CSCI 2110 Data Structures and Algorithms

# Module 9: Hashing and Hash Tables

DALHOUSIE UNIVERSITY

# MOTIVATION

- Name the three most important and common operations that you would perform on any data structure.
- **SEARCH, SEARCH AND SEARCH!**
- How did we do in terms of the time complexity of search on different data structures?
  - Unordered List: **O(n)**
  - Ordered List: **$O(\log_2 n)$** (because of binary search)
  - Binary Tree: **O(n)** (because it can be just a linear set of nodes)
  - Binary Search Tree: **$O(\log_2 n)$** if the tree is complete (balanced) – Good tree!
    **O(n)** if the tree is not balanced – Bad tree!
  - Heaps: **O(n)** key can be anywhere on the tree
- **CAN WE DO BETTER???**

# Can we aim for O(1) complexity for search?

- Let’s set ourselves a tall order: O(1) for search (at least on the average).

- We know that in an array, given its index, we can retrieve an element in O(1) time.

- Can we look back to the array for help?

- How can we put a set of keys onto an array so that the search complexity is O(1)?

# Can we aim for O(1) complexity for search?

- Suppose we have an arbitrary set of keys 2, 8, 7, 4, 0.
- Remember that each key is associated with an object: if you get the key, you get the object.
- If we store the keys in the array arbitrarily, we need O(n) time → Sequential search

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8. | 9 |
|---|---|---|---|---|---|---|---|---|---|
| **2** | **8** | **7** | **4** | **0** | | | | | |

- If we store the keys in the array sorted, we still need $O(\log_2 n)$ time → Binary search

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8. | 9 |
|---|---|---|---|---|---|---|---|---|---|
| **0** | **2** | **4** | **7** | **8** | | | | | |

- **BUT WHAT IF WE <u>MAKE THE KEY THE INDEX</u>?**
- **Time for search: O(1) (insert exclamations here).**

# Here's the idea:

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |

***Keys:***
***2, 8, 7, 4,0***

| | |
|---|---|
| 0 | |
| 1 | null |
| 2 | |
| 3 | null |
| 4 | |
| 5 | null |
| 6 | null |
| 7 | |
| 8 | |
| 9 | null |

# What is the catch?

- In our example, we chose the keys to be 2, 8, 7, 4, 0 and hence we declared an array of size 10.
- **What if the keys are 1, 43, 102, 131,15143 and 19992?**
- Would you declare an array of size 20000?
- It appears that whatever you gained in time complexity was lost in space complexity.
- Can we have the best of both – good time complexity and good space complexity?
- ***The trick: MAP the key to an index!***
- **The process: HASHING**
- **The data structure: HASH TABLE**

# Hashing and Hash Table - Definition

- A hash table is a storage array.
- A hash function maps a key to a specific index on the hash table.
- Hashing is the process of storing objects in the hash table using a hash function.

A simple hash function or mapping function can be written as:

*key % N*

where N is the table size.

For example, if the table size is 10, then we get our index as follows:

*index = key % 10*

# Example

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |

Suppose we have these 5 keys:
9876, 234, 5672, 89990, 6777

We choose a table of size 10 (enough to fit the number of keys).

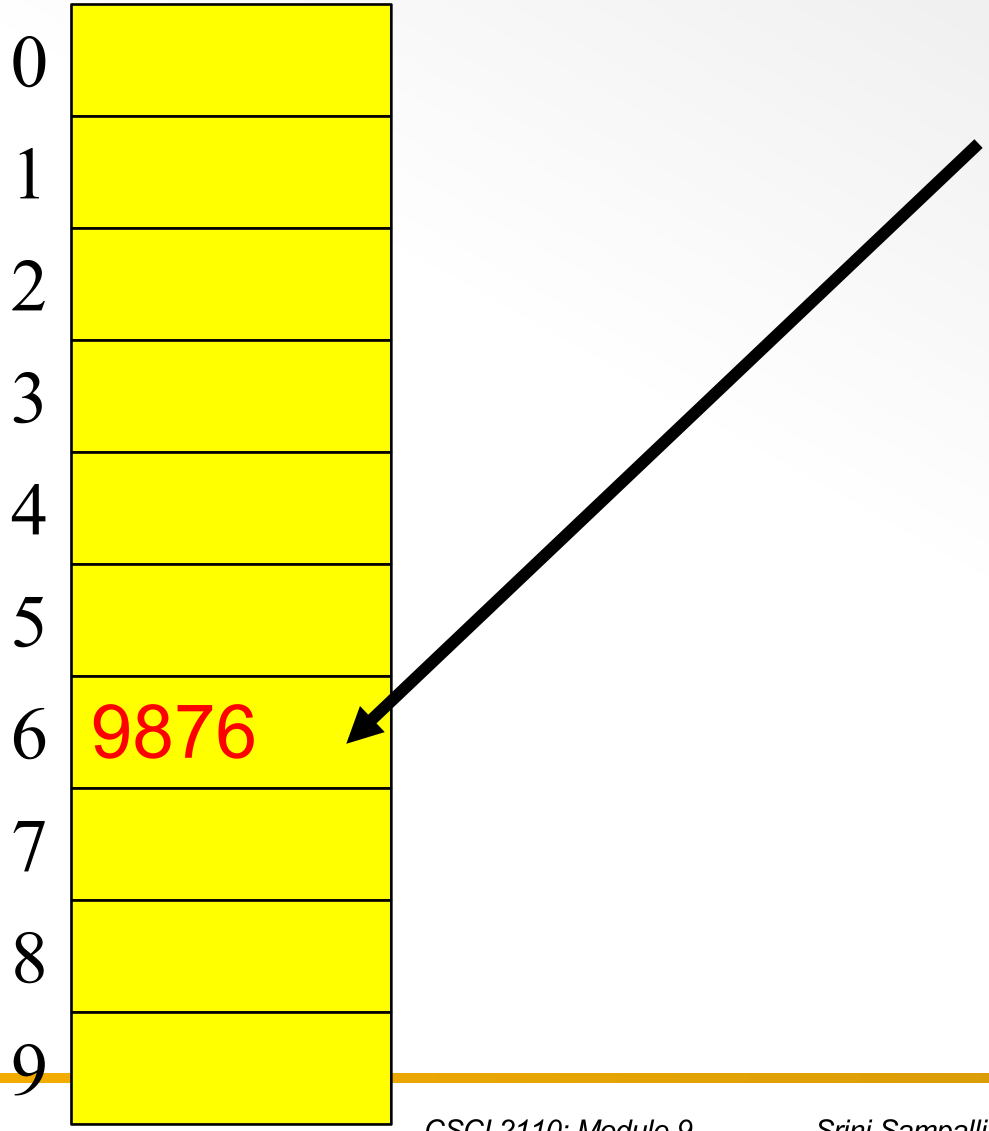
Then we apply the hash function

$$index = key \ \% \ 10$$

to map the keys to the table.

# Example

***Keys:***
***9876, 234, 5672, 89990, 6777***

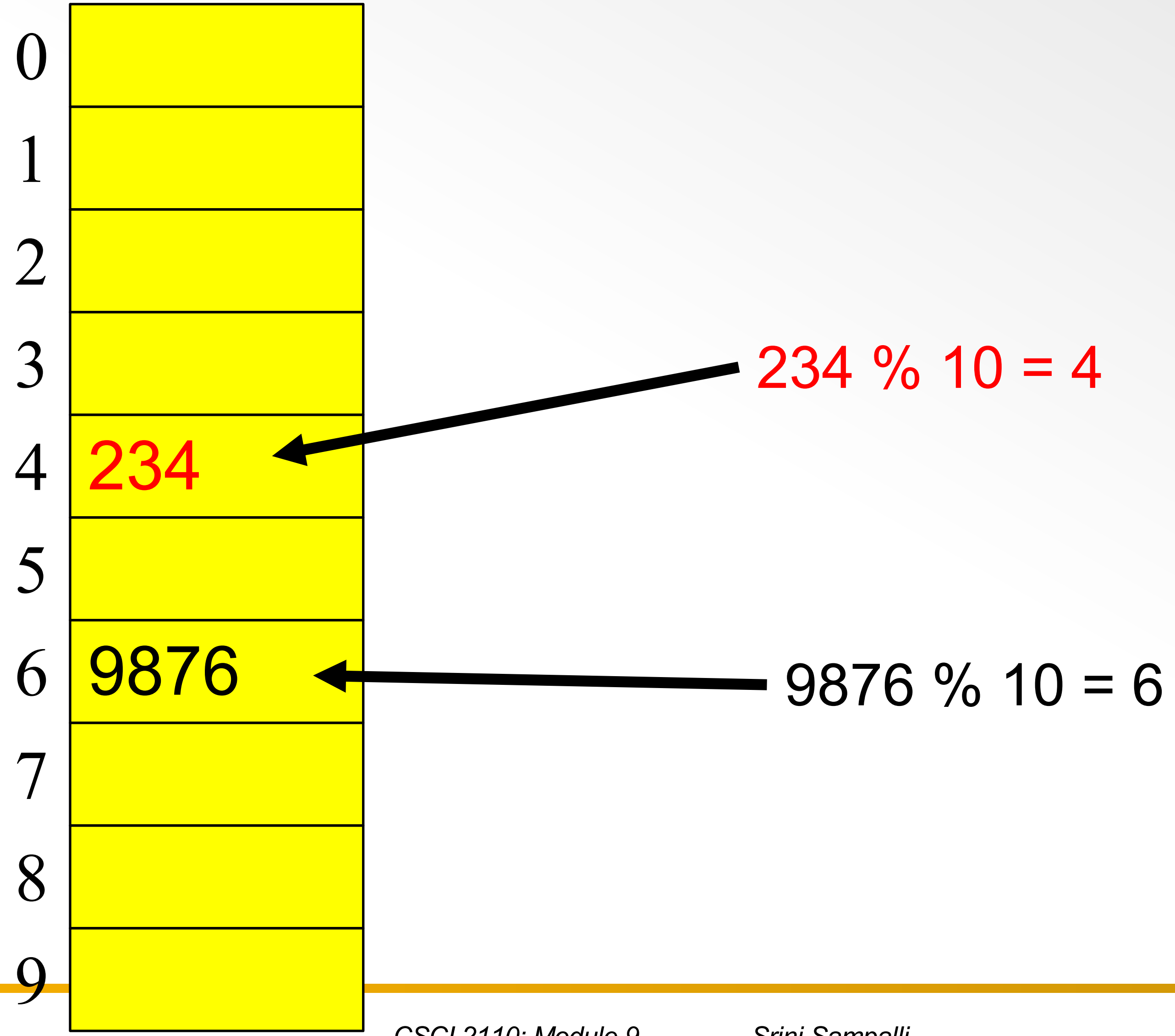



9876 % 10 = 6

Key 9876 goes to location 6

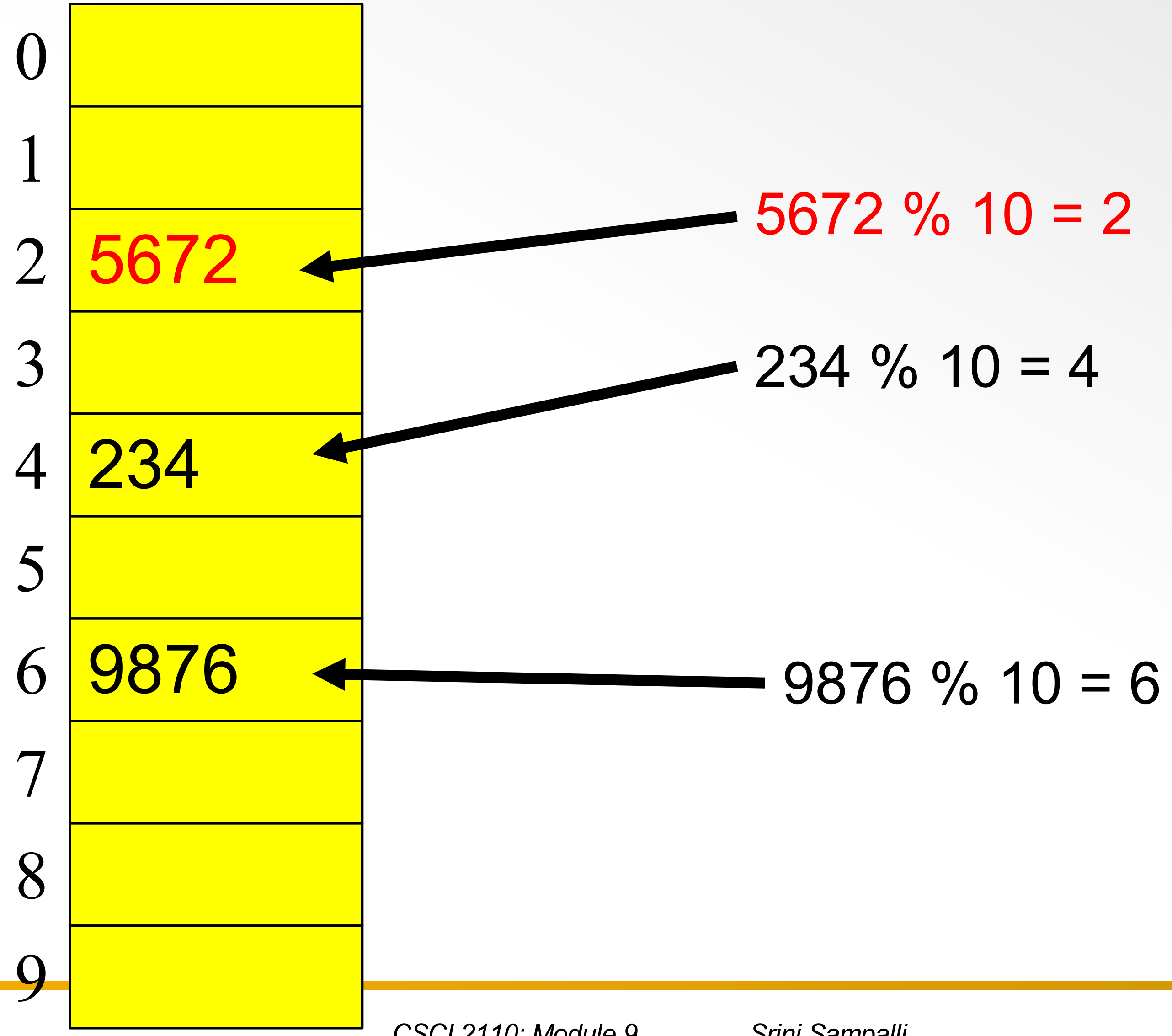
# Example

***Keys:***
***9876, 234, 5672, 89990, 6777***



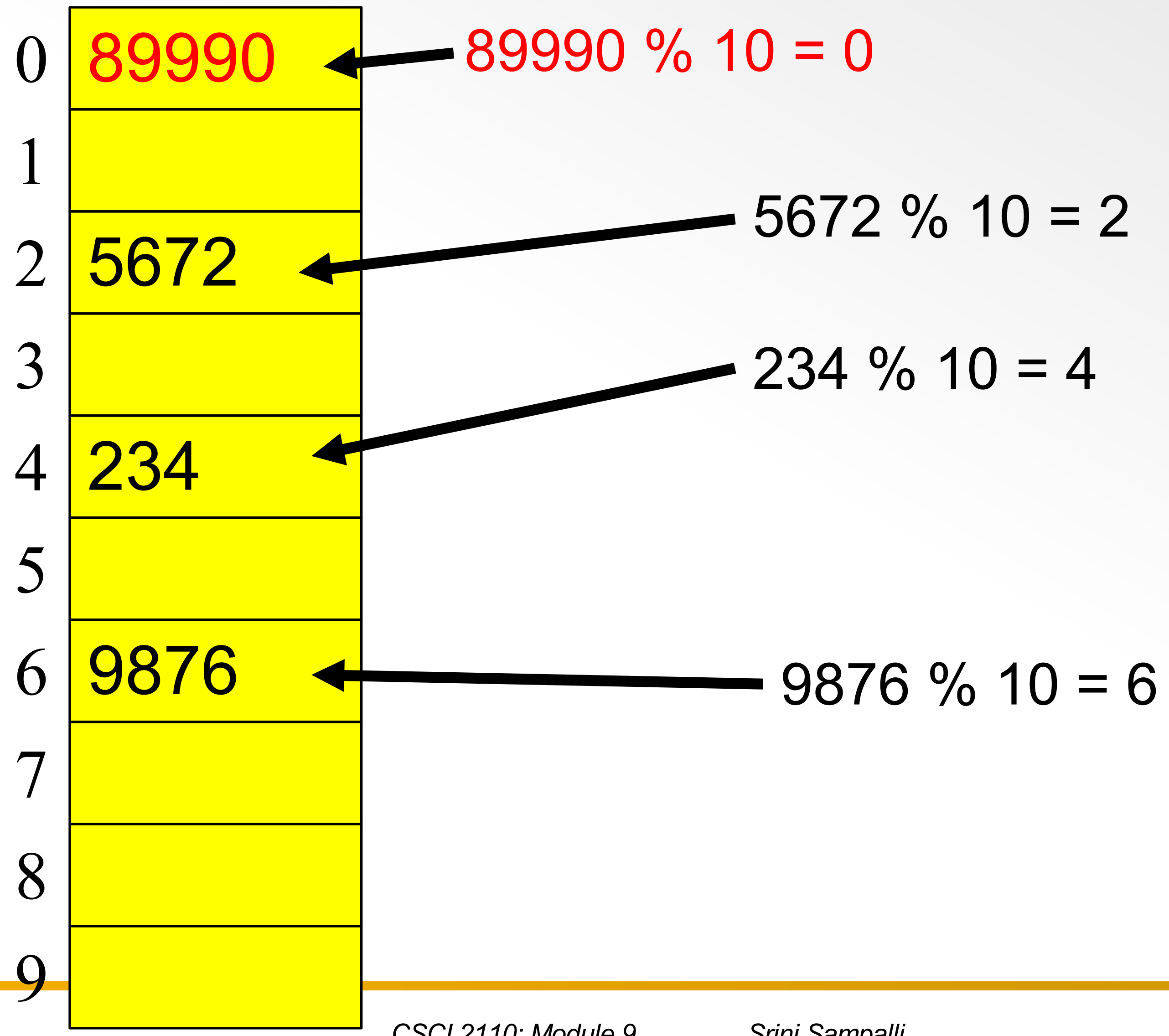
# Example

***Keys:***
***9876, 234, 5672, 89990, 6777***



# Example

***Keys:***
***9876, 234, 5672, 89990, 6777***



# Example

***Keys:***
***9876, 234, 5672, 89990, 6777***

| | |
|---|---|
| 0 | 89990 |
| 1 | |
| 2 | 5672 |
| 3 | |
| 4 | 234 |
| 5 | |
| 6 | 9876 |
| 7 | 6777 |
| 8 | |
| 9 | |

89990 % 10 = 0

5672 % 10 = 2

234 % 10 = 4

9876 % 10 = 6

6777 % 10 = 7

# Now look at the search operations!

| | |
|---|---|
| 0 | 89990 |
| 1 | |
| 2 | 5672 |
| 3 | |
| 4 | 234 |
| 5 | |
| 6 | 9876 |
| 7 | 6777 |
| 8 | |
| 9 | |

# Now look at the search operations!

| | |
|---|---|
| 0 | 89990 |
| 1 | |
| 2 | 5672 |
| 3 | |
| 4 | 234 |
| 5 | |
| 6 | 9876 |
| 7 | 6777 |
| 8 | |
| 9 | |

*Search for key 234*
*Apply the hash function*
*234 %10 = 4*
*Go to location 4*
*Found!*
*O(1) time complexity!*

# Now look at the search operations!

| | |
|---|---|
| 0 | 89990 |
| 1 | |
| 2 | 5672 |
| 3 | |
| 4 | 234 |
| 5 | |
| 6 | 9876 |
| 7 | 6777 |
| 8 | |
| 9 | |

*Search for key 234*
*Apply the hash function*
*234 %10 = 4*
*Go to location 4*
*Found!*
*O(1) time complexity!*

*Search for key 878*
*Apply the hash function*
*878 %10 = 8*
*Go to location 8*
*Not found!*
*O(1) time complexity!*

# Now look at the search operations!

| | |
|---|---|
| 0 | 89990 |
| 1 | |
| 2 | 5672 |
| 3 | |
| 4 | 234 |
| 5 | |
| 6 | 9876 |
| 7 | 6777 |
| 8 | |
| 9 | |

*Search for key 234*
*Apply the hash function*
*234 %10 = 4*
*Go to location 4*
*Found!*
*O(1) time complexity!*

*Search for key 878*
*Apply the hash function*
*878 %10 = 8*
*Go to location 8*
*Not found!*
*O(1) time complexity!*

*Search for key 312*
*Apply the hash function*
*312 %10 = 2*
*Go to location 2*
*Not found!*
*O(1) time complexity!*

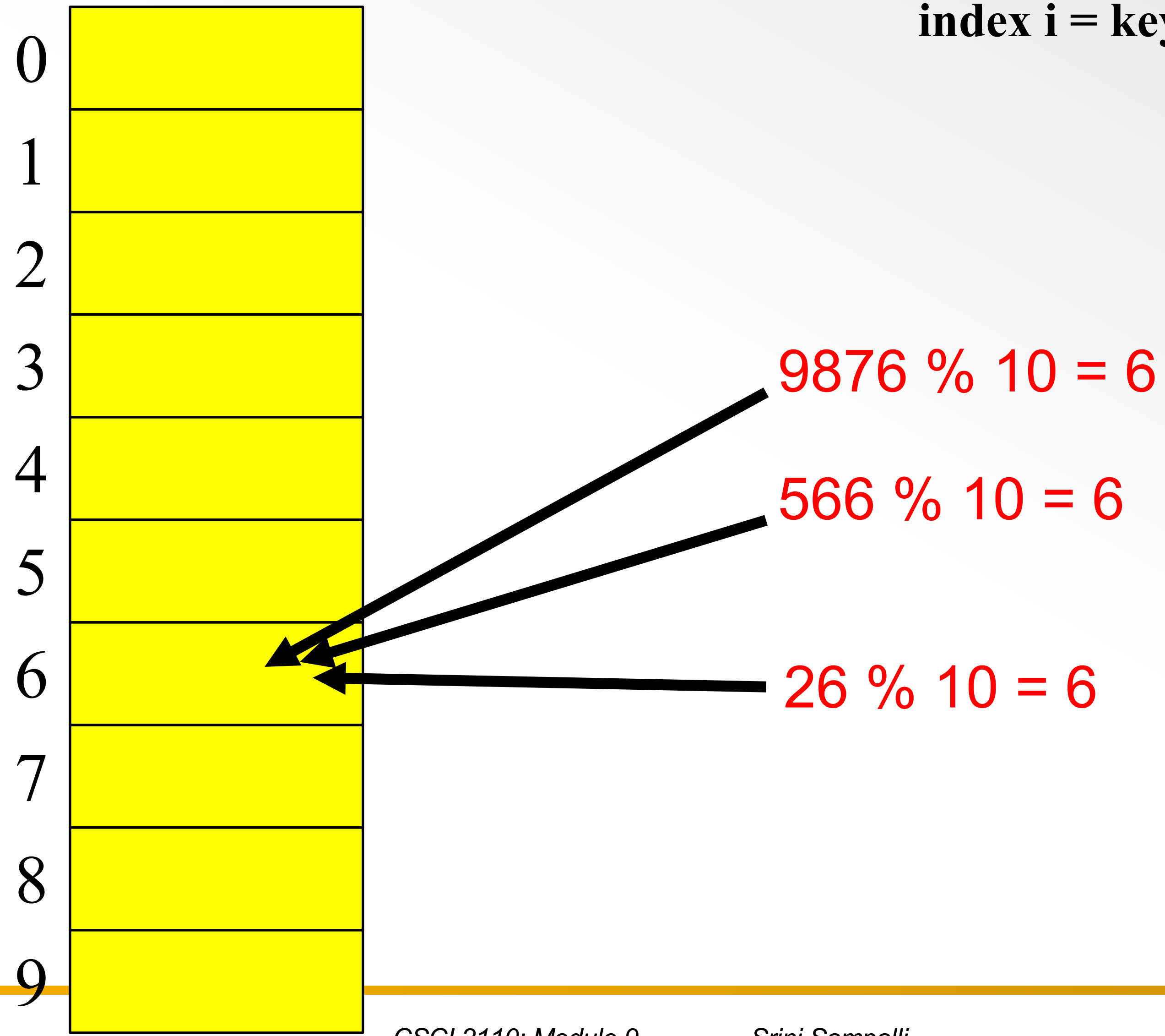
***Problem:***
***What if two or more keys map to the same index?***

This is bound to happen since we are mapping a large range of keys into a small table.

# Example

***Keys:***
***9876, 566 and 26***

**index i = key % 10**




# The Hash Clash Problem

***If two or more keys map to the same index, it is called a Hash Clash or a Hash Collision.***

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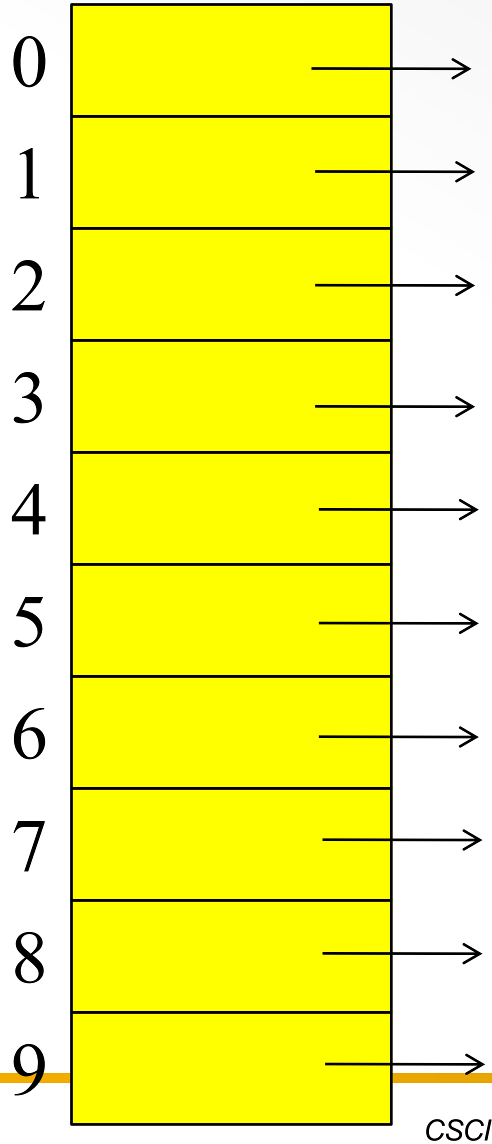
***If two or more keys map to the same index, it is called a Hash Clash or a Hash Collision.***

***Two Solutions:***

- ***OPEN HASHING (SEPARATE CHAINING):***
- ***CLOSED HASHING***

# Open Hashing or Separate chaining

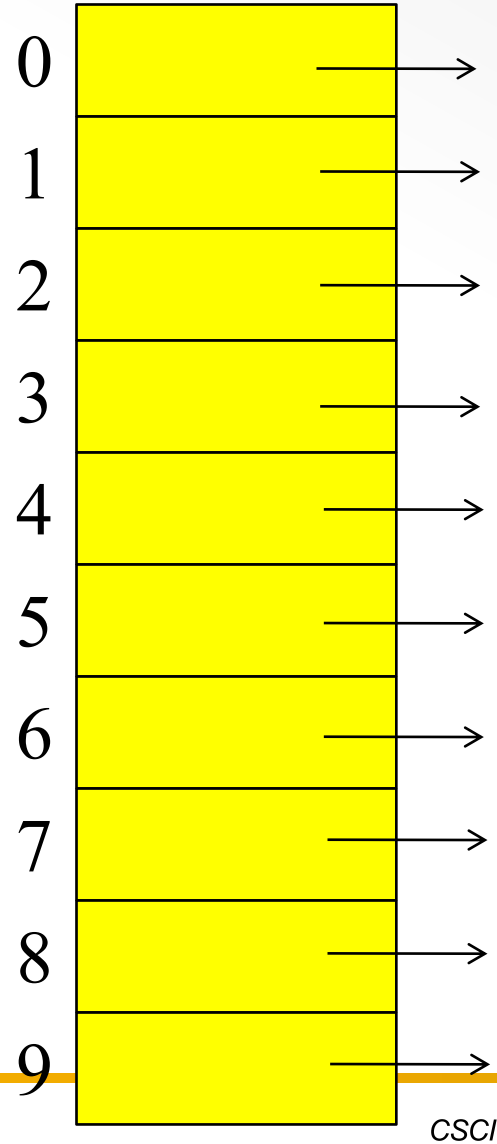
## Concept: Create an ArrayList of Linked Lists




***Keys:***
***9876, 234, 5672, 89990, 6777, 366, 7446, 212, 4676***

# Open Hashing or Separate chaining

## Concept: Create an ArrayList of Linked Lists

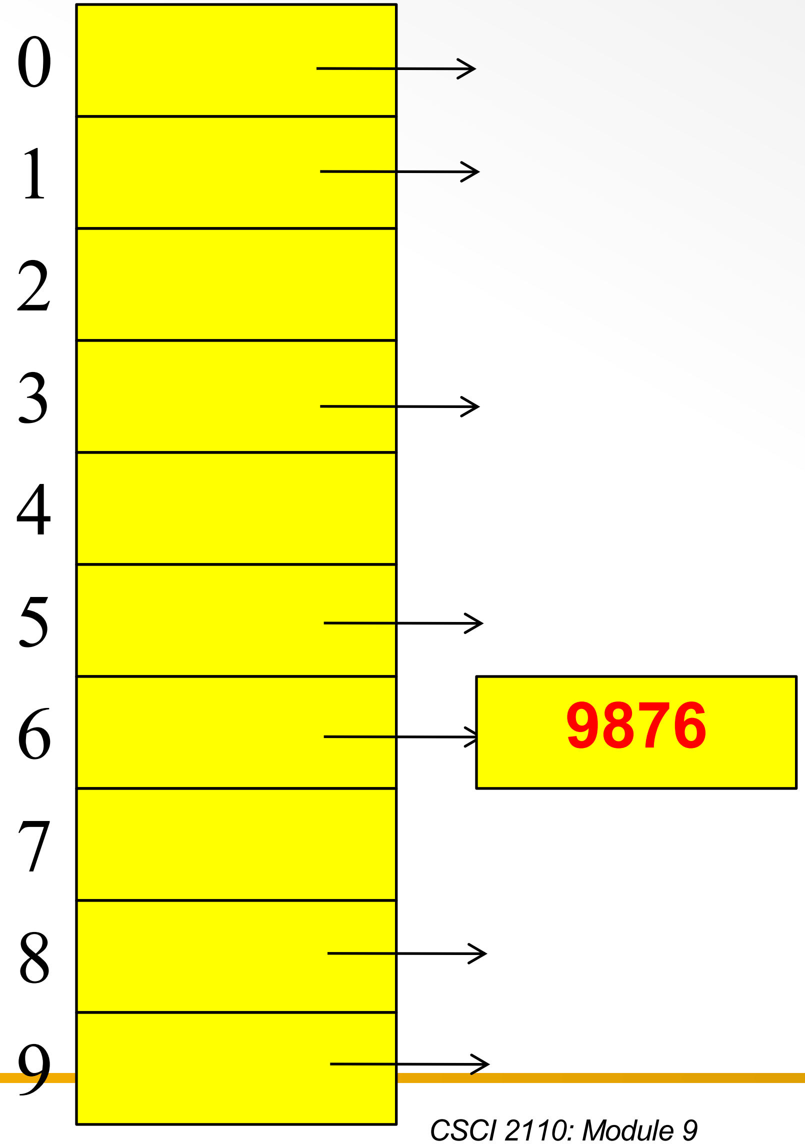



***Keys:***

***9876, 234, 5672, 89990, 6777, 366, 7446, 212, 4676***

# Open Hashing or Separate chaining

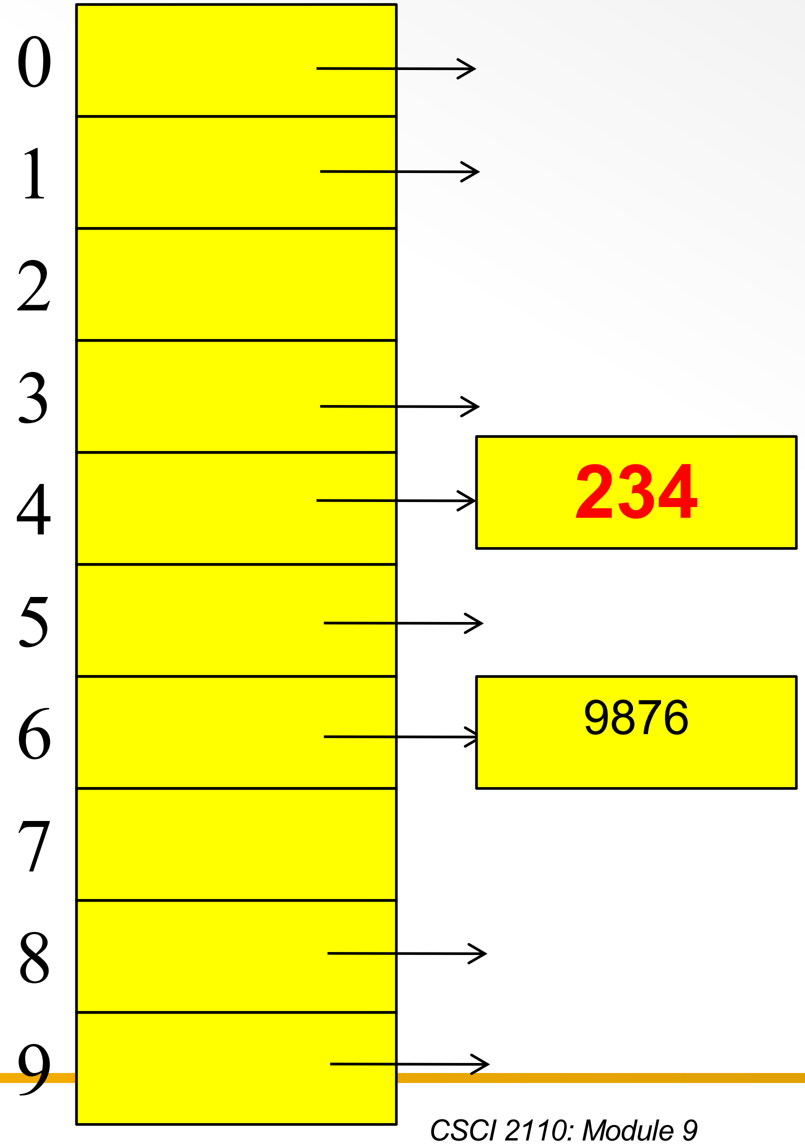
## Concept: Create an ArrayList of Linked Lists




***Keys:***

***9876, 234, 5672, 89990, 6777, 366, 7446, 212, 4676***

# Open Hashing or Separate chaining
## Concept: Create an ArrayList of Linked Lists




***Keys:***
***9876, 234, 5672, 89990, 6777, 366, 7446, 212, 4676***

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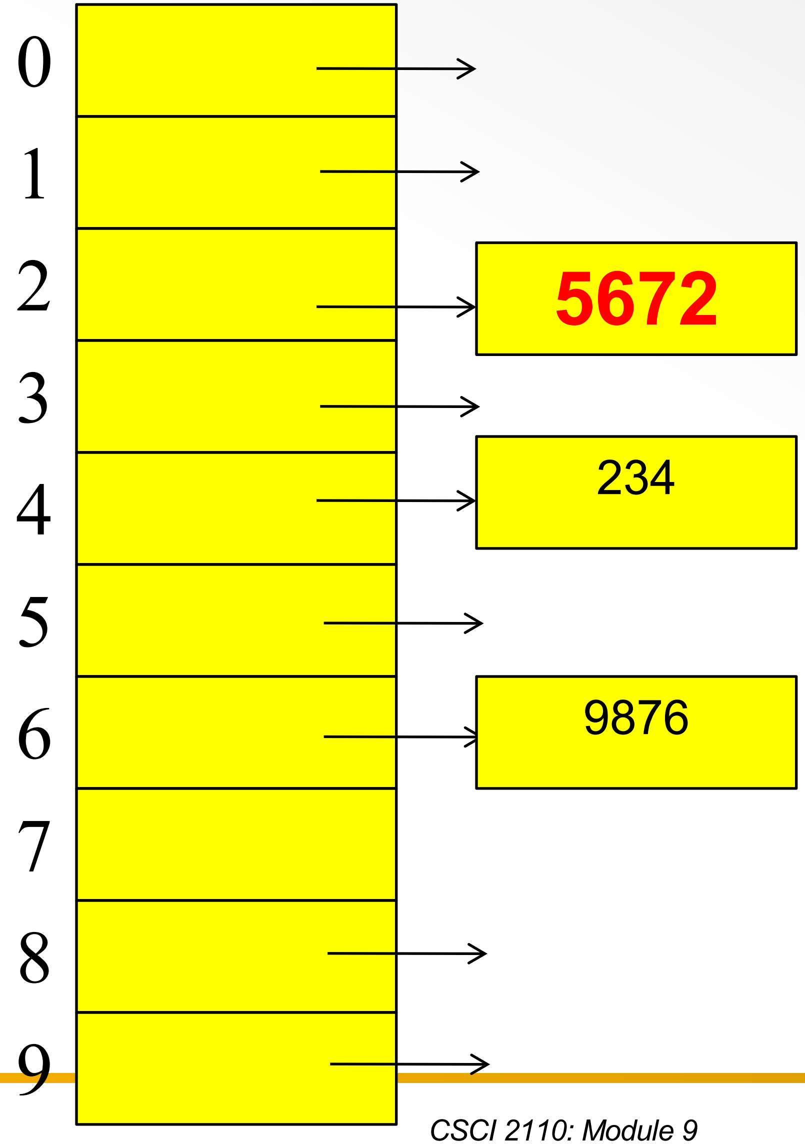
## Concept: Create an ArrayList of Linked Lists




***Keys:***

***9876, 234, 5672, 89990, 6777, 366, 7446, 212, 4676***

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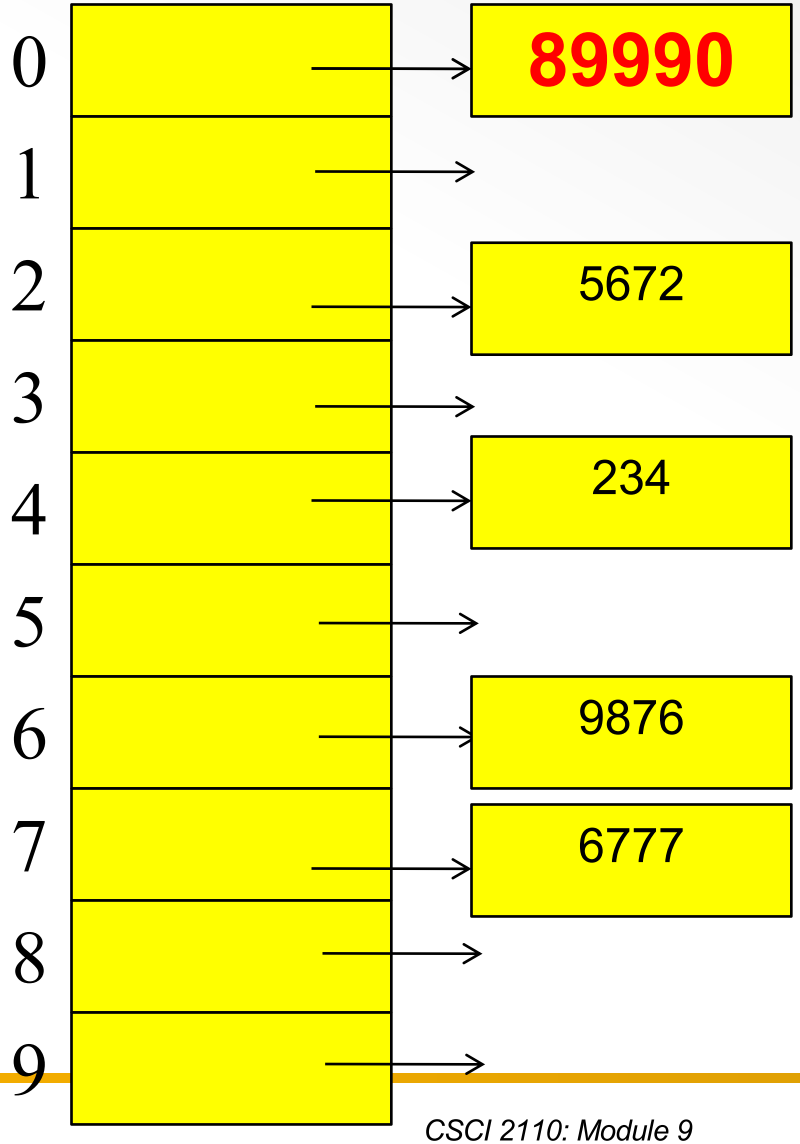
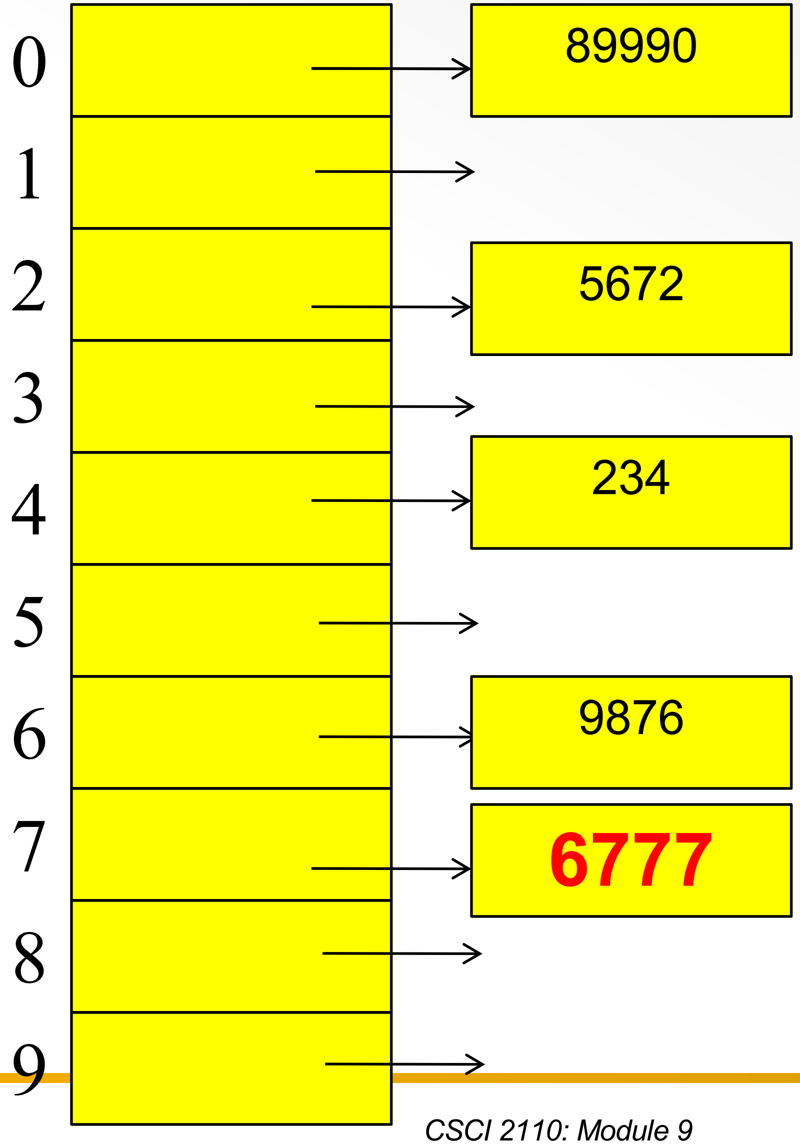
## Concept: Create an ArrayList of Linked Lists




***Keys:***

***9876, 234, 5672, 89990, 6777, 366, 7446, 212, 4676***

# Open Hashing or Separate chaining
## Concept: Create an ArrayList of Linked Lists

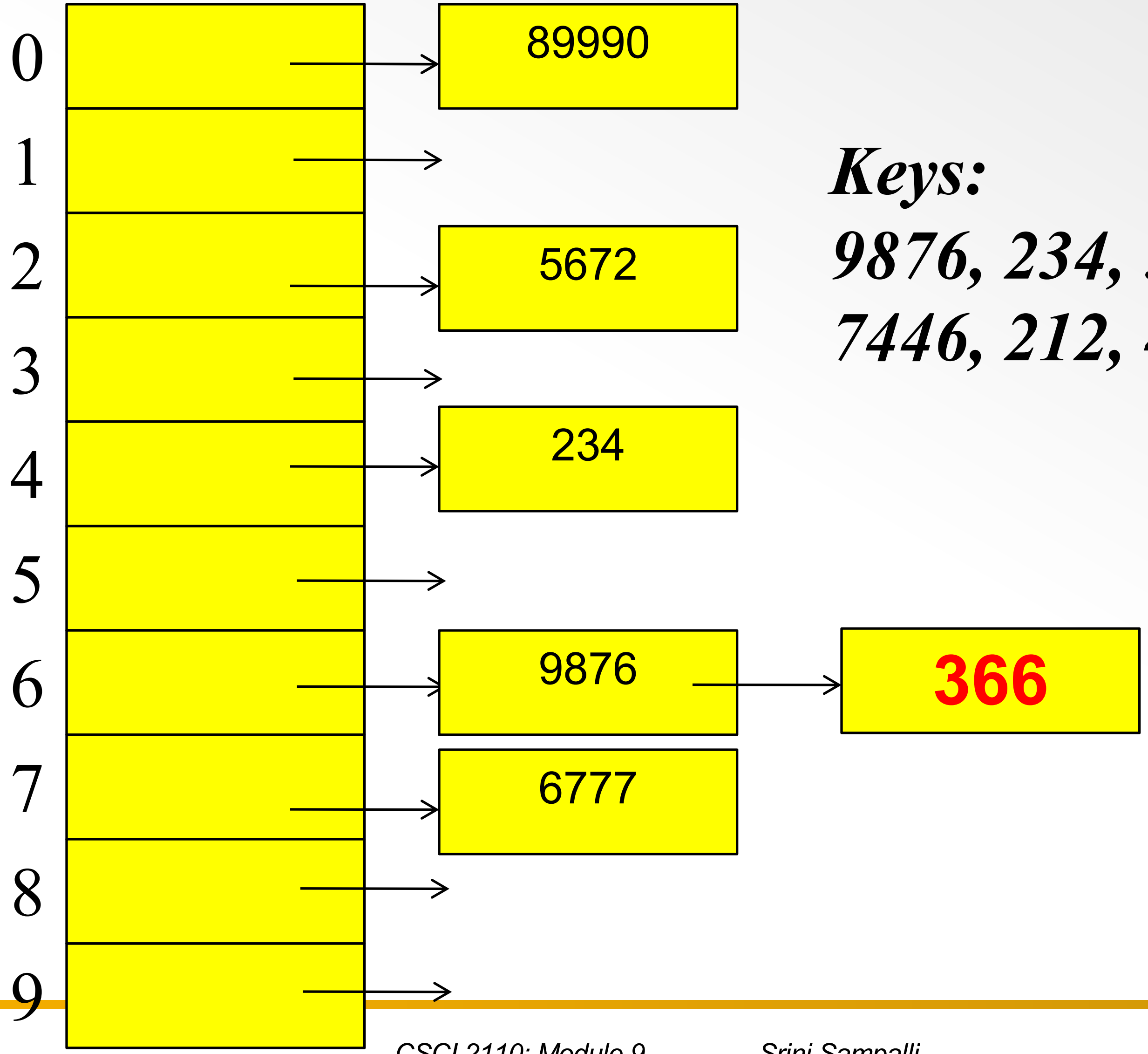



***Keys:***

***9876, 234, 5672, 89990, 6777, 366, 7446, 212, 4676***

# Open Hashing or Separate chaining

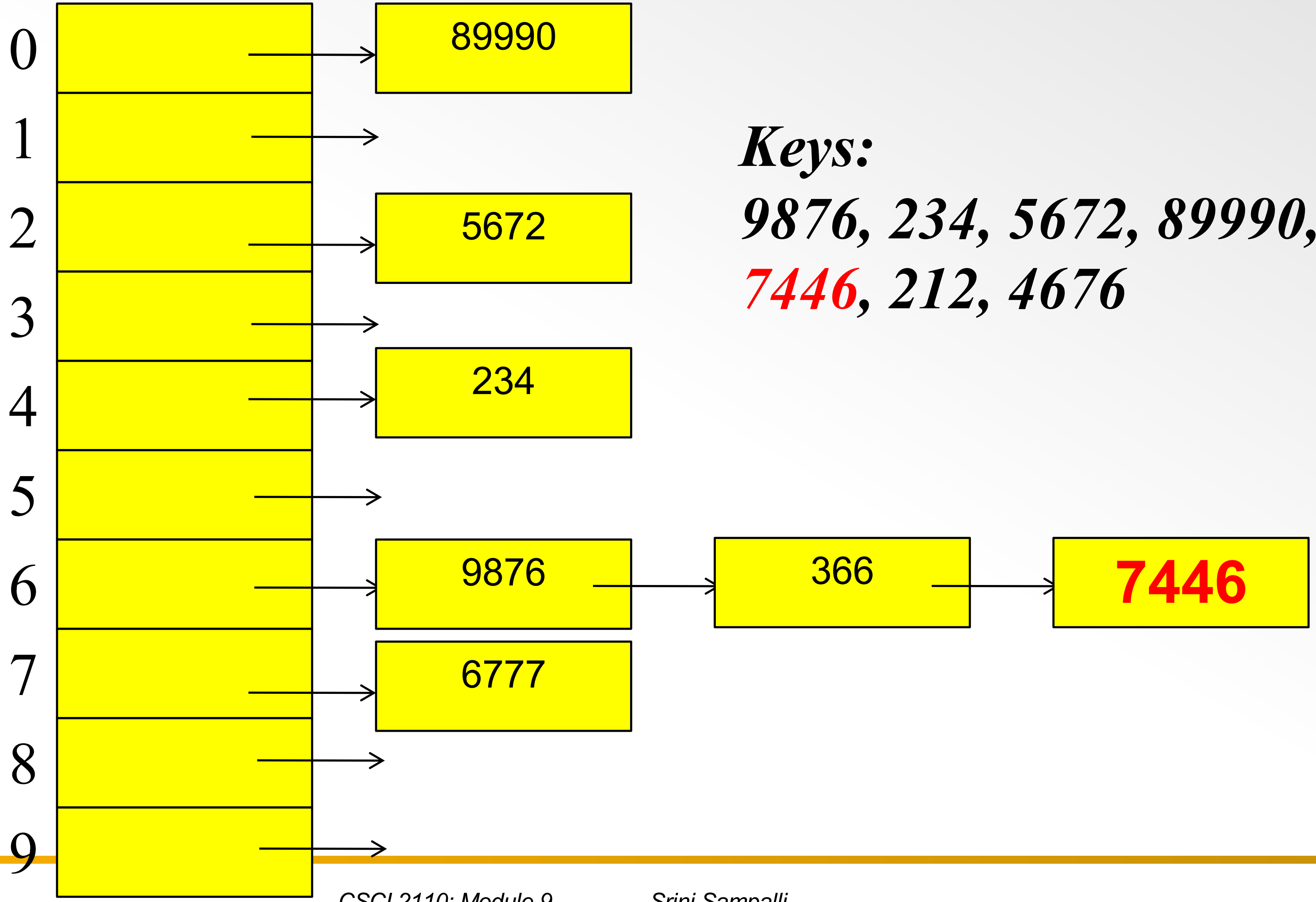
## Concept: Create an ArrayList of Linked Lists




***Keys:***

***9876, 234, 5672, 89990, 6777, 366, 7446, 212, 4676***

# Open Hashing or Separate chaining
## Concept: Create an ArrayList of Linked Lists

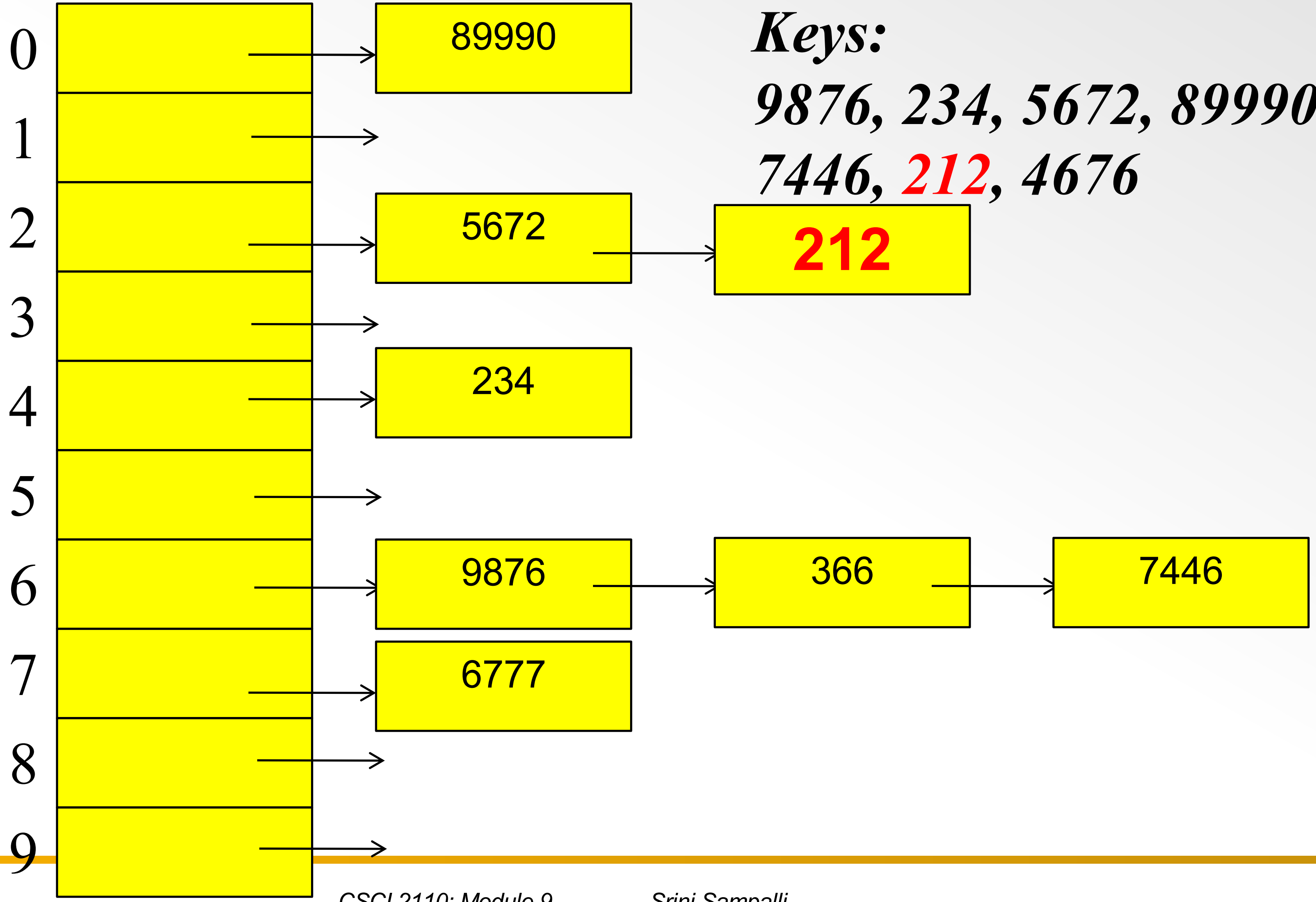



***Keys:***

***9876, 234, 5672, 89990, 6777, 366, 7446, 212, 4676***

# Open Hashing or Separate chaining

## Concept: Create an ArrayList of Linked Lists




***Keys:***

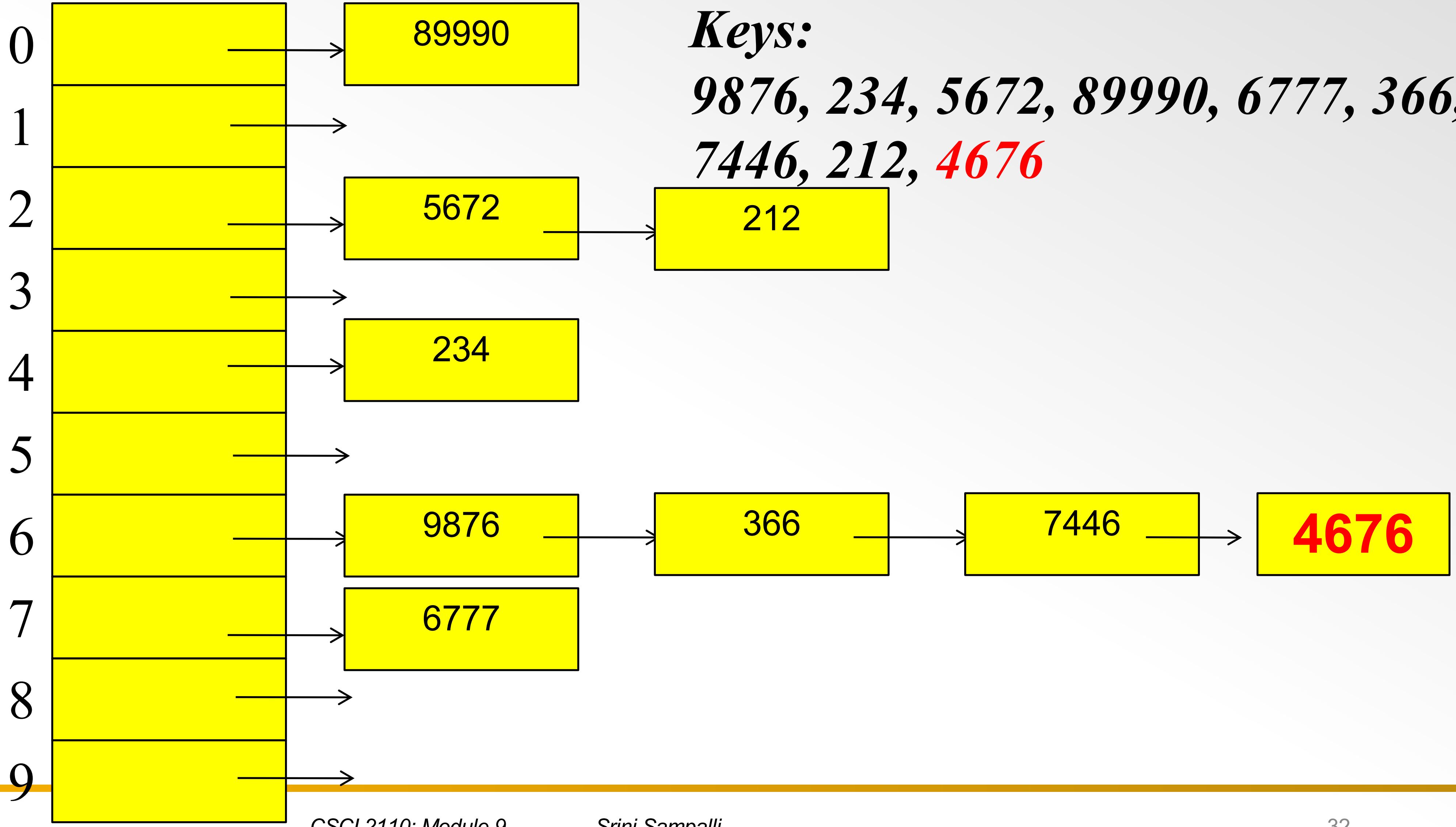
***9876, 234, 5672, 89990, 6777, 366, 7446, 212, 4676***

# Open Hashing or Separate chaining

## Concept: Create an ArrayList of Linked Lists

***Keys:***

***9876, 234, 5672, 89990, 6777, 366, 7446, 212, 4676***



## MORE EXAMPLES WITH OPEN HASHING

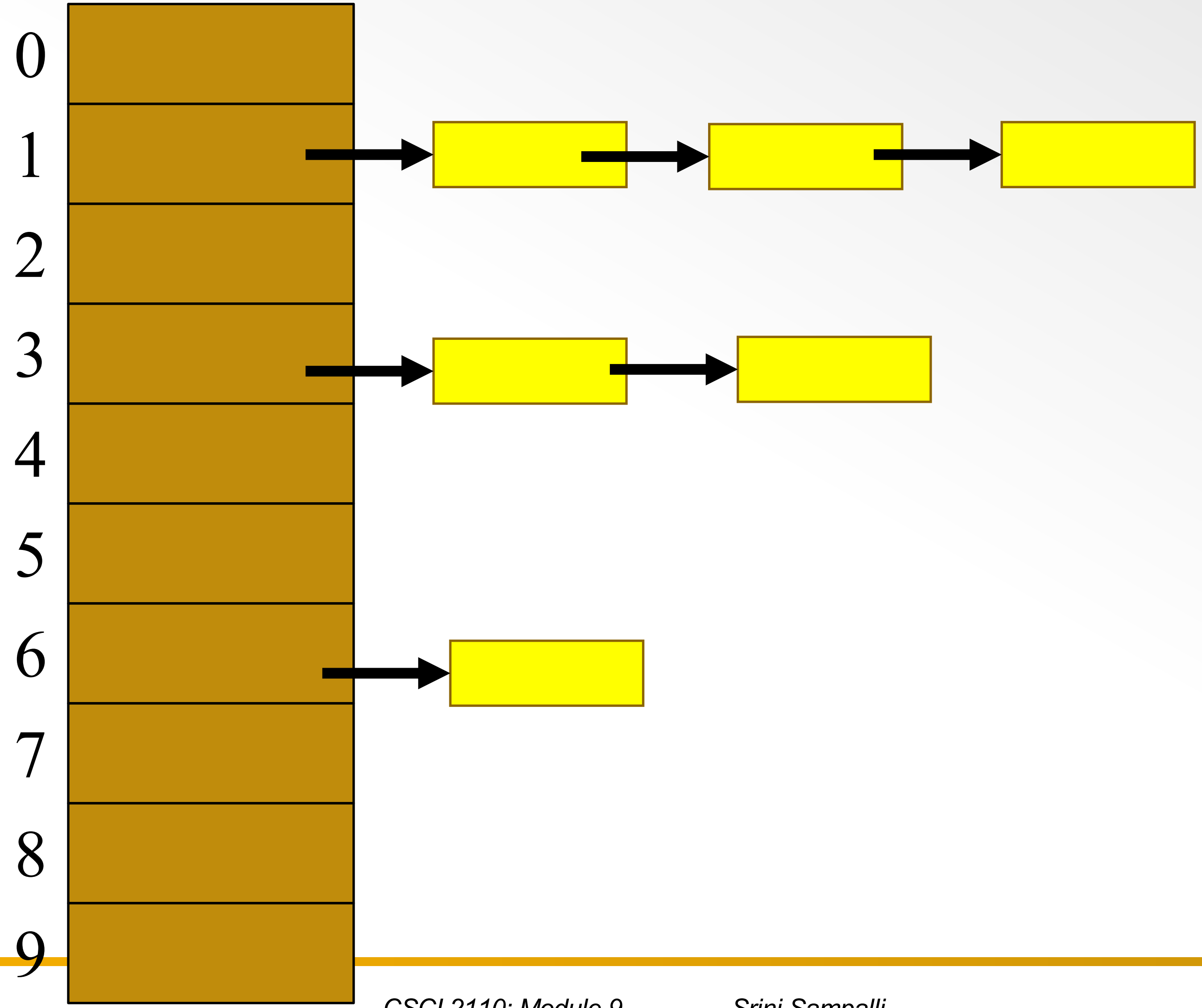
1. **Draw the hash table with open hashing resulting from the insertion of the following sequence of keys**
   **50, 700, 76, 85, 92, 73, 101**
   **Assume that the size of the table is 7 and the hash function is key mod 7**

**2. Draw the hash table with open hashing resulting from the insertion of the following sequence of keys (words):**
**SKILL, ABILITY, KNOWLEDGE, HARDWORK, ATTITUDE**

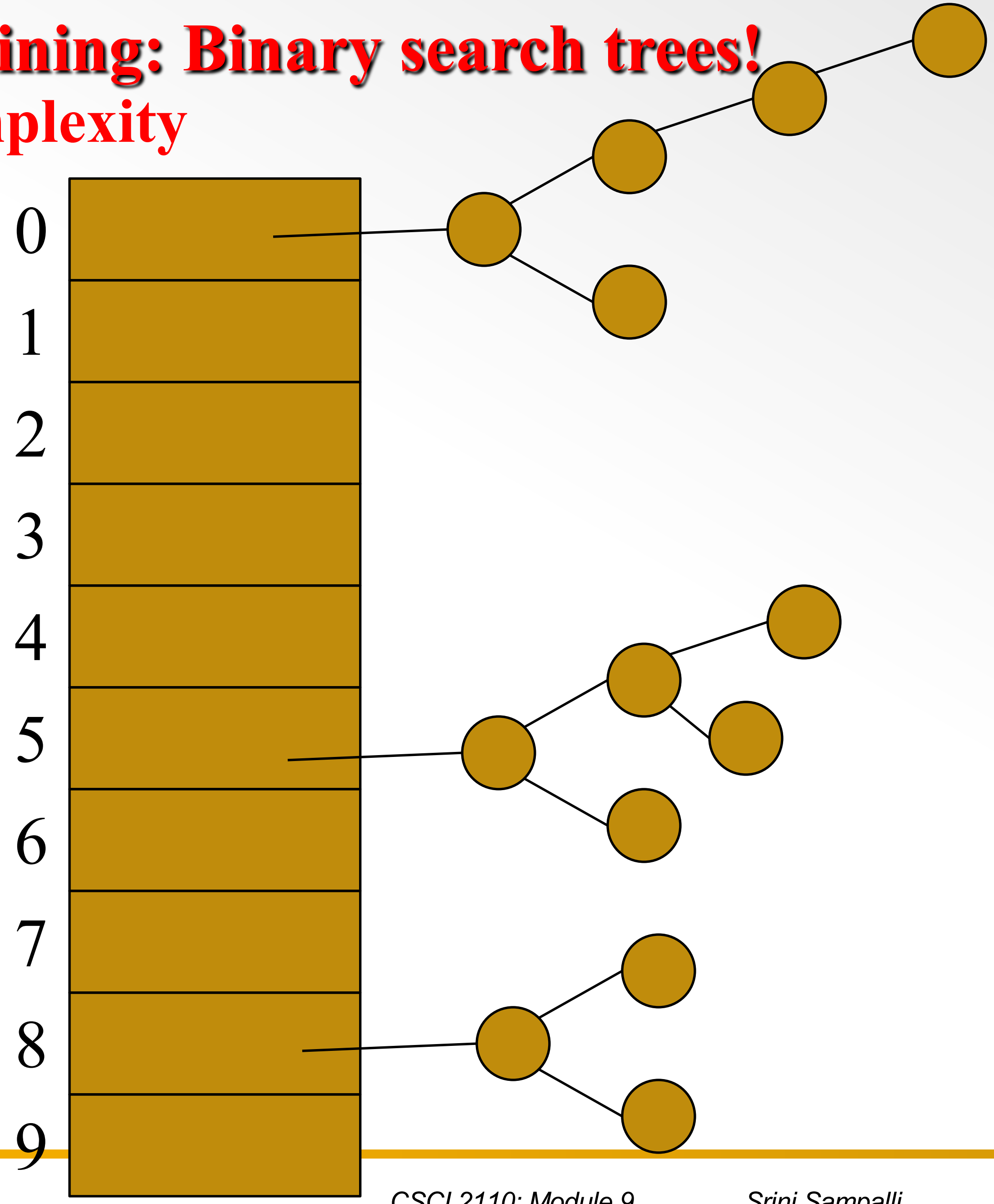
**Assume that the size of the table is 5 and the hash function is defined as follows: add the positions of the word's letters in the alphabet (A=1, B=2, etc.) and compute the sum's remainder after division by 5.**

| | | |
|---|---|---|
| **SKILL** | **= 19 + 11 + 9 + 12 + 12** | **= 63** |
| **ABILITY** | **= 1 + 2 + 9 + 12 + 9 + 20 + 25** | **= 78** |
| **KNOWLEDGE** | **= 11 + 14 + 15 + 23 + 12 + 5 + 4 + 7 + 5** | **= 96** |
| **HARDWORK** | **= 8 + 1 + 18 + 4 = 23 + 15 + 18 + 11** | **= 98** |
| **ATTITUDE** | **= 1 + 20 = 20 + 9 + 20 + 21 + 4 + 5** | **= 100** |

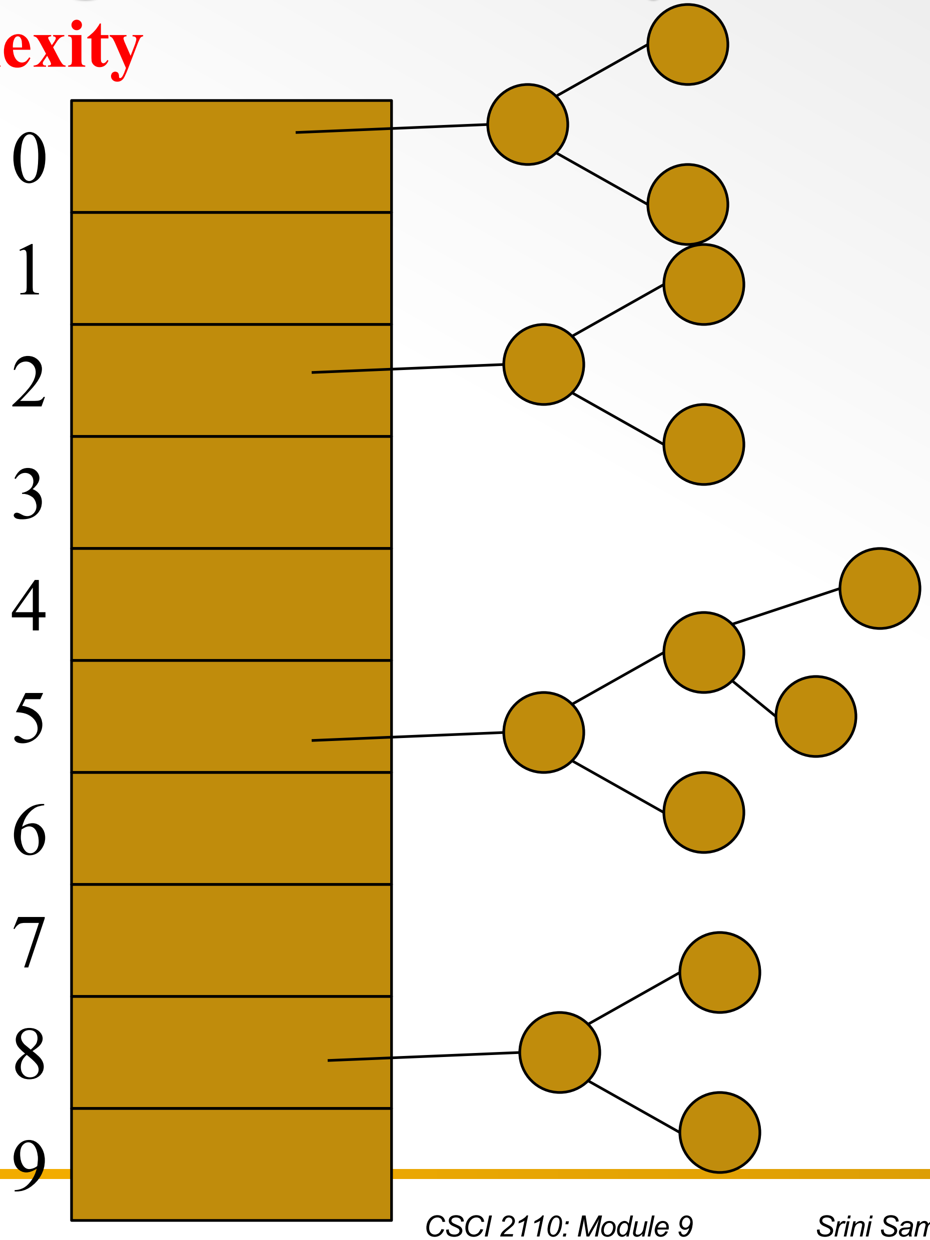
# Separate chaining: An arraylist of linked lists - Search Complexity



# Separate chaining: Binary search trees!
## - Search Complexity



# Separate chaining: Balanced Binary Search Trees!
## - Search Complexity



## 2. Closed Hashing

*Concept:* Hash tables store keys directly within the array. A collision is resolved by trying alternative locations within the array until a free spot is found.

There are two important techniques for trying alternative locations:

a) Linear Probing
b) Quadratic Probing

### ***a) Linear Probing:***

Find the index i where the key should be mapped using the hash function.

If a collision occurs at index i, try index i+1, i+2, i+3, etc. until an empty spot is found.

If you reach the end of the array, wraparound.

*Example 1:*

*Example 2:*

*Problems with linear probing:*

## b) Quadratic Probing:

Find the index i where the key should be mapped using the hash function.

If a collision occurs at index i, try index $i+1^2$, $i+2^2$, $i+3^2$, etc. until an empty spot is found.

If you reach the end of the array, wraparound.

*Example 1:*

*Example 2:*

*Example 3:*

## HASH FUNCTIONS

We mainly considered a table of size 10 and a hash function h(key) = key % 10.
In general, we can write the hash function as

h(key) = key%N where N is the table size.

***Example:*** Insert the keys 9, 22, 10, 31, and 48 using the hash function h(key) = key%N where N = 16 (table size).

The above is just one example of a hash function.
A host of other hash functions can be used.

In general, a good hash function:

- Must have O(1) time complexity
- Must distribute keys uniformly (should not favour one location over another).

## APPLICATIONS OF HASHING IN CRYPTOGRAPHY

1. **PASSWORD STORAGE**

2. **FILE MODIFICATION DETECTOR**

3. **DIGITAL SIGNATURES**

4. **BLOCKCHAIN**

**LOAD FACTOR**

**TIME COMPLEXITY OF HASHING**

Best case complexity for search:

Worst case complexity for search:

Average case complexity for search (depends upon the load factor):

**REHASHING**

When more and more keys are inserted, the current table size may not suffice. At that time, a *rehashing* is done – a new table double its size is created, and all the keys are remapped to the new table.

**DOUBLE HASHING**

The key is hashed twice to map it.

## HASH MAP IN JAVA

Java has a class called HashMap (java.util.HashMap) that lets you store and process keys and values associated with keys in a hash table.

Some important methods in the HashMap class

| | |
|---|---|
| put(K key, V value) | Adds a key and value to the hash table |
| get (K key) | Returns the value associated with the key; returns null if the key has no value associated with it. |
| containsKey(K key) | Returns true if the key is found |
| values() | Returns a collection of values in the table |
| keySet() | Returns the set of keys in the table |
| remove(K key) | Deletes the entry with key K |
| isEmpty() | Returns true if table is empty |

```java
//Simple illustration of hashmap in java
//Creates a hashmap of student id numbers as keys and student names as records.
import java.util.HashMap;
import java.util.Scanner;
import java.io.*;

public class HashMapDemo
{
	public static void main(String[] args) throws IOException
	{
		HashMap<Integer, String> studentRecord = new HashMap<Integer,String>();
		Integer id;
		String name;
		Scanner keyboard = new Scanner(System.in);
		System.out.print("Enter the filename to read from: ");
		String filename = keyboard.nextLine();

		File file = new File(filename);
		Scanner inputFile = new Scanner(file);

		while (inputFile.hasNext())
		{
			id = Integer.parseInt(inputFile.next());
			name = inputFile.nextLine();
			studentRecord.put(id, name);
		}

		inputFile.close();
		System.out.println(studentRecord.values());
		System.out.println(studentRecord.keySet());

		System.out.print("Enter key: ");
		id = keyboard.nextInt();
		if (studentRecord.containsKey(id)){
			name = studentRecord.get(id);
			System.out.println(id + "\t" + name  + "  found");
		}
		else
			System.out.println(id + " not found");
	}
}
```

Input file:

```
10245 James
23450 Jack
10398 Amar
10009 Boris
51430 Amy
69087 Brenda
88700 Zirui
67568 Xu
22229 Nick
17171 Chandra
```

Sample Run:

```
Enter the filename to read from: students.txt
[ Chandra,  Nick,  Xu,  Jack,  Boris,  Amar,  Amy,  James,  Brenda,  Zirui]
[17171, 22229, 67568, 23450, 10009, 10398, 51430, 10245, 69087, 88700]

Enter the search key: 10398
10398    Amar found

Process completed.
```

**Two alternative classes: TreeMap and LinkedHashMap**

*TreeMap stores the records in a tree data structure and retrieves them in a sorted order. Default sorting is done on the keys.*

import java.util.TreeMap;
…

TreeMap<Integer,String> studentRecord = new TreeMap<Integer, String>();

Sample run:
Enter the filename to read from: students.txt
[ Boris, James, Amar, Chandra, Nick, Jack, Amy, Xu, Brenda, Zirui]
[10009, 10245, 10398, 17171, 22229, 23450, 51430, 67568, 69087, 88700]

Enter the search key: 23450
23450 Jack found

*LinkedHashMap stores the records using a doubly linked list. Retrieval is in the order of insertion.*

import java.util.LinkedHashMap;
…

LinkedHashMap<Integer,String> studentRecord = new LinkedHashMap<Integer, String>();

Sample run:
Enter the filename to read from: students.txt
[ James, Jack, Amar, Boris, Amy, Brenda, Zirui, Xu, Nick, Chandra]
[10245, 23450, 10398, 10009, 51430, 69087, 88700, 67568, 22229, 17171]

Enter the search key: 10009
10009 Boris found

```
import java.util.*;

public class HashMapDemo4 {

    public static void main(String [] args) {


      Scanner keyboard = new Scanner(System.in);

      // create empty map
      HashMap<String,ArrayList<Integer>> data =
          new HashMap<String,ArrayList<Integer>>();

      // read words, one on each line, until “end”
      String word = keyboard.next();
      int lineNumber = 1;
      while (!word.equals("end")) {

       // if the word is not yet in map
            if (data.get(word) == null) {

            // create a new list and add it to the map
            ArrayList<Integer> lines = new ArrayList<Integer>();
            lines.add(lineNumber);
            data.put(word,lines);
          }
          else {

            // get the existing list for this word
            ArrayList<Integer> lines = data.get(word);

            // add the line number to that list
            lines.add(lineNumber);
          }

          // get the next word
          word = keyboard.next();
          lineNumber += 1;
      }

      // print the map
      for (String w : data.keySet()) {
          System.out.println(w + " - " + data.get(w));
      }

    }

}
```