



K.N. Toosi University of Technology

Student name: Mostafa Latifian

Student ID: 40122193

Professor: Dr. Hamidreza Taghirad

Course: Linear Control System

Research Subject: The reason why we use the derivative if a row becomes zero in the Routh-Hurwitz

Contents

1. instruction	3
2. Theory of Routh-Hurwitz	3
3. Derivative Application in the Routh-Hurwitz Criterion	3
4. conclusion	4

1. instruction

The Routh-Hurwitz criterion is the systematic way of finding the stability of control systems through the observation of sign changes in the first column of the Routh array. The criterion is derived from the observation that the number of poles in the right-half plane in the system characteristic equation is equal to the number of sign changes in the first column of the Routh array.

But the significant challenge comes when constructing the Routh array: sometimes there may be a row of zeros. Such an occurrence of a zero row shows that there are repeated or multiple roots of the characteristic equation of the system, which can make the stability analysis challenging. Continuing directly with the application of the Routh-Hurwitz criterion in this case may produce an unsuitable analysis.

In resolving this issue, a derivative of the polynomial constructed from the row that is written above the zero row is used. The derivative offers a way of acquiring useful information regarding the repeated roots, thereby facilitating further analysis. The aim of this paper is to explain the theoretical foundation behind the application of the derivative and demonstrate how the method assists in completing the Routh array as well as correctly assessing system stability.

2. Theory of Routh-Hurwitz

The Routh array is created using the coefficients of the system's characteristic polynomial. This applies to a polynomial structured in a specific form:

$$a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0 = 0$$

The Routh array is formed by first dividing the coefficients into two rows: one with the even-powered coefficients of s and the other with the odd-powered coefficients. The subsequent rows are calculated by applying some formulas on the elements of the previous rows, and what is obtained is a set of polynomials that represent the locations of the poles of the system.

The challenge occurs when a row is completely filled with zeros. This usually suggests that the system has repeated or multiple roots, which can be difficult to pinpoint just by looking at the array. To address this, we calculate the derivative of the polynomial created by the non-zero elements in the row above the zero row. This derivative gives us the essential information needed to continue building the Routh array.

3. Derivative Application in the Routh-Hurwitz Criterion

If there is a zero row, then the process is as follows:

1. Locate the Zero Row: When there is a row in the Routh array with all elements equal to zero, then there are repeated roots.
2. Differentiate the row above it: The polynomial built from the row above the zero row is differentiated with respect to s .
3. Place the Derivative in the Array: The result of differentiation forms the next row of the Routh array, providing useful information about repeated roots.

4. Complete the Routh Array Construction: After placing the derivative, the array can be completed and the stability analysis continued by determining the sign changes in the first column.

This approach ensures that the stability analysis holds, even in the case of having more than one root.

4. conclusion

In summary, the Routh-Hurwitz criterion is a basic tool for determining the stability of a system. The appearance of a zero row in the Routh array, however, indicates the presence of repeated roots, and the analysis procedure becomes complicated. One may gain knowledge about these repeated roots using the derivative of the row above the zero row, thus facilitating the procedure of stability analysis. This process ensures that the Routh array is filled and supports the analysis of the stability of a system even when the system has repeated roots. It is crucial to learn and apply this technique to perform successful stability analysis of higher-order systems.