Worksheet #3
Solution

[1] $f(x,y) = x^2 + y^2$

1) $\nabla f(x,y) = \begin{pmatrix} 2x \\ 2y \end{pmatrix} = 0$ $\quad 2x = 0 \quad 2y = 0 \quad x = 0 \quad y = 0$ $\quad$ Stationary Point $(0,0)$

2) $\nabla^2 f = H = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = \underset{+}{2}, \underset{+}{4}$ $\quad$ Positive Definite

at $(0,0) \longrightarrow$ local min.

[2] $f(x_1, x_2) = x_1^2 + 2x_1x_2 + 2x_2^2$

1) $\nabla f(x_1,x_2) = \begin{pmatrix} 2x_1 + 2x_2 \\ 2x_1 + 4x_2 \end{pmatrix} = 0$ $\quad x_1 = 0 \quad x_2 = 0$ $\quad$ S.P $(0,0)$

2) $\nabla^2 f = H = \begin{vmatrix} 2 & 2 \\ 2 & 4 \end{vmatrix} = \underset{+}{2}, \underset{+}{4}$ $\quad$ Positive Definite

at $(0,0) \longrightarrow$ local min.

[3] $f(x,y) = \frac{1}{3}x^3 + y^3 - x - y$

1) $\nabla f(x,y) = \begin{pmatrix} x^2 - 1 \\ 3y^2 - 1 \end{pmatrix}$ $\quad$ 2) $H = \begin{vmatrix} 2x & 0 \\ 0 & 6y \end{vmatrix}$

$x = \pm 1$, $y = \pm\frac{1}{\sqrt{3}}$ $\longrightarrow$ we have 4 Stationary Points.

$(1, \frac{1}{\sqrt{3}})$ $(1, \frac{-1}{\sqrt{3}})$ $(-1, \frac{1}{\sqrt{3}})$ $(-1, \frac{-1}{\sqrt{3}})$

① $H(1, \frac{1}{\sqrt{3}}) = \begin{vmatrix} 2 & 0 \\ 0 & 6/\sqrt{3} \end{vmatrix} = \underset{+}{2}, \underset{+}{12/\sqrt{3}}$ $\quad$ P.D

local min

② $H(1, \frac{-1}{\sqrt{3}}) = \begin{vmatrix} 2 & 0 \\ 0 & -6/\sqrt{3} \end{vmatrix} = \underset{+}{2}, \underset{-}{-12/\sqrt{3}}$ $\quad$ saddle