

Learning Meters of Arabic Poems with Deep Learning

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CERTIFICATION OF APPROVAL

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Deep Learning

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Thesis Outline

The coming chapters are arranged as follows:

- Chapter 1: Presents some basic introduction and background knowledge as regards the Arabic Poem and its definitions. Also, it contains details about the Arabic language and some feature used during our work.
- Chapter 2: Introduces the essential pre-processing steps, and the justification for their need. Pre-processing steps are data extraction, data cleansing and data format.
- Chapter 3: introduces the data encoding techniques used and the effect of each one. Also, it contains some comparisons between the three techniques used.
- Chapter 4: presents the model's details and how we chose the model and the architecture and hyper-parameters details.
- Chapter 5: Results and discussion.
- Chapter 6: Conclusion and future work

ABSTRACT

People can easily determine whether a piece of writing is a poem or prose, but only specialists can determine the class of poem.

In this thesis, We built a model that can classify poems according to their meters; a forward step towards machine understanding of Arabic language.

A number of different deep learning models are proposed for poem meter classification. As poems are sequence data, then recurrent neural networks are suitable for the task. We have trained three variants of them, LSTM, GRU with different architectures and hyper-parameters. Because meters are a sequence of characters, then we have encoded the input text at the character-level, so that we preserve the information provided by the letters succession directly fed to then models. Besides, We introduce a comparative study on the difference between binary and one-hot encoding regarding their effect on the learning curve. We also introduce a new encoding technique called *Two-Hot* which merges the advantages of both *Binary* and *One-Hot* techniques.

Artificial Intelligence currently works to do the human tasks such as our problem here. Our target in this thesis is to achieve the human accuracy which will make it easy for anyone to know the meter for any poem without referring to the language experts or to study the whole field to achieve it.

In this thesis, We will explain how to use the deep learning to classify the Arabic poem to classes. Also, explain in details the feature of Arabic poem and how to deal with this features. Besides, We explain how can anyone work with Arabic text encoding with a dynamic way to encode the text at the character level and deal with the Arabic text feature example the *Tashkeel*.

Chapter 1

INTRODUCTION

Arabic is the fifth most widely spoken language¹. It is written from right to left. Its alphabet consists of 28 primary letters, and there are 8 more derived letters from the basic ones, so the total count of Arabic characters is 36 characters. The writing system is cursive; hence, most letters join to the letter that comes after them, a few letters remain disjoint.

1.1 Arabic Poetry

Arabic poetry (الشعر العربي) is the earliest form of Arabic literature. It dates back to the Sixth century. Poets have written poems without knowing exactly what rules which make a collection of words a poem. People recognize poetry by nature, but only talented ones can write poems. This was the case until *Al-Farahidi* (718–786 CE) has analyzed the Arabic poetry, then he came up with that the succession of consonants and vowels produce patterns or *meters*, which make the music of poetry. He has counted them fifteen meters. After that, a student of *Al-Farahidi* has added one more meter to make them sixteen. Arabs call meters بحور which means “seas”. The study of Arabic Poems classification is named **Al-Arud** (العروض). It takes too much time for anyone to be an expert in this field.

1.2 Deep Learning

Deep Learning also named Deep Neural Network is part of Machine Learning algorithms. Deep Learning is trying to simulate the human brain into Neural dependency. Using Deep Learning, we can achieve better learning results from the data. Deep Neural Network needs a huge amount of data to achieve the expected learning curve

¹ according to the 20th edition of *Ethnologue*, 2017

and results. It also needs a massive amount of computation to build the networks which are based on an artificial neural network. We used the Recurrent Neural Network (RNN) to work on the Arabic Text which shown its ability to achieve outstanding performance over the text problem data. We also used LSTM to solve the long dependency issue in RNN. We will go deep into the Background section (add deep learning section reference).

1.3 Thesis Objectives

In this study, we work to classify the poem and utilize the latest technologies check the class of poem. We also worked to achieve near human expert results which make our work is a breakthrough in the field concerning the results compared to the current achieved results. Figure 1.1 shows the steps,

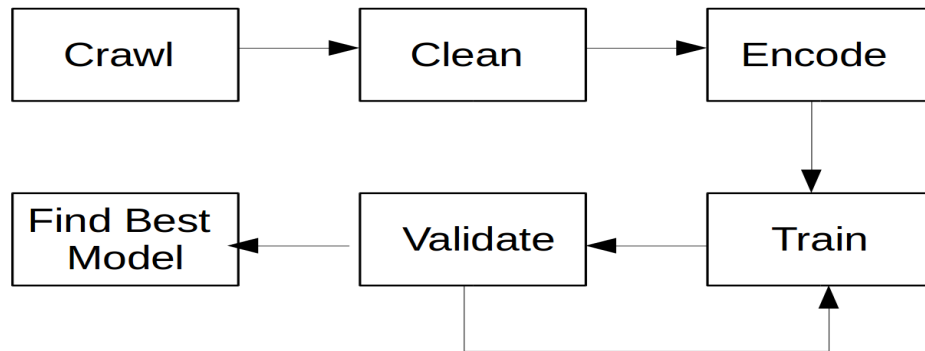


Figure 1.1: Thesis Working Steps.

- Crawling the data from the available sources with labeling.
- Clean and transform the data.
- Encode the data into a way to be input to the model to work on it. We used many encoding methods and compared each of them.
- Train the RNN model into the cleaned data.
- Validate and test the model.
- Enhance the model.

Chapter 2

BACKGROUND

Each Arabic letter represents a consonant, which means that short vowels are not represented by the 36 characters, for this reason, the need of *diacritics* rises. *Diacritics* are symbols that comes after a letter to state the short vowel accompanied by that letter. There are four diacritics َ ُ ِ ْ which represent the following short vowels /a/, /u/, /i/ and *no-vowel* respectively, their names are *fat-ha*, *dam-ma*, *kas-ra* and *sukun* respectively. The first three symbols are called *harakat*. Table 2.1 shows the 4 diacritics on a letter.

Diacritics	without	fat-ha	kas-ra	dam-ma	sukun
Shape	د	دَ	دِ	دُ	دْ

Table 2.1: *Diacritics on the letter د*

There are two more sub-diacritics made up of the basic four to represent two cases:

Definition 1 Shadaa

to indicate the letter is doubled. Any letter with shaddah (ّ) the letter should be duplicated: first letter with a constant (sukoon) and second letter with a vowel (haraka) [?]; Table 2.2 shows the dal with shadda and the original letters.

Diacritics	letter with Shadda	letters without shadaa
Shape	دّ	دُ دَ

Table 2.2: *Shadaa diacritics on the letter د*

Definition 2 Tanween

is doubling the short vowel, and can convert Tanween fathah, Tanween dhammah or Tanween kasrah by replacing it with the appropriate vowel (dhammah, fathah or kasrah) then add the Noon letter with constant to the end of the word [؟]. Table 2.3 shows the difference between the original letter and the letter with Tanween

Diacritics	letter with tanween	letters without tanween
Tanween Fat-ha	دَ	دَ + نْ
Tanween Dam-ma	دُ	دُ + نْ
Tanween Kas-ra	دِ	دِ + نْ

Table 2.3: Tanween diacritics on the letter د

Arabs pronounce the sound /n/ accompanied *sukun* at the end the indefinite words, that sound corresponds to this letter نْ, it is called *noon-sakinah*, however, it is just a phone, it is not a part of the indefinite word, if a word comes as a definite word, no additional sound is added. Since it is not an essential sound, it is not written as a letter, but it is written as *tanween* ْ ُ ِ. *Tanween* states the sound *noon-sakinah*, but as you have noticed, there are 3 *tanween* symbols, this because *tanween* is added as a diacritic over the last letter of the indefinite word, one of the 3 *harakat* accompanies the last letter, the last letter's *harakah* needs to be stated in addition to the sound *noon-sakinah*, so *tanween* is doubling the last letter's *haraka*, this way the last letter's *haraka* is preserved in addition to stating the sound *noon-sakinah*; for example, نْ + رَجُلٌ is written رَجُلٌ and نْ + رَجُل is written رَجُل.

Those two definition, Definition 1 and Definition 2 will help us to reduce the dimension of the letter's feature vector as we will see in *preparing data* section.

Diacritics makes short vowels clearer, but they are not necessary. Moreover, a phrase without full diacritics or with just some on some letters is right linguistically, so it is allowed to drop them from the text.

In Unicode, Arabic diacritics are standalone symbols, each of them has its own unicode. This is in contrast to the Latin diacritics; e.g., in the set {ê, é, è, ë, , , }, each combination of the letter *e* and a diacritic is represented by one unicode.

2.1 Arabic Arud Science

reference is the book to be added

Definition 3 *Arud*

In Arabic Arud natively has many meanings (the way, the direction, the light clouds and Mecca and Madinah¹). Arud is the science which studies The Arabic Poem meters and the rules which confirm if the Poem is sound meters & broken meters. It named Arud because some people said he put this science in Arud place العَرُوض with fat-ha, not with dam-ma such as the science name العُرُوض between Mecca and Madinah.

The Author of this science is *Al-Farahidi* (718 786 CE) has analyzed the Arabic poetry; then he came up with that the succession of consonants and vowels produce patterns or *meters*, which make the music of poetry. He was one of the famous people who know The melodies and the musical parts of speech. He has counted them fifteen meters. After that, a student of *Al-Farahidi* has added one more meter to make them sixteen. Arabs call meters بحور which means "seas." Poets have written poems without knowing exactly what rules which make a collection of words a poem.

The Reasons which makes *Al-Farahidi* put this science is

- Protect the Arabic Poems from the broken meters.
- Distinguish between the original Arabic Poem and the non-poem or from the prose.
- Make the rules clear and easy for anyone who needs to write a poem.

Some people said that the one-day *Al-Farahidi* was walking into the metal-market and he was said some of the poems and for some reasons the knock of the metals matched the musical sound of the poem he was saying then he got an idea to explore the Arud of the poems.

¹Mecca and Madinah are two cities in Saudi Arabia.

2.1.1 Feet Representation

A meter is an ordered sequence of feet. Feet are the basic units of meters; there are ten of them.

Definition 4 Feet

A Foot consists of a sequence of **Sukun** (Consonants) represented as (0) and **Harakah** (Vowels) (1). Traditionally, feet are represented by mnemonic words called *tafail* تفاعيل.

Feets consists of three parts (Reasons أسباب, Wedge وتد, Breaks فواصل).

- **Reasons** (أسباب): It has two types
 1. **Light** (سبب خفيف) which happens when we have the first letter is harakah and the second is sukun (/0) example (هَبْ, لَمْ).
 2. **Heavy** (سبب ثقيل) which happens when we have two harakah letter (/ /) example (لَكَ, بِكَ).
- **Wedge** (تد): It has two types
 1. **Combined Wedge** (وتد مجموع) which happens when we have two harakah letters followed by sukun (/ / 0) example (مَشَى, عَلَى).
 2. **Separated Wedge** (وتد مفروق) which happens when we have two harakah and in between a sukun letter (/ 0 /) example (مُنْذُ, مَضْرُ).
- **Breaks** (فواصل): It has two types
 1. **Small Break** (فاصلة صغرى) which happens when we have three harakah letters followed by a sukun letter (/ / / 0) example (ذَهَبُوا, سُفُنًا).
 2. **Big Break** (فاصلة كبرى) which happens when we have four harakah letters followed by a sukun letter (/ / / / 0) example (جَعَلَهُمْ).²

² Some of Arab linguistic scientist assume the small Breaks as a combination between big reason and small reason. Same for the Big Breaks assumed to be a combination between Big reason and Combined Wedge. So, they didn't assume we have three types of feet it is only pure two and any other feets constructed from this two. In this thesis we assume there are three feets .

2.1.1.1 Rules for Arabic Letters Representation

Arabic letter has one general rule in the poem representation which is we represent only the letters which is (spoken) not the written which means the letters with phonatics not the written. We have give the below rules as a results of the general rule.

- Any letter with *harakah* represented as (/).
- Any letter with *sukun* represented as (0).
- Any letter with shaddah represented by two letters the first one will be *sukun* and the second letter will be *harakah* represented as (0/) example (مُحَمَّد) will be (/0/0/0).
- Any letter with tanween represented by two letters the first one is *haraka* (/) and the second is *sukun*.
- Alef without hamze (همزة الوصل) and Wow Aljmaa are not represented example (وَأَعْلَمُوا) will be (/0/0/0)
- If we have a letter which is not written but (spoken) so, we will represent it example (هذا) it include Alef but not written (هاذا) the representation will be (/0/0).
- If we have *Meem Aljamaa* with *harakah* so, it represented with *Mad* example (هُمْ) will be (/0/0) .
- *Alef Mad* (آ) will be two letters *Alef with harakah* and *Alef with sukun* example (آدَمُ) will be (/0//).
- if the verse ended with *harkah* we will add *sukun* to it.

Exampele: (note: the below representation is not complete)

وَجَدْتُ النَّاسَ مَيِّتًا مِثْلَ حَيٍّ *** بِحُسْنِ الذِّكْرِ أَوْ حَيًّا كَمَيِّتٍ
وَجَدْنَنَا سَمِيتْنِمْتُ لَحْيَيْنِ *** بِحُسْنِ ذِكِّ رَاوَحَيْنِ كَمَيِّتَيْنِ

2.1.2 Arabic Poetry Feet

Arabic poetry feet has ten tafa'il تفاعيل (scansion) any poem constructed from these feet. They are eight from writing (syntax) perspective, But it ten in the rules.

#	Feet	Scansion	Construction
1	فَعُولُنْ	0/0//	combined wedge (فعو) and small reason (لن)
2	مَفَاعِيلُنْ	0/0/0//	combined wedge (مفا) and two light reasons (عي) (لن)
3	مُفَاعَلَتُنْ	0///0//	combined wedge (مفا), heavy reason (عل) and light reason (تن)
4	فَاعِلَاتُنْ	0/0//0/	light reason (فا), combined wedge (علا) and light reason (تن)
5	فَاعٍ لَا تُنْ	0/0//0/	Separated wedge (فاع) and two light reason (لا) (تن) ³
6	فَاعِلُنْ	0//0/	light reason (فا) and combined wedge (علن)
7	مُتَفَاعِلُنْ	0//0///	heavy reason (مت), light reason (فا) and combined wedge (علن)
8	مَفْعُولَات	0//0///	two light reason (مف) (عو) and separated wedge (لات)
9	مُسْتَفْعِلُنْ	0//0/0/	two light reason (مس) (تف) and combination wedge (علن)
10	مُسْتَفْعٍ لُنْ	0//0/0/	light reason (مس), separated wedge (تفع) and light reason (لن) ⁴

Table 2.4: The ten feet of the Arabic meters.

³We separated the letters (ع) and (لا) in (فاع لاتن) to show that this part is separated wedge and distinguish between this feet and (فاع لاتن) which contains combined wedge.

⁴We separated the letters (ع) and (ل) in (مستفع لن) to show that it ends with a separated wedge and distinguish between this feet and (مستفع لن) which contains combined wedge

Definition 5 Meter

Poetic meters define the basic rhythm of the poem. Each meter is described by a set of ordered feet which can be represented as ordered sets of consonants and vowels [?].

ولد الهدى فالكائنات ضياء *** وفم الزمان تبسم وثناء انشاء
الروح والملا الملائك حوله *** للدين والدنيا به بشراء

Definition 6 Arabic Verse

refers to "poetry" as contrasted to prose. Where the common unit of a verse is based on meter or rhyme, the common unit of prose is purely grammatical, such as a sentence or paragraph⁵. A verse know as Bayt in Arabic بيت

Definition 7 Shatr

A verse consists of two halves, each of them is called shatr and carries the full meter. We will use the term shatr to refer to a verse's half; whether the right or the left half.

Definition 8 Poem

is a set of verses has the same meter and rhyme.

⁵ [https://en.wikipedia.org/wiki/Verse_\(poetry\)](https://en.wikipedia.org/wiki/Verse_(poetry)).

2.1.3 Arabic Poetry Meters

2.1.3.1 Al-Taweel الطويل

Why it named Al-Taweel? *Al-Taweel is named Al-Taweel for two reasons; first, It is the longest meter between all meters. Second, It starts with Wedge then Reasons and Wedge is longer than Reasons. So, it named Al-Taweel. We need here to note later in the encoding section we will pad all other meters by zeros to make it all the same length. Example if the max Bayt is 82 so, any Bayt less than 82 will be padded by zeros to have the same length.*

tafa'il

فعلون مفاعيلن فعولن مفاعيلن *** فعولن مفاعيلن فعولن مفاعيلن

Example:

إذا جادَ أقوامٌ بمالٍ رأيتَهُمُ *** يَجودونَ بالأرواحِ مِنْهُمُ بلا بُخلٍ
0/0/0// 0/0// 0/0/0// 0/0// *** 0//0// 0/0// 0/0/0// 0/0//
فعلون مفاعيلن فعولن مفاعيلن *** فعولن مفاعيلن فعولن مفاعيلن

2.1.3.2 Al-Madeed المديد

Why it named Al-Madeed?

Al-Madeed is named because of the reasons الأسباب is represented in all its seven parts of tafa'il, One in the first part and the other in the second part. So, it named Madeed مديداً.

tafa'il

فاعلاتن فاعِلن فاعلاتن *** فاعلاتن فاعِلن فاعلاتن

Example:

يَا لَيْكُرْ أَنْشُرُوا لِي كُليباً يَا لَيْكُرْ أَيْنَ أَيْنَ الْفِرَارُ
0/0//0/ 0//0/ 0/0//0/ *** 0/0//0/ 0//0/ 0/0//0/
فاعلاتن فاعِلن فاعلاتن *** فاعلاتن فاعِلن فاعلاتن

2.1.3.3 Al-Baseet البسيط

Why it named Al-Baseet?

- Al-Baseet there is a different idea behind this name
- The Reasons الأسباب expanded into it is tafa'il. So, We will find at the beginning of every part two reasons so; it named Al-Baseet.
- The other reasons which may be the more logic are the harkat الحركات expanded in its tafa'il.

tafa'il

مستفعّلن فعّلن مستفعّلن فاعلن *** مستفعّلن فاعلن مستفعّلن فاعلن

Example:

لَيْسَ الْجَمَالُ بِأَثْوَابٍ تُزَيَّنُنَا *** إِنَّ الْجَمَالَ جَمَالُ الْعِلْمِ وَالْأَدَبِ
0/// 0//0/0/ 0/// 0//0/0/ *** 0/// 0//0/0/ 0/// 0//0/0/
مستفعّلن فعّلن مستفعّلن فاعلن *** مستفعّلن فاعلن مستفعّلن فاعلن

2.1.3.4 Al-Wafer الوافر

Why it named Al-Wafer? Al-Wafer there are different ideas behind this name

- There is much harakat in its parts because there is no tafa'il has part includes harakat more than the word مفاعلتن.
- There are many parts to its base.

tafa'il

مفاعلتن مفاعلتن مفاعلتن *** مفاعلتن مفاعلتن مفاعلتن

Example:

إِذَا بَلَغَ الْفَطَامَ لَنَا صَبِيٌّ *** تَخَرَّجَ لَهُ الْجَبَابِرُ سَاجِدِينَ
0/0// 0//0// 0//0// *** 0/0// 0//0// 0//0//
مفاعلتن مفاعلتن فعولن *** مفاعلتن مفاعلتن فعولن

2.1.3.5 Al-Kamel الكامل

Why it named Al-Kamel? *Al-Kamel is named because its harakat is fully integrated and it is 30 harakah which is not similar to any other Bahr has this numbers of harakat. However, Al-wafer has much harakat in its parts but not the same number as Al-Kamel. Al-Wafer has the harakat but it not overwritten into its source but Al-Kamel it is written into its source أصله. So, Al-Kamel in Arabic named due to it is more integrated أَكْمَل than Al-Wafer.*

tafa'il

متفاعِلن متفاعِلن متفاعِلن *** متفاعِلن متفاعِلن متفاعِلن

Example:

وَلَقَدْ شَفَا نَفْسِي وَأَبْرَأْتُ سَقَمَهَا *** قِيلُ الْفَوَارِسِ وَيَاكَ عَنَتَرُ أَقْدَمُ
0//0//0// 0//0//0// 0//0//0// *** 0//0//0// 0//0//0// 0//0//0//
متفاعِلن مستفعلن متفاعِلن *** مستفعلن متفاعِلن متفاعِلن

2.1.3.6 Al-Hazaj الهزج

Why it named Al-Hazaj? *Al-Hazaj is named because of the sound reverberation in its parts. Al-Hazaj in Arabic is sound frequency. So, due to the sound reverberation, it named AL-Hazaj. Also, Because every part ends for two reasons so, it made some of sound like a piece of music to be Al-Hazaj.*

tafa'il

مفاعِلِن مفاعِلِن *** مفاعِلِن مفاعِلِن

Example:

أَيَا مَنْ لَامَ فِي الْحُبِّ *** وَلَمْ يَعْلَمْ جَوَى قَلْبِي
0/0/0// 0/0/0// *** 0/0/0// 0/0/0//
مفاعِلِن مفاعِلِن *** مفاعِلِن مفاعِلِن

2.1.3.7 Al-Rejz الرجز

Why it named Al-Rejz? *Al-Rejz is named Al-Rezj because it constructed from three parts. If there is an animal and someone pull this animal by one leg and the animal walk into three legs in Arabic named Rejz رجز, this is the reason it named Rejz because of it has three parts similar than animal pulled by one leg and walk into three legs. Also, in Arabic, if we have a camel when it stands up has some disturbances due to it sick or has any issue it named Rejz Camel جمل رجز, and in Arabic disturbance إضطراب means Rejz for example.*

tafa'il

مستفعلن مستفعلن مستفعلن *** مستفعلن مستفعلن مستفعلن

Example:

دار لسلمي إذ سلمي جارة *** قفر ترى آياها مثل الزّبر
0//0/0/ 0//0/0/ 0//0/0/ *** 0//0/0/ 0//0/0/ 0//0/0/
مستفعلن مستفعلن مستفعلن *** مستفعلن مستفعلن مستفعلن

2.1.3.8 Al-Raml الرمل

Why it named Al-Raml? *Al-Raml is named Al-Raml because Al-Raml is a type of the singing which constructed from this Bahr. Another reason is the Wedges is appeared in between the Reasons so; it named Al-Raml due to the diversity between the Reasons and the wedges inside the tafa'il.*

tafa'il

فاعلاتن فاعلاتن فاعلاتن *** فاعلاتن فاعلاتن فاعلاتن

Example:

إنما الدنيا غرور كلها *** مثل لمغ الشال في أرض الفقار
0//0//0/ 0//0//0/ 0//0//0/ *** 0//0/ 0//0//0/ 0//0//0/
فاعلاتن فاعلاتن فاعلن *** فاعلاتن فاعلاتن فاعلاتن

2.1.3.9 Al-Sarea السريع

Why it named Al-Sarea? *Al-Sarea is named Al-Sarea in Arabic Al-Sarea means the fastest. It named these because its speed in teste الزوق or its parts التقطيع. The other reason because its three parts have 7 reasons and the reasons are faster than the wedges.*

tafa'il

مستفعلن مستفعلن مفعولات *** مستفعلن مستفعلن مفعولات

Example:

أزمان سلمى لا يرى مثلها الر *** راؤون في شام ولا عراق
00//0/ 0//0/0/ 0//0/0/ *** 0//0/ 0//0/0/ 0//0/0/
مستفعلن مستفعلن مفعولات *** مستفعلن مستفعلن مفعولات

2.1.3.10 Al-Monsareh المنسرح

Why it named Al-Monsareh? *to be written later :) .*

tafa'il

مستفعلن مفعولات مستفعلن *** مستفعلن مفعولات مستفعلن

Example:

إن ابن زيد لا زال مستعملا *** للخير يفشي في مصره العرفا
0//0/ 0//0/0/ 0//0/0/ *** 0//0/0/ 0//0/0/ 0//0/0/
مستفعلن مفعولات مستفعلن *** مستفعلن مفعولات مفتعلن

2.1.3.11 Al-Khafeef الخفيف

Why it named Al-Khafeef? *Al-Khafeef name in Arabic means light. The reason behind the name is its wedge last harkah connected to its reason so, it became light خفت. The other reason is its light in teste الزوق or its parts التقطيع because it has a part with three reasons and the reasons are lighter than wedges.*

tafa'il

فاعلاتن مستفع لن فاعلاتن *** فاعلاتن مستفع لن فاعلاتن

Example:

حل أهلي ما بين دوني فبادو *** لي وحلت علوية بالسخال
0/0//0/ 0//0/0/ 0/0//0/ *** 0/0//0/ 0//0/0/ 0/0//0/
فاعلاتن مستفع لن فاعلاتن *** فاعلاتن مستفع لن فاعلاتن

2.1.3.12 Al-Modarea المضارع

Why it named Al-Modarea? *Al-Modarea in Arabic means the present. It named by this name because it is the present version of the Al-Hazaj. Also, This Bahr wasn't famous in Arabic Poem and there weren't any popular peams or poetry used this Bahr before.*

tafa'il

مفاعلين فاع لاتن *** مفاعلين فاع لاتن

Example:

كأن لم يكن جديراً *** بحفظ الذي أضاعا
0/0//0/ /0/0// *** 0/0//0/ /0/0//
مفاعيل فاع لاتن *** مفاعيل فاع لاتن

2.1.3.13 Al-Moktadeb المقتضب

Why it named Al-Moktadeb ? *Al-Moktadeb in Arabic means reproduced from another thing الإقتطاع and because this Bahr word is all appeared into Al-Monsareh in all its words but also there is a difference in the order of the parts. So, it named Al-Moktadeb because it seems to reproduce from Al-Monsareh. We will have another section which will focus on the relation between the Bohor.)*

tafa'il

مفعولات مستفعلين *** مفعولات مستفعلين

Example:

حف كأسها الحبب *** فهي فضة ذهب
0///0/ /0//0/ *** 0///0/ /0//0/
فاعلات مفتعلن *** فاعلات مفتعلن

2.1.3.14 Al-Mojtaz المجتزأ

Why it named Al-Mojtaz? *Al-Mojtaz in Arabic name is similar meaning for Al-Moktadeb it means reproduced from another thing الإجتزأ أو الإقتطاع and because these Bahr words are all appeared into Al-khafeef in all its words but also there is a difference in the order of the parts. So, it named Al-Mojtaz because it seems it reproduced from Al-khafeef إجتزأ من بحر الخفيف.*

tafa'il

مستفع لن فاعلاتن *** مستفع لن فاعلاتن

Example:

أنت الذي ولدتك *** أسماء بنت الحباب
0/0//0/ 0//0/0/ *** /0/// 0//0/0/
مستفع لن فاعلاتن *** مستفع لن فاعلاتن

2.1.3.15 Al-Motaqareb المتقارب

Why it named Al-Motaqareb? *Al-Motaqareb in Arabic means convergent and is named by this name because the wedges are convergent to each other this is because between every two wedges one reason, so the wedges are convergent to each other. Also, another reason is its parts are similar to each other, so it named Al-Motaqareb. It has eight parts.*

tafa'il

فعولن فعولن فعولن فعولن فعولن فعولن فعولن فعولن

Example:

فَأَمَّا تَمِيمٌ، تَمِيمٌ بِنُ مُرٍّ *** فَالْفَاهُ الْقَوْمُ رَوَّيَ، نِيَامَا
0/0// 0/0// 0/0// 0/0// *** 0/0// 0/0// 0/0// 0/0//
فعولن فعولن فعولن فعولن فعولن فعولن فعولن فعولن

2.1.3.16 Al-Motadarek المتدارك

Why it named Al-Motadarek? There are different ideas behind this name.

- Al-Motadarek in Arabic means explored because Al-Farahidi forgets this Bahr and his student Al-Akhfash Al-Awsat has explored it named by this name.
- Al-Motadarek in Arabic also means followed by something يُدرك, and because this Bahr came after Al-Motaqareb, it names Al-Motadarek.

tafa'il

فَاعِلُنْ فَاعِلُنْ فَاعِلُنْ فَاعِلُنْ *** فَاعِلُنْ فَاعِلُنْ فَاعِلُنْ فَاعِلُنْ

Example:

لَمْ يَدَعْ مَنْ مَضَى لِلَّذِي قَدْ غَبَرَ *** فَضَّلَ عِلْمَ سَوَى أَخْذِهِ بِالْأَثَرِ
0//0/ 0//0/ 0//0/ 0//0/ *** 0//0/ 0//0/ 0//0/ 0//0/
فَاعِلُنْ فَاعِلُنْ فَاعِلُنْ فَاعِلُنْ *** فَاعِلُنْ فَاعِلُنْ فَاعِلُنْ فَاعِلُنْ

Chapter 3

DATASET

We have scrapped the Arabic dataset from two big poetry websites: ¹الديوان الموسوعة, ²الشعرية. Both are merged into one large dataset. It is important to note that the verses' diacritic states are not consistent, this means that a verse can carry full, semi diacritics or it can carry nothing. The total number of verses is 1,862,046 poetic verses; each verse is labeled by its meter, the poet who wrote it, and the age which it was written in. There are 22 meters, 3701 poets and 11 ages; and they are Pre-Islamic, Islamic, Umayyad, Mamluk, Abbasid, Ayyubid, Ottoman, Andalusian, era between Umayyad and Abbasid, Fatimid and modern. We are only interested in the 16 classic meters which are attributed to *Al-Farahidi*, and they are the majority of the dataset with a total number of 1,722,321 verses³.

3.1 Preparing Data

3.1.1 Data Cleaning

¹alldiwan.net

²poetry.tcaabudhabi.ae

³<https://www.github.com/tahamagdy>

Chapter 4

DATA ENCODING

4.0.1 Arabic Poem Encoding

4.0.1.1 One-Hot encoding

4.0.1.2 Binary Encoding

4.0.1.3 Two-Hot encoding

Chapter 5

MODEL TRAINING

Chapter 6

RESULTS AND DISCUSSION

Chapter 7

CONCLUSION AND FUTURE WORK

7.1 Future Work

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APPENDICES

APPENDIX A

Phase Correlation Theory

Let $D_1(x, y)$ and $D_2(x, y)$ be the dilated images to be registered, the Fourier transform for both $F_1(u, v)$ and $F_2(u, v)$ is given by:

$$\begin{aligned} F_k(u, v) &= \mathcal{F}\{D_k(x, y)\} \\ &= \int_{y=-\infty}^{y=\infty} \int_{x=-\infty}^{x=\infty} D_k(x, y) \exp(-i2\pi\omega xy) dx dy \end{aligned} \quad (7.1)$$

where, \mathcal{F} is the Fourier operator, K denotes image 1 or 2, ω is the frequency (in hertz), x and y are the spatial domain coordinates, u and v are the frequency domain coordinates of the two images.

Given two images of size $N \times M$ shifted against each other, according to the Fourier shift property, their Fourier becomes:

$$F_2(u, v) = F_1(u, v) \exp\left(-i2\pi\left(\frac{u\Delta x}{M} + \frac{v\Delta y}{N}\right)\right) \quad (7.2)$$

The Normalized Cross Power Spectrum ($C(u, v)$) is defined as:

$$C(u, v) = \frac{F_1(u, v) \cdot F_2(u, v)^*}{|F_1(u, v) \cdot F_2(u, v)^*|} \quad (7.3)$$

where ‘ \cdot ’ denotes the element-wise product, ‘ $*$ ’ denotes the complex conjugate.

Using equation ??:

$$C(u, v) = \frac{F_1(u, v) \cdot F_1(u, v)^* \exp\left(i2\pi\left(\frac{u\Delta x}{M} + \frac{v\Delta y}{N}\right)\right)}{\left|F_1(u, v) \cdot F_1(u, v)^* \exp\left(i2\pi\left(\frac{u\Delta x}{M} + \frac{v\Delta y}{N}\right)\right)\right|} \quad (7.4)$$

Since the phase term of $F_1(u, v) \cdot F_1(u, v)^*$ is zero, only the magnitude remains, i.e. $F_1(u, v) \cdot F_1(u, v)^* = |F_1(u, v) \cdot F_1(u, v)^*|$ and since the magnitude of any complex exponential is 1, the equation drops to:

$$C(u, v) = \frac{|F_1(u, v) \cdot F_1(u, v)^*| \exp\left(i2\pi\left(\frac{u\Delta x}{M} + \frac{v\Delta y}{N}\right)\right)}{|F_1(u, v) \cdot F_1(u, v)^*|}$$

$$= \exp\left(i2\pi\left(\frac{u\Delta x}{M} + \frac{v\Delta y}{N}\right)\right) \quad (7.5)$$

the inverse Fourier transform of which is a delta function, i.e. a single peak.

The Normalized Cross Correlation (c) equals:

$$c = \mathcal{F}^{-1}\{C\} = \delta(x + \Delta x, y + \Delta y) \quad (7.6)$$

The shift in x and y between the two images $(\Delta x, \Delta y)$ takes the location of the maximum peak in c , such that:

$$(\Delta x, \Delta y) = \underset{x,y}{\operatorname{argmax}}\{c\} \quad (7.7)$$