 4 \quad f(n) = \Theta(n^2)$

Using Case 1

$T(n) = \Theta(n^{\log_b a}) = \Theta(n^{\log_4 a})$

By unifying log bases to simplify

Comparison $T(n) = \Theta(n^{\lg \sqrt{a}})$

$\therefore \sqrt{a} < 7$

$a < 49$

$\therefore$ The largest integer would be $\boxed{48}$