

Diagnosing Bias vs. Variance

In this section we examine the relationship between the degree of the polynomial d and the underfitting or overfitting of our hypothesis.

- We need to distinguish whether **bias** or **variance** is the problem contributing to bad predictions.
- High bias is underfitting and high variance is overfitting. Ideally, we need to find a golden mean between these two.

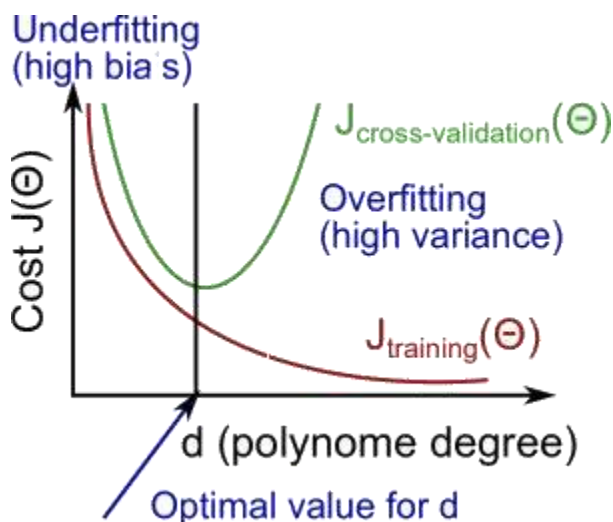
The training error will tend to **decrease** as we increase the degree d of the polynomial.

At the same time, the cross validation error will tend to **decrease** as we increase d up to a point, and then it will **increase** as d is increased, forming a convex curve.

High bias (underfitting): both $J_{\text{train}}(\Theta)$ and $J_{\text{CV}}(\Theta)$ will be high. Also, $J_{\text{CV}}(\Theta) \approx J_{\text{train}}(\Theta)$.

High variance (overfitting): $J_{\text{train}}(\Theta)$ will be low and $J_{\text{CV}}(\Theta)$ will be much greater than $J_{\text{train}}(\Theta)$.

This is summarized in the figure below:



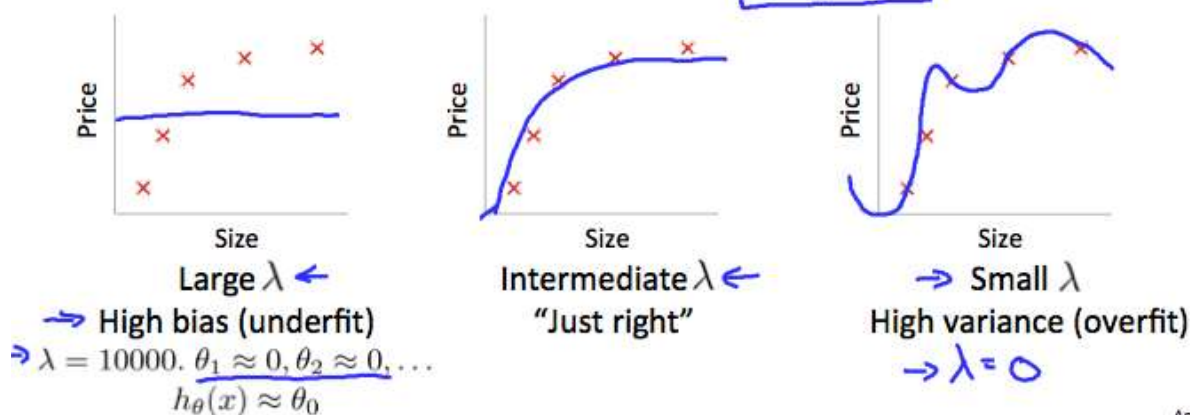
Regularization and Bias/Variance

Note: [The regularization term below and through out the video should be $\frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$ and **NOT** $\frac{\lambda}{2m} \sum_{j=1}^m \theta_j^2$]

Linear regression with regularization

Model: $h_{\theta}(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$ ←

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + \frac{\lambda}{2m} \sum_{j=1}^m \theta_j^2$$
 ←



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In the figure above, we see that as λ increases, our fit becomes more rigid. On the other hand, as λ approaches 0, we tend to overfit the data. So how do we choose our parameter λ to get it 'just right'? In order to choose the model and the regularization term λ , we need to:

1. Create a list of lambdas (i.e. $\lambda \in \{0, 0.01, 0.02, 0.04, 0.08, 0.16, 0.32, 0.64, 1.28, 2.56, 5.12, 10.24\}$);
2. Create a set of models with different degrees or any other variants.
3. Iterate through the λ s and for each λ go through all the models to learn some Θ .
4. Compute the cross validation error using the learned Θ (computed with λ) on the $J_{CV}(\Theta)$ **without** regularization or $\lambda = 0$.
5. Select the best combo that produces the lowest error on the cross validation set.
6. Using the best combo Θ and λ , apply it on $J_{test}(\Theta)$ to see if it has a good generalization of the problem.

Learning Curves

Training an algorithm on a very few number of data points (such as 1, 2 or 3) will easily have 0 errors because we can always find a quadratic curve that touches exactly those number of points. Hence:

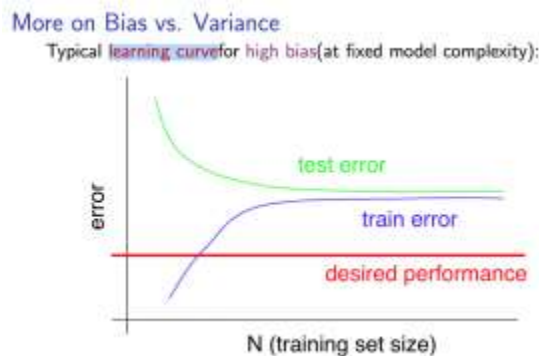
- As the training set gets larger, the error for a quadratic function increases.
- The error value will plateau out after a certain m , or training set size.

Experiencing high bias:

Low training set size: causes $J_{\text{train}}(\Theta)$ to be low and $J_{\text{CV}}(\Theta)$ to be high.

Large training set size: causes both $J_{\text{train}}(\Theta)$ and $J_{\text{CV}}(\Theta)$ to be high with $J_{\text{train}}(\Theta) \approx J_{\text{CV}}(\Theta)$.

If a learning algorithm is suffering from **high bias**, getting more training data will not **(by itself)** help much.



Experiencing high variance:

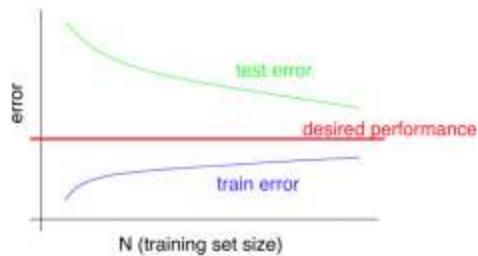
Low training set size: $J_{\text{train}}(\Theta)$ will be low and $J_{\text{CV}}(\Theta)$ will be high.

Large training set size: $J_{\text{train}}(\Theta)$ increases with training set size and $J_{\text{CV}}(\Theta)$ continues to decrease without leveling off. Also, $J_{\text{train}}(\Theta) < J_{\text{CV}}(\Theta)$ but the difference between them remains significant.

If a learning algorithm is suffering from **high variance**, getting more training data is likely to help.

More on Bias vs. Variance

Typical learning curve for high variance (at fixed model complexity):



Deciding What to do Next Revisited

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Our decision process can be broken down as follows:

- **Getting more training examples:** Fixes high variance
- **Trying smaller sets of features:** Fixes high variance
- **Adding features:** Fixes high bias
- **Adding polynomial features:** Fixes high bias
- **Decreasing λ :** Fixes high bias
- **Increasing λ :** Fixes high variance.

Diagnosing Neural Networks

- A neural network with fewer parameters is **prone to underfitting**. It is also **computationally cheaper**.
- A large neural network with more parameters is **prone to overfitting**. It is also **computationally expensive**. In this case you can use regularization (increase λ) to address the overfitting.

Using a single hidden layer is a good starting default. You can train your neural network on a number of hidden layers using your cross validation set. You can then select the one that performs best.

Model Complexity Effects:

- Lower-order polynomials (low model complexity) have high bias and low variance. In this case, the model fits poorly consistently.
- Higher-order polynomials (high model complexity) fit the training data extremely well and the test data extremely poorly. These have low bias on the training data, but very high variance.
- In reality, we would want to choose a model somewhere in between, that can generalize well but also fits the data reasonably well.