Problem 1

(15pt) (Simple harmonic motion) Consider a unit mass connected to a support through a spring whose spring constant is unity. If z measures the displacement of the mass from equilibrium, then

$$\ddot{z} + z = 0$$

1. (5pt) Write down the state space form of the dynamical system.
2. (5pt) Determine the equilibrium of the system.
3. (5pt) Is the equilibrium stable or unstable? Justify your answer.

Problem 2

(10pt) Consider the linear system in state space

$$\begin{aligned} \dot{x}_1 &= -2x_1 + 2x_2 \\ \dot{x}_2 &= x_1 - x_2 \end{aligned}$$

Introduce a state transformation $z = Tx$ such that the system can be represented by $\dot{z} = \Lambda z$ where Λ is either a diagonal matrix or a Jordan matrix.

Problem 3

(25pt) Read the attached reading material. Chapter 2. System modeling from Feedback Systems: An Introduction for Scientists and Engineers by Karl Johan Astrom and Richard M. Murray.
section 2.2 and 2.4.
finish the following exercise.

2.6 (Normalized oscillator dynamics) Consider a damped spring–mass system with dynamics

$$m\ddot{q} + c\dot{q} + kq = F.$$

Let $\omega_0 = \sqrt{k/m}$ be the natural frequency and $\zeta = c/(2\sqrt{km})$ be the damping ratio.

(a) Show that by rescaling the equations, we can write the dynamics in the form

$$\ddot{q} + 2\zeta\omega_0\dot{q} + \omega_0^2 q = \omega_0^2 u, \tag{2.35}$$

where $u = F/k$. This form of the dynamics is that of a linear oscillator with natural frequency ω_0 and damping ratio ζ.

(b) Show that the system can be further normalized and written in the form

$$\frac{dz_1}{d\tau} = z_2, \qquad \frac{dz_2}{d\tau} = -z_1 - 2\zeta z_2 + v. \tag{2.36}$$

The essential dynamics of the system are governed by a single damping parameter ζ. The *Q-value* defined as $Q = 1/2\zeta$ is sometimes used instead of ζ.

hint: state space transform.
Question: What is the equilibrium of the system without input force? Is it stable or unstable equilibrium? Justify your reasoning.
Practice homework (not graded): Use matlab ode function to simulate the system given a chosen initial state and a constant force or zero force. Describe the behavior of the system based on different parameter you have selected for this system.