SOLUTIONS

CSCI 2110
DATA STRUCTURES AND ALGORITHMS

TOPICS AND PRACTICE QUESTION BANK FOR FINAL EXAM

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Last Name M-P: Rowe 1020
Last Name Q-Z: Killam MacMechan Aud.

NOTE:
FINAL EXAM IS ON MONDAY, DECEMBER 18th, AT 12 NOON
DURATION: 2 HOURS 30 MINUTES
LOCATION: ROWE 1028, ROWE 1020, AND KILLAM MACMECHAN AUDITORITUM
FORMAT: CLOSED BOOK, PROCTORED EXAM ON BRIGHTSPACE WITH RESPONDUS LOCKDOWN BROWSER

DETAILED BREAKDOWN OF TOPICS

Module 5: RECURSION

- What is a recursive definition?
- Recursion: Definition, Advantages, Base and Glue Cases
- Examples: Writing recursive methods
- Examples: Tracing recursive methods

Module 6: BINARY TREES

- Definition and Terminology
- Strictly binary and complete binary trees
- Traversals: Preorder, Inorder, Postorder, Level Order
- Recursive definitions for a binary tree
- Binary Tree Class: Methods and Complexities
- Application: Huffman coding

Module 7: BINARY SEARCH TREES

- Definition and Examples
- Operations – Search, Insert and Delete
- Binary Search Tree Class: Methods and Complexities

Module 8: HEAPS

- Definition and Examples
- Operations – Insert and DeleteMax
- Heap Class: Methods and Complexities
- HeapSort
- Updatable Heaps

Module 9: HASHING AND HASH TABLES

- Motivation
- Hashing, Hash Function and Hash Table Concept with Examples
- Hash Clash
- Solution to Hash Clash: Open Hashing or Separate Chaining
- Solution to Hash Clash: Closed Hashing – Linear Probing and Quadratic Probing
- Miscellaneous Concepts in Hashing
- HashMap in Java with examples

Module 10: GRAPHS

- Definition and Motivation
- Graph Terminology with Examples
- Graph Representation: Adjacency Matrix and Adjacency List
- Graph Algorithms
- Graph Traversals: Depth First and Breadth First
- Topological Sorting
- Shortest Path Algorithms

NOTES:

1. For the, BinaryTree class, BinarySearchTree class, and Heap class, you should know how the methods work. For programming questions pertaining to these classes, you will be given the list of methods from the class and their meanings. You may be asked to develop other methods using these methods. You may be given code snippets and asked to fill in the missing lines of code. You may also be given code snippets and asked to trace the output.

2. You will not be asked to draw any diagrams on the Brightspace exam portal. Instead, you may have to draw diagrams on a scratch sheet of paper that will be supplied to you, and then enter the answers in text on the Brightspace exam portal.

3. No cheat sheets or any reference material allowed.

4. You may bring a basic non-programmable calculator. A calculator will also be enabled on the Lockdown Browswer.

CSCI 2110 DATA STRUCTURES AND ALGORITHMS
PRACTICE QUESTION BANK FOR THE FINAL EXAM

Question 1. Multiple Choice Set - For each of the following questions, select the most appropriate answer:

1. Suppose that a recursive method implements the recursive definition given below:

```
square (1) = 1
square (N) = square (N-2) + 2*N – 1
```

Assuming that the definition has been implemented correctly, how many times will the method be called if the main method calls square(5)?

   a. 5
   b. 3
   c. 6
   d. 1

Square (5)
↳ Square (3)
↳ Square (1)

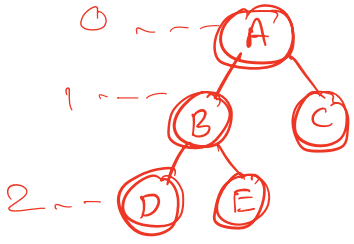
2. The number of branches from the root to a node in a binary tree is called
   a. height of the node
   b. depth of the node
   c. height of the root
   d. depth of the root

3. A binary tree, in which all levels are filled, except possibly the last level, which is filled from left to right, is called a
   a. Binary Search Tree
   b. Complete Binary Tree
   c. Strictly Binary Tree
   d. Leveled Tree

7. Which of the following statements about binary trees is NOT true?
   a. Every binary tree has at least one node.
   b. Every non-empty tree has exactly one root node.
   c. Every node has at most two children.
   d. Every non-root node has exactly one parent.

The next three questions pertain to a binary tree with 5 nodes. The root is A. A has a left child B and right child C. B has a left child D and a right child E. There are no other nodes.

8. Which of the following traversals yields ABCDE?
   a. Inorder
   b. Level order
   c. Post order
   d. Pre order
   e. Two of the above



9. Which of the following is an inorder traversal of the tree?
   a. ABCDE
   b. ABDEC
   c. DBEAC
   d. DEBAC
   e. None of the above

Inorder : Left - Root - Right

10. The height of the tree is

a. 0
b. 1
c. 2
d. 3
e. None of the above

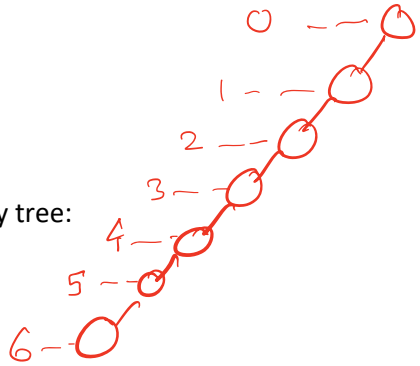
Inorder: L - Root - R
Preorder: Root - L - R
Postorder: L - R - Root

11. Which of the following properties are obeyed by all three tree – traversals?

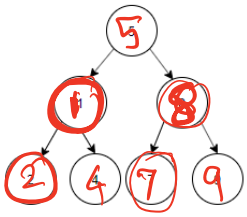
a. Left subtrees are visited before right subtrees
b. Right subtrees are visited before left subtrees
c. Root node is visited before left subtree
d. Root node is visited before right subtree

12. Suppose you have a binary tree with exactly 7 nodes. What is the maximum height that the tree could have?

a. 2
b. 3
c. 6
d. 7



13. Consider the following binary tree:



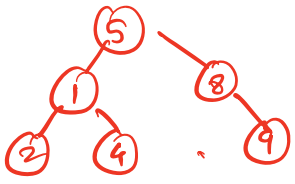
Which of the following statements is true?

I. It is a complete binary tree. ✓
II. It is a strictly binary tree. ✓
III. It is a binary search tree. ✗

a. I only
b. I and II
c. I, II and III
d. I and III

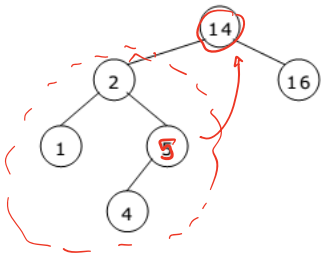
14. Suppose node with key 7 is NOT THERE in the binary tree in the above question, then which of the following statements is true?

I. It is a complete binary tree. ✗
II. It is a strictly binary tree. ✗
III. It is a binary search tree. ✗



a. I only
b. I only
c. III only
d. I and III
e. None of the statements is true.

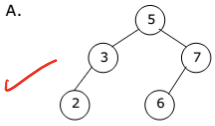
15. For the binary search tree shown below, suppose the root is deleted and replaced with **a node in** the left subtree of the root, what will be the new root?



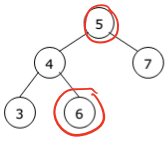
a. 1
b. 2
c. 4
d. 5

16. Which among the following is not a binary search tree?

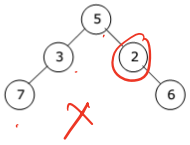
A.



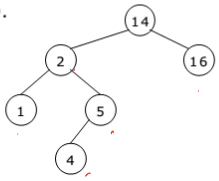
C.



B.



D.



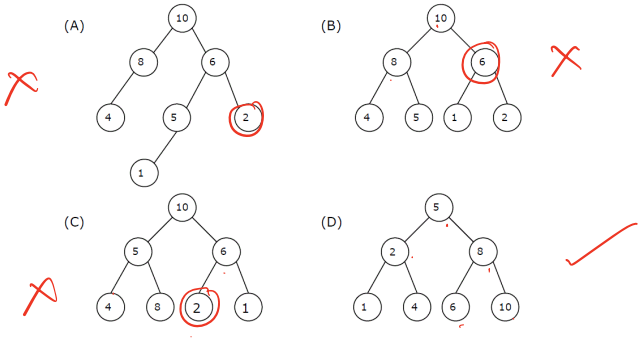
a. A and D
b. B only
c. C only
d. B and C

17. Which of the following statements is true for a binary search tree?

I. It must be a complete binary tree.
II. It must be a strictly binary tree.
III. The key x at a node must be larger than ~~or equal~~ to all the keys in the left subtree of the node.
IV. The key x at a node must be smaller than all the keys in the right subtree of the node.

a. I, II and III
b. I, II and IV
c. III only
d. III and IV

18. Which of the following is a binary search tree?



a. A
b. B
c. C
d. D
e. C and D

19. Which of the following statements is true for a heap?

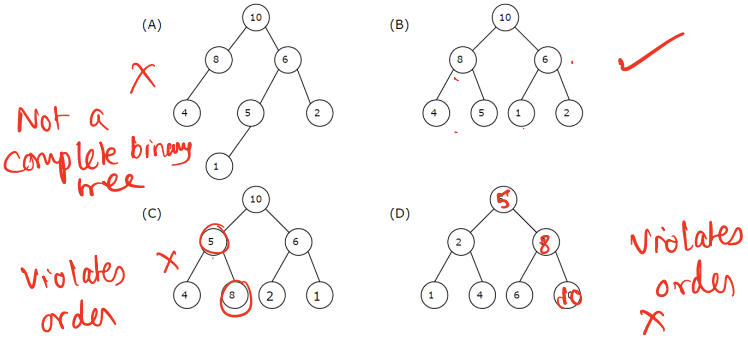
I. It must be a complete binary tree.
II. It must be a strictly binary tree.
III. The key x at a node must be larger than or equal to all the keys in the left and right subtrees of the node.
IV. The key x at a node must be larger than all the keys in the left and right subtrees of the node.

a. I, II and III
b. I, II and IV
c. I and III
d. I and IV

20. What is the worst-case time complexity of building a heap with n items?

a. O(n)
b. O(log n)
c. O(1)
d. O(n log n)

21. Which of the following is a heap?



a. B only
b. B, C and D
c. A and B
d. B and C
e. A and C

22. Consider the following heap:

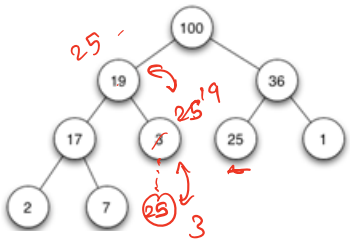


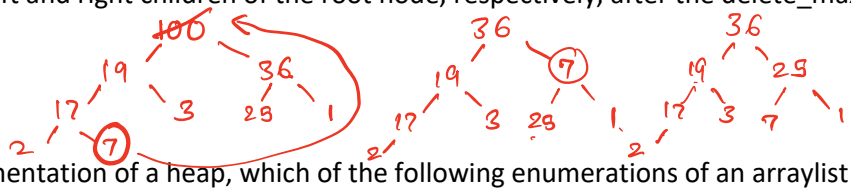
Figure 1

Suppose key = 25 is inserted according to the Sift-up algorithm. What are the keys in the left and right children, respectively, of the root node after the insertion?

a. 19 and 25
b. 25 and 19
c. 25 and 36
d. 36 and 25

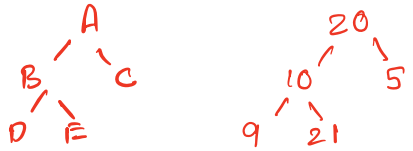
23. Consider the same heap shown in Figure 1. Suppose we do a delete_max operation according to the Sift-down algorithm. What are the left and right children of the root node, respectively, after the delete_max?

a. 19 and 25
b. 25 and 19
c. 25 and 36
d. 36 and 25



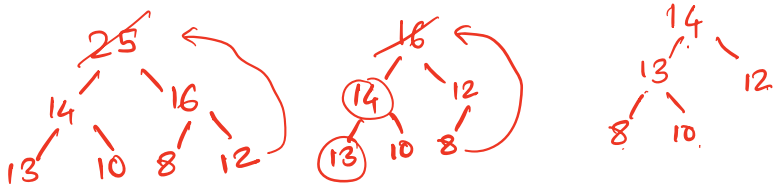
24. Assuming an arraylist implementation of a heap, which of the following enumerations of an arraylist represents a valid heap? (For keys which are letters/strings, assume that lower the alphabet in lexicographic ordering, lower the priority).

a. [A, B, C, D, E]
b. [10, 10, 10, 10, 10]
c. [20, 10, 5, 9, 21]
d. None of the above



25. Suppose a heap is represented by the following arraylist: [25, 14, 16, 13, 10, 8,12]. What will be the content of the arraylist after two successive delete_max operations using the Sift-down algorithm?

a. [16, 13, 10, 8, 12]
b. [14, 13, 12, 8, 10]
c. [14, 13, 8, 10, 12]
d. [16, 14, 13, 10, 8]



26. Given the following input (4322, 1334, 1471, 9679, 1989, 6171, 6173, 4199) and the hash function x mod 10, which of the following statements are true?

i. 9679, 1989, 4199 hash to the same value ✓
ii. 1471, 6171 hash to the same value ✓
iii. All elements hash to the same value ✗
iv. Each element hashes to a different value ✗

a. i only
b. ii only
c. iii only
d. iv only
e. i and ii

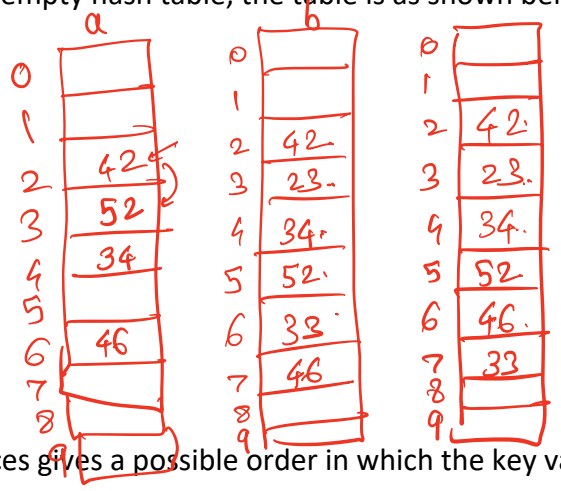
27. Given a hash table T with 25 slots that stores 2000 elements, the load factor α for T is _____

a. 80
b. 0.0125
c. 50,000
d. 1.25

$$\text{Load factor} = \frac{\#\text{ keys}}{\text{Table size}} = \frac{2000}{25} = 80$$

28. A hash table of length 10 uses open addressing with hash function h(k)=k mod 10, and linear probing. After inserting 6 values into an empty hash table, the table is as shown below.

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

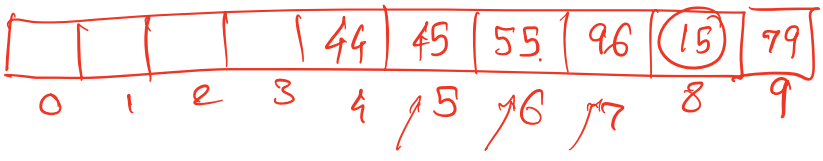


Which one of the following choices gives a possible order in which the key values could have been inserted in the table?

a. 46, 42, 34, 52, 23, 33 ✗
b. 34, 42, 23, 52, 33, 46 ✗
c. 46, 34, 42, 23, 52, 33 ✓
d. 42, 46, 33, 23, 34, 52 ✗

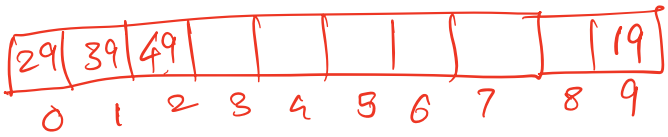
29. A hash function h defined h(key)=key mod 10, with linear probing, is used to insert the keys 44, 45, 79, 55, 96, 15 into a table indexed from 0 to 9. What will be the location of key 15?

a. 5
b. 6
c. 7
d. 8



30. Suppose keys 19, 29, 39 and 49 are hashed into a hash table of size 10 in sequence with a hash function key % table_size, and closed hashing with linear probing is used, at what location will 49 be placed?

a. 0
b. 1
c. 9
d. 2



31. In cryptographic hashing, which of the following is correct?

a. The hash of the password is transmitted to the server and the server compares the hash with the stored value.

b. Hashing is not used for password checking.

c. The server sends the hash of the password to the user and the user compares the hash with the stored value.

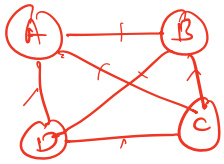
d. The password is transmitted to the server and the server computes the hash of the password.

32. An undirected graph G has four vertices. Its adjacency matrix representation has all 0's in its **MAIN** diagonal elements and all 1's in its non-diagonal elements. How many edges does the graph G have?

a. 3
b. 6
c. 8
d. 9

$$\frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$$

	A	B	C	D
A	0	1	1	1
B	1	0	1	1
C	1	1	0	1
D	1	1	1	0



33. Which of the following is statements is correct with respect to graph representation?

I. Adjacency list representation is better than adjacency matrix for sparse graphs (graphs with large number of vertices and less number of edges)
II. Finding whether there is an edge between any two nodes in a graph is easier with Adjacency List than with Adjacency Matrix.
III. Finding all the edges of a given vertex is easier with Adjacency Matrix than with Adjacency List.

a. I only
b. I and II
c. I and III
d. II only
e. III only

34. Which of the following data structures is useful to traverse a graph using breadth first search?

a. Unordered List
b. Binary Tree
c. Stack
d. Queue

35. Which of the following would be the Depth First Traversal of the following graph?

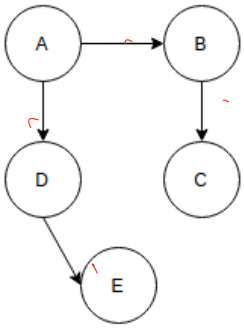
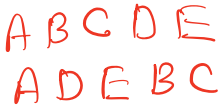
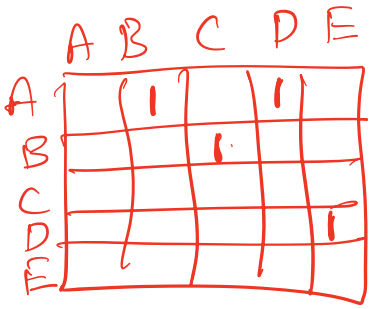


Figure 2

I. ABCDE
II. ADEBC
III. ABDCE
IV. ADBEC

a. I only
b. I and II
c. III only
d. III and IV





36. Consider the graph in Figure 2. How many zeros will be there in the adjacency matrix representation of the graph?

a. 4
b. 5
c. 21
d. 12

37. Consider the graph in Figure 2. The indegree and outdegree of vertex A are, respectively,

a. 1 and 1
b. 2 and 0
c. 0 and 2
d. 1 and 0

QUESTION NO. 2 (RECURSION)

2.1 Consider the following recursive method:

```
public static void showMe(int n){
      if (n>10)
            System.out.print(n);
      else
            showMe(n+3);
      }
}
```

Assuming that the main method has been implemented correctly, what will be displayed if the main method calls showMe(5)?

showMe(5)
↳ showMe(8)
↳ showMe(11)

Answer: 11

2.2

```
public static void showMe(int n){
      if (n<10){
            System.out.print(n + " ");
            showMe(n+3);
      }
      else
            return;
}
```

Answer: 5 8

Assuming that the main method has been implemented correctly, what will be displayed if the main method calls showMe(5)?

showMe(5)
↳ showMe(8)
↳ showMe(11)

2.3 Suppose that a recursive method implements the recursive definition given below:
func (0) = 0
func(N) = func(N-2) + N
Assuming that the definition has been implemented correctly, what will be displayed if the main method has a statement System.out.println(func(8))?

func(8)
↳ func(6) + 8
↳ func(4) + 6
↳ func(2) + 4
↳ func(0) + 2

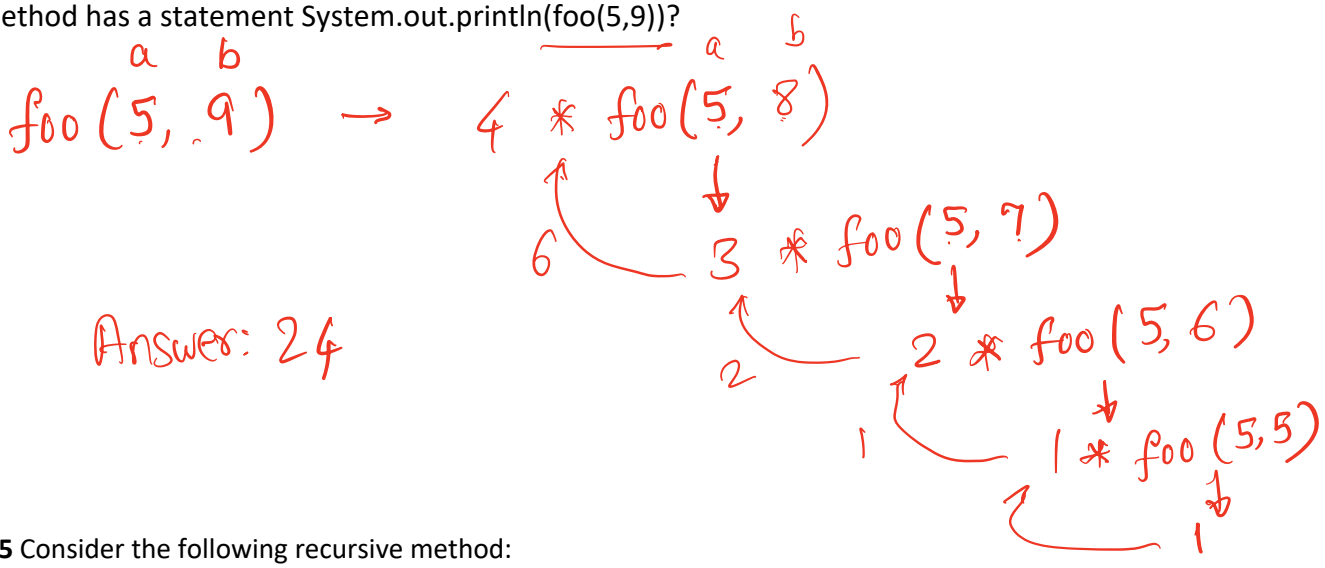
20 ← 12 ← 6 ← 2

Answer: 20

2.4 Consider the following recursive method:

```
public static int foo(int a, int b){

                if (b<=1 || b<=a)
                      return 1;
                else
                      return (b-a) * foo(a, b-1);
}
```

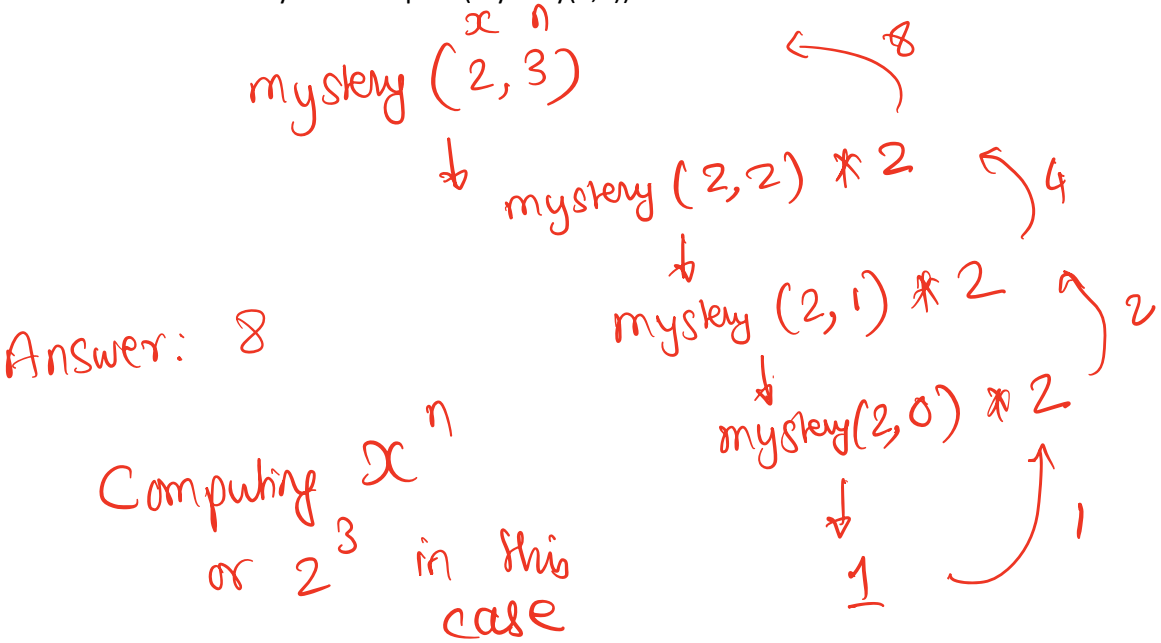
Assuming that the main method has been implemented correctly, what will be displayed if the main method has a statement System.out.println(foo(5,9))?



2.5 Consider the following recursive method:

```
public static int mystery(int x, int n){
          if (n>0)
               return (mystery(x, n-1)*x);
          else
               return 1;
}
```

Assuming that the main method has been implemented correctly, what will be displayed if the main method has a statement System.out.print(mystery(2,3))?



2.6 Write a recursive method public static int count(int n) { ....} to count the number of digits in a given positive integer. For example, count(3456) should return 4, count(901101) should return 6, and count(4) should return 1.

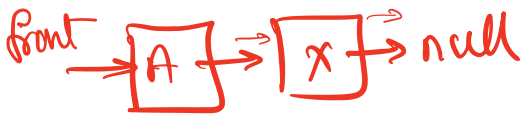
public static int count(int n){
// continue

```
if (n/10 == 0)
        return 1;
else    return 1 + count(n/10);
}
```

3456
1 + count(345)
1 + count(34)
1 + count(3)
1

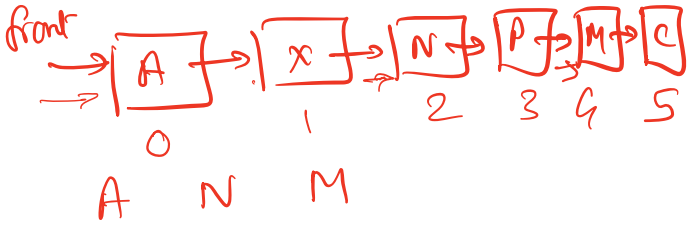
2.7: Write a recursive method for displaying the data in the even-numbered nodes (0, 2, 4, etc.) in a linked list. Assume that the nodes are numbered from 0. For example, if the linked list is → A→X→N→P→M→C, it should display

A N M



Assume that the LinkedList<T> class has the following structure.

```
public class LinkedList<T>
{
        private Node<T> front;
        private int count;

        public LinkedList()
        {
                front = null;
                count=0;
        }
```



A

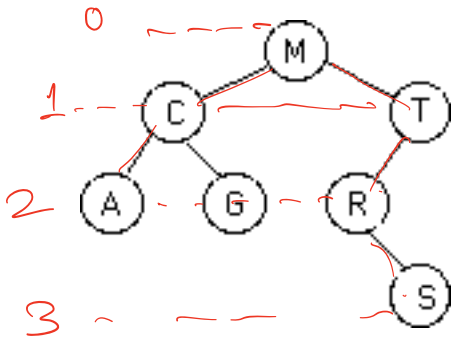
You may also use the methods in the Node class.

```
public void displayEven(Node<T> front)
{   if (front == null)
        return;
    else if (front.getNext() == null)
        System.out.print(front.getData() + " ");
    else {  System.out.print(front.getData() + " ");
            displayEven(front.getNext().getNext());
    }
}
```

A N M

QUESTION NO. 3

You are given the following binary tree:



i) Which are the leaf nodes? A, G, S

ii) Which are the internal nodes? M, C, T, R

iii) What is the longest distance (between any two nodes) in the tree? 5 A → S

iv) Which are the nodes at Level 2? A, G, R

v) What is the height of the tree? 3

vi) What are the leaf nodes in the right subtree of the root node? S

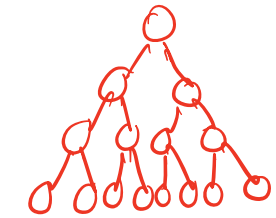
vii) What is the left subtree of the left child of the right child of the root node? Null

viii) Is it a strictly binary tree? Why or why not? Not. T & R have only one child

ix) Is it a complete binary tree? Why or why not? Not. Last 2 levels incorrect.

QUESTION NO. 4

4.1 What is the maximum number of nodes in a complete binary tree of height = 3? In general, what is the maximum number of nodes in a complete binary tree of height h?



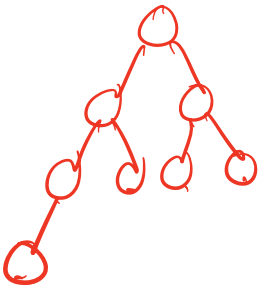
$2^{h+1} - 1$

h = 0 h = 1 h = 2 h = 3

1 3 7 15

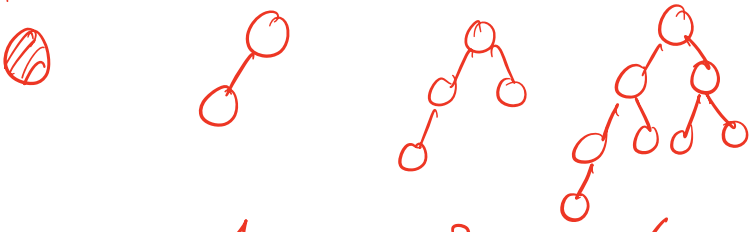
h = 3 #nodes = 15

4.2 What is the minimum number of leaves in a complete binary tree of height = 3? In general, what is the minimum number of nodes in a complete binary tree of height h?



min: 4 leaves

h = 0 h = 1 h = 2 h = 3



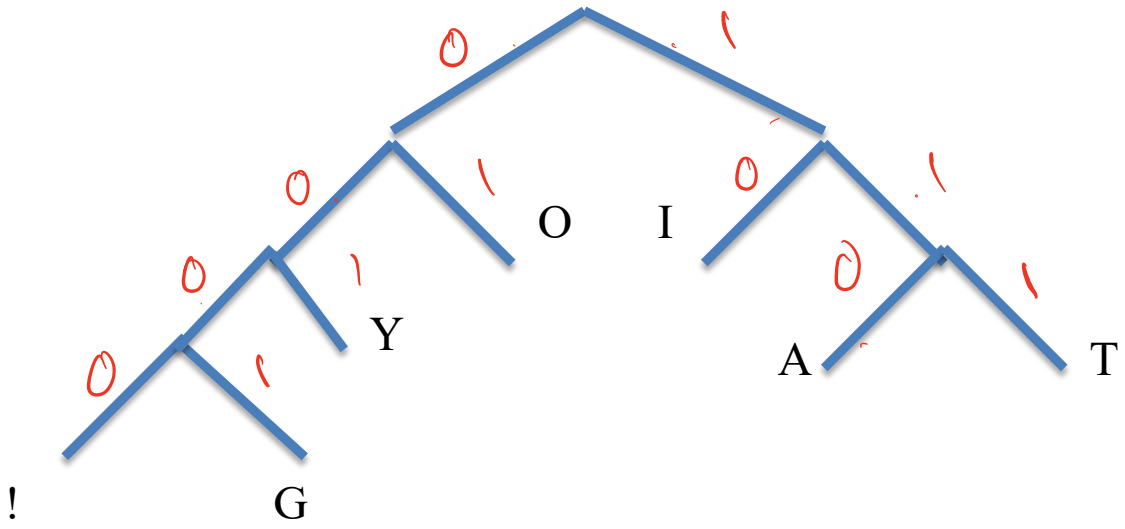
1 1 2 4

if h = 0, min. #leaves = 1

if h > 0, min. #leaves = 2^{h-1}

QUESTION NO. 5

The following tree was generated using the Huffman technique.



The following message was generated using the codes from the above tree.

0011100010000 10 000101111 1011100000000

Decode the message. Leave the spaces as they are.

YAY! I GOT IT!!

QUESTION NO. 6

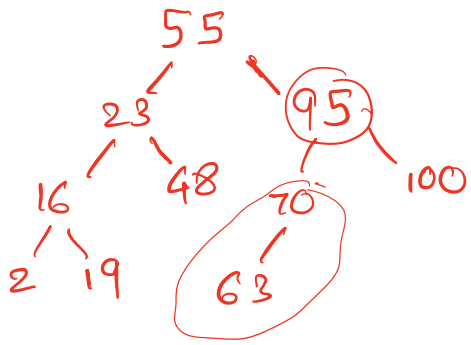
Write a recursive method to count the number of nodes in a binary tree. Assume that the method is implemented within the Binary Tree class.

```
public int countNodes(BinaryTree<T> tree)
{
    if (tree == null)
        return 0;

    else
        return 1 + countNodes(tree.getLeft())
                 + countNodes(tree.getRight());

}
```

QUESTION NO. 7

Starting from an empty **binary search tree,** suppose that the following operations are performed ***in sequence***.

- Insert 55, 23, 48, 95, 16, 2, 70, 63, 100, 19



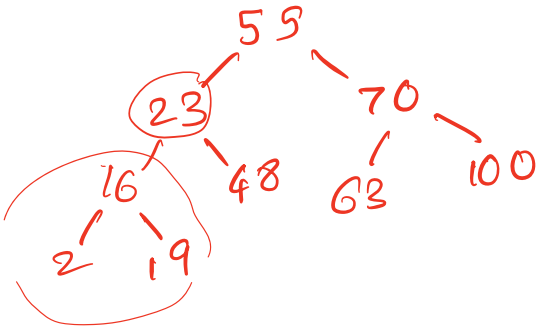
a) What is the root node?

b) What are the leaf nodes?

c) What are the left and right child nodes of the root?

a) 55
b) 2, 19, 48, 63, 100
c) 23, 95

- Delete the right child of the root node

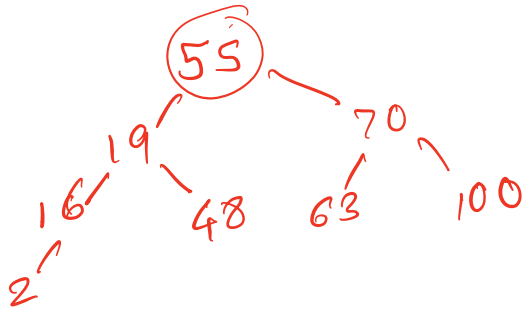


a) What are the leaf nodes?

b) What are the left and right child nodes of the root?

a) 2, 19, 48, 63, 100
b) 23, 70

- Delete the left child of the root node

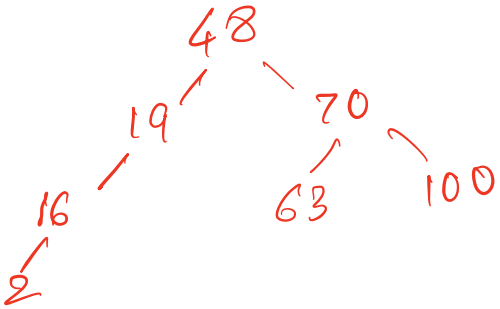


a) What are the leaf nodes?

b) What are the left and right child nodes of the root?

a) 2, 48, 63, 100
b) 19, 70

- Delete the root node



a) What is the new root node?

b) What are the leaf nodes?

c) What are the left and right child nodes of the root?

a) 48
b) 2, 63, 100
c) 19, 70

QUESTION NO. 8 (HEAPS)

Starting from an empty heap, do the following operations *in sequence.*

a) Insert 12, 19, 10, 4, 23, 7, 45, 8, 15

b) Delete_max

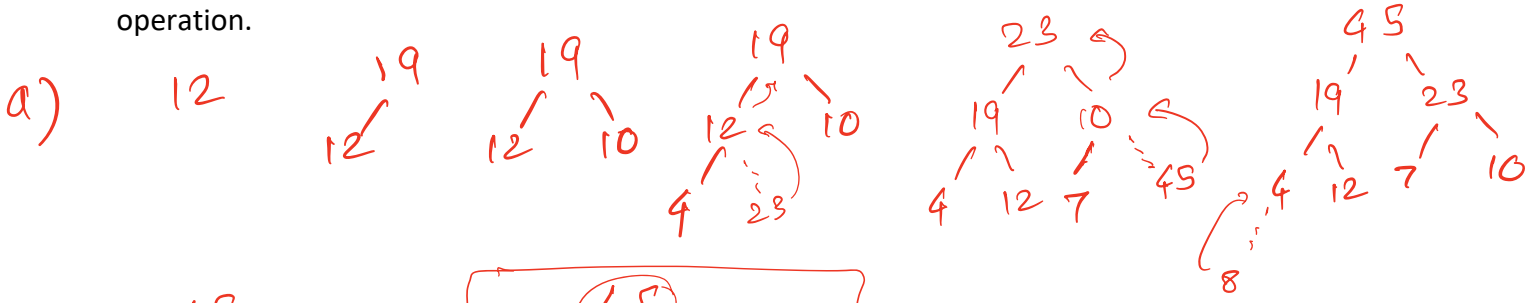
c) Insert 22

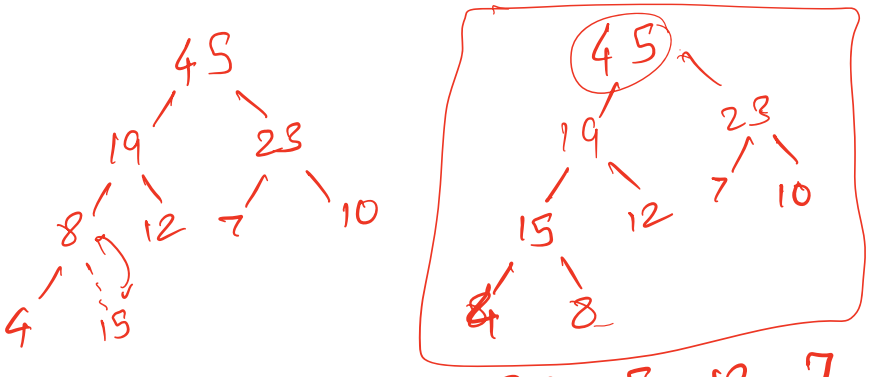
d) Delete_max

e) Delete_max five times.

Assuming an arraylist implementation of the heap, write the enumeration of the arraylist after each operation.

a)

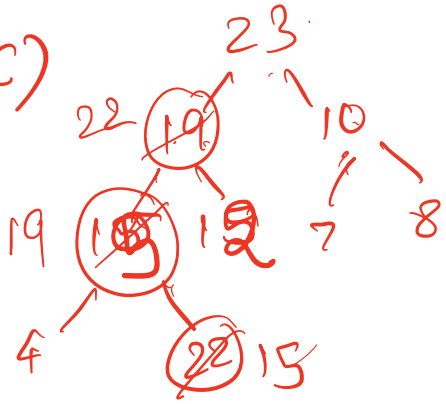




[45, 19, 23, 15, 12, 7, 10, 4, 8]

b) [23, 19, 10, 15, 12, 7, 8, 4]

c)



[23, 22, 10, 19, 12, 7, 8, 4, 15]

d) [22, 19, 10, 15, 12, 7, 8, 4]

e) Try on your own [8, 4, 7]

QUESTION NO. 9 (HASHING)

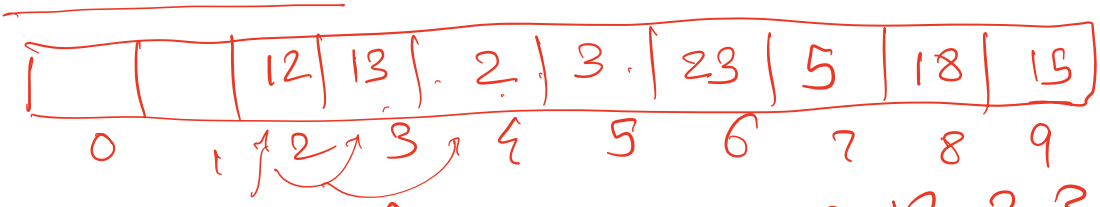
9.1 The keys 12, 18, 13, 2, 3, 23, 5 and 15 are inserted into an initially empty hash table of length 10 using closed hashing and linear probing. The hash function is h(k) = k mod 10.

What is the resultant hash table? Write your answer as a sequence of numbers separated by commas and a '-' for an empty array slot.

For example, if your resulting hash table is the array (Note: This is not the correct answer!)

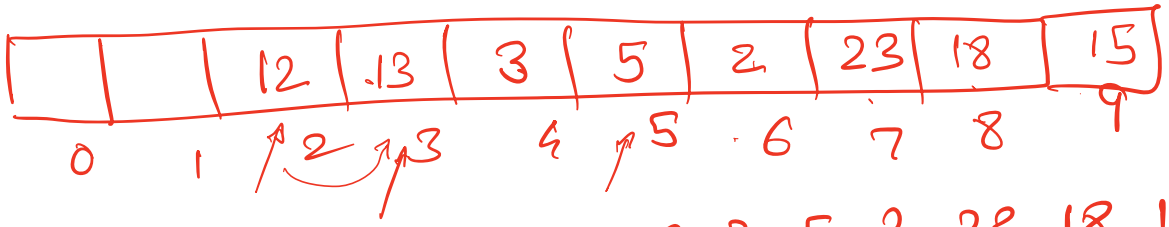
12		13		18	2	3	23	5	15

You would write it your answer as follows:
12, - , 13, - , 18, 2, 3, 23, 5, 15



Answer: —, —, 12, 13, 2, 3, 23, 5, 18, 15

9.2 Repeat the above question but this time use quadratic probing.



Answer: —, —, 12, 13, 3, 5, 2, 23, 18, 15

9.3 Consider a hash table of size seven, with starting index zero, and a hash function (3x + 4) mod 7, where x is the key. Assuming the hash table is initially empty, the sequence of keys 1, 3, 8, 10 is inserted into the table using closed hashing and linear probing?

What is the resultant hash table? Write your answer as a sequence of numbers separated by commas and a '-' for an empty array slot.

key: 1 $(3*1+4) \bmod 7 = 0$

3 $(3*3+4) \bmod 7 = 3$

8 $(3*8+4) \bmod 7 = 0 \rightarrow 1$

10 $(3*10+4) \bmod 7 = 6 \rightarrow 0 \rightarrow 1 \rightarrow 2$

Answer: 1, 8, 10, -, -, 3

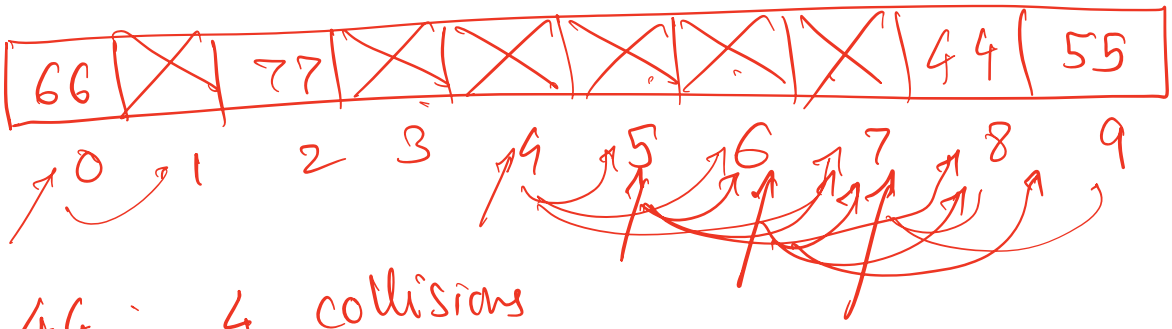
9.4 The figure below shows the current state of a 10-element hash table, with the occupied cells shown with an XX. Assuming that the hash function is key mod 10. The following keys are inserted in sequence:

44, 55, 66, 77

	XX		XX	XX	XX	XX	XX		
0	1	2	3	4	5	6	7	8	9

a) What is the resulting table if Linear probing is used? What is the total number of collisions?

b) What is the resulting table if Quadratic probing is used? What is the total number of collisions?

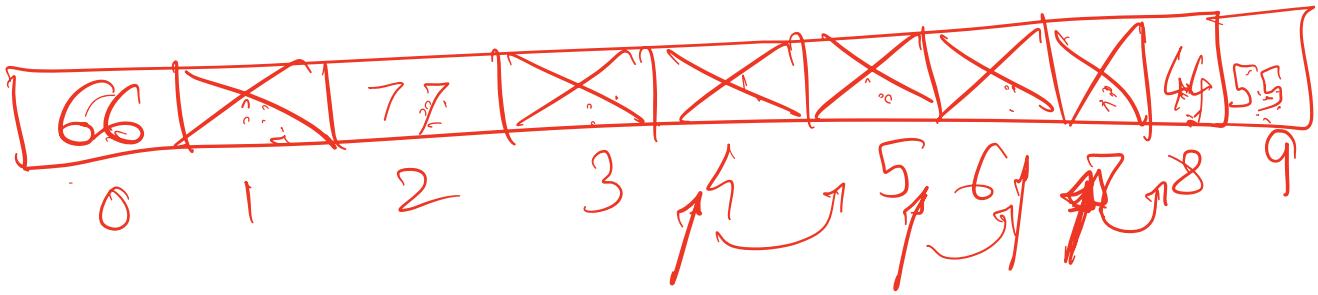


44 : 4 collisions

55 : 4 collisions Total : 17 collisions

66 : 4 collisions

77 : 5 collisions



44 : 1 → 2 collisions

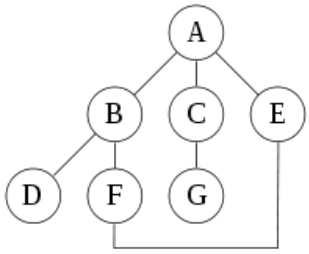
55 : 1 → 2 collisions Total: 11 collisions

66 : 1 → 2 collisions

77 : 1 1 1 1 1 → 5 collisions

QUESTION NO. 10:

For the graph given below
a) State whether the graph is weighted or unweighted, directed or undirected, connected or unconnected, cyclic or non-cyclic.
b) List all vertices with degree = 3.
c) List a cycle in the graph, if it exists.
d) List all possible simple paths from A to D.
e) Determine the DFS and BFS traversals from node A.



a) Unweighted, Undirected Connected Cyclic

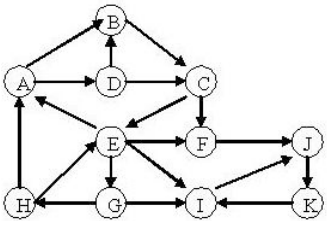
b) A, B

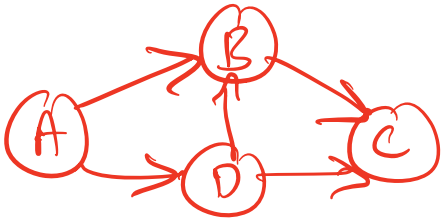
c) A E F B A

d) ABD, AEFBD

e) DFS: A B D F E C G
BFS: A B C E D F G

QUESTION NO. 11:
For the following graph, which node(s) has (have) the highest indegree? What is the highest indegree? Which node(s) has (have) the highest outdegree? What is the outdegree? Is the subgraph connecting vertices A, B, C, D strongly connected or weakly connected?





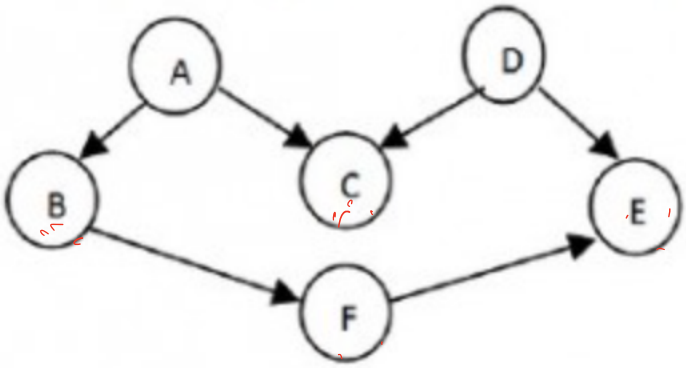
Weakly connected because, for e.g., B → A not possible.

	A	B	C	D	E	F	G	H	I	J
Indegree	2	2	2	1	2	2	1	1	3	2
Outdegree	2	1	2	2	4	1	2	2	1	1

Highest indegree : I, 3
Highest outdegree : E, 4

QUESTION NO. 12

For the graph shown below, find one solution that lists the vertices so that the dependency is not violated. (Run the topological sorting algorithm and assign topological numbers to the vertices. Show steps).



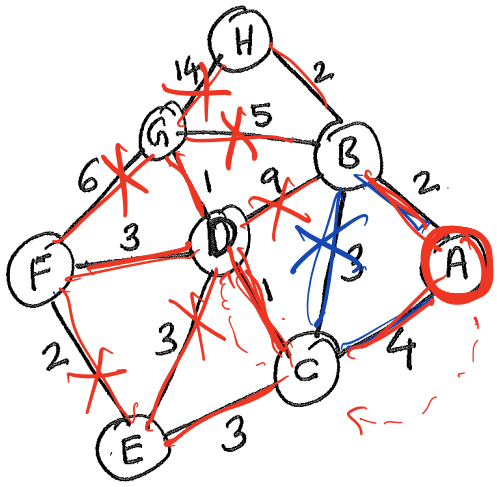
	A	B	C	D	E	F
PRED	0	1	2	0	2	1
	0	1 0			1 0	0

QUEUE: ~~A~~ ~~D~~ ~~B~~ ~~C~~ F E

TOPNUM	1	2	3	4	5	6
	A	D	B	C	F	E

QUESTION NO. 13

Using Dijkstra's algorithm, find the shortest paths from node A to all other nodes in the following graph, Show steps.



A	B	C	D	E	F	G	H
B,2	A,2	A,4	B,9	C,3	D,3	B,5	B,2
C,4	C,3	B,3	C,1	D,3	E,2	D,1	G,4
	D,9	D,11	E,3	F,2	G,6	F,6	
	G,5	E,3	F,3			H,14	
	H,2		G,1				

Step NO	C list	T list	Remarks
1	A, 0, –	B, 2, B C, 4, C	Add A to C list. Its neighbours to T list
2	B, 2, B	C, 4, C D, 11, B G, 7, B H, 4, B	Move B to C list Retain old entry for C New entries for D, G, H
3	C, 4, C	D, 5, C E, 7, C G, 7, B H, 4, B	Move C to C list Change entry for D New entry for E
4	H, 4, B	G, 7, B D, 5, C E, 7, C	Move H to C list Retain old G

5	D, 5, C	E, 7, C F, 8, C G, 6, C	Move D to Clist New entry for F change entry for G
6	G, 6, C	F, 8, C E, 7, C	Move G to Clist
7	E, 7, C	F, 8, C	Move E to Clist
8	F, 8, C	—	Move F to Clist

Shortest Paths