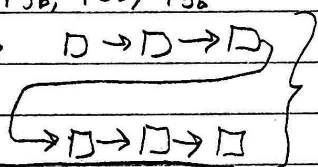
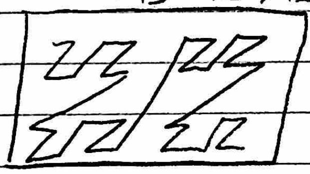
Interactive Comp. Graphics

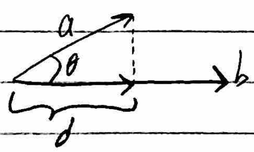
Background/Prerequisits

Raster Images

- v/h → resolution
- interleaved → rgb, rgb, rgb or rgba, rgba, rgba
  rgb, rgb, rgb rgba, ------
- Scanline order →  typical
- Swizzled order  → improves locality & cache performance
- high freq. detail: details lost when going from high res to low res
- Alpha α : opacity
- RGBA → 8 bits/channel will yeild 32 bit (1 word) per pixel

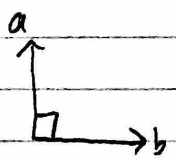
Vectors

- Dot Prod.



$a \cdot b = a_x b_x + a_y b_y + a_z b_z$

if $b = 1$, $d = a \cdot b$
else, $\boxed{a \cdot b = |a||b|\cos(\theta)}$

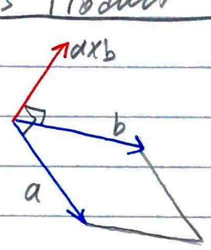


if $\theta = 90°$, $a \cdot b = 0$

Dot Product Properties

- $a \cdot b = b \cdot a$ — Symmetric
- $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ — associative
- $ka \cdot b = a \cdot kb = k(a \cdot b)$ — Commutative / distributive

Cross Product



Properties

$$a \times b = -(a \times b)$$
$$(ka) \times b = k(a \times b)$$
$$a \times (b + c) = a \times b + a \times c$$

Affine Transformations – 2D

- simplest type of transformations
  - Translation $\quad p' = p + t$
  - Scale $\quad p' = Sp$
  - Rotation
$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = p_x \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + p_y \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \quad \text{(about origin)}$$

or

$$\begin{bmatrix} p'_x \\ p'_y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

Singular Value Decomp. (SVD)

- for ANY matrix, it's SVD is:

$M = USV^T$
- U → orthogonal (rotation) → $P^T P = I$ or $P^T = P^{-1}$
- S → diagonal (Scale)
- V^T → orthogonal (rotation)

Series of Transforms

$$p' = RSRSSRSRRS\,p + t$$
$$= Mp + t$$

↳ translation makes it complicated;

$\ldots\ p' = M_2(M_1 p + t) \ldots$ etc.

homogeneous

$$\begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \alpha p_x \\ \alpha p_y \\ \alpha p_z \\ \alpha \end{bmatrix}$$

- to get around this complexity ... Homogeneous Coords.

Homogeneous Coordinates

- cheat by adding an extra dimension

$$\begin{bmatrix} p_x' \\ p_y' \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} p_x + t_x \\ p_y + t_y \end{bmatrix}$$

$$\begin{bmatrix} p_x' \\ p_y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

So now, $p' = SRTRTSRTp$

$p' = Mp$

$$\begin{bmatrix} p_x' \\ p_y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & c & e \\ b & d & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

3D Affine Transformations

- for homogeneous coordinates: $\longrightarrow$ $M = \begin{bmatrix} a & d & g & j \\ b & e & h & k \\ c & f & i & l \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- Scale: $\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, trans: $\begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- Rotation: $\longrightarrow$ now need to specify a rotation axis

R_x: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

R_z: $\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

R_y: $\begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

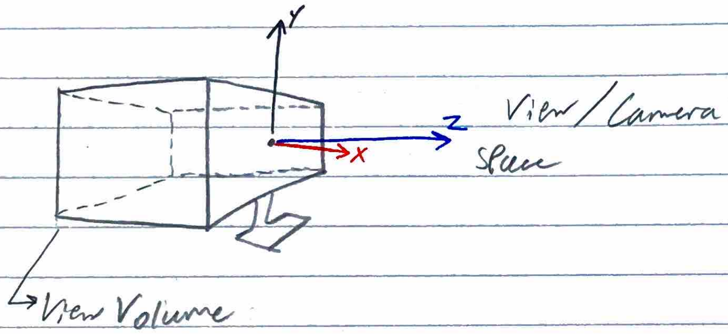
$\rightarrow$

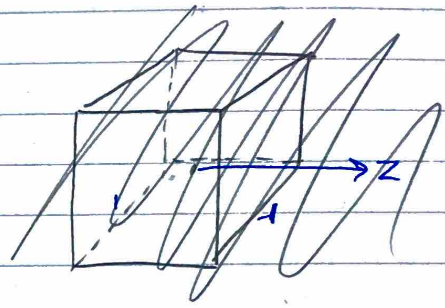
- Order Matters!!!

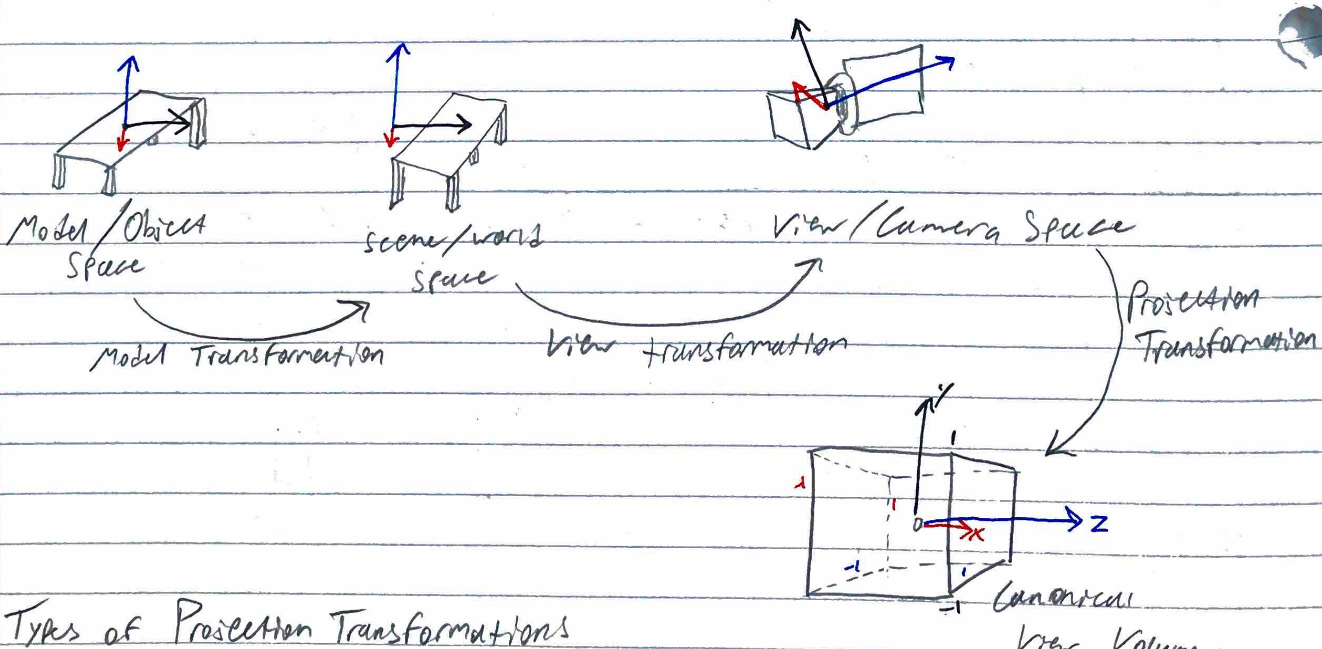
$$R_z(\alpha)\,R_y(\beta)\,R_x(\gamma)$$
$$\neq R_y(\beta)\,R_z(\alpha)\,R_x(\gamma)$$

order is left to right

Viewing (Projections & Transforms)

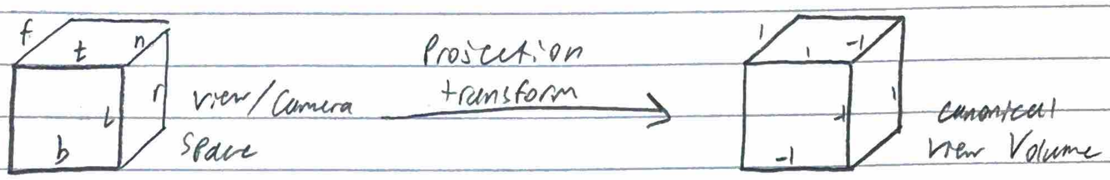






Types of Projection Transformations
- Orthographic Projection
- Perspective

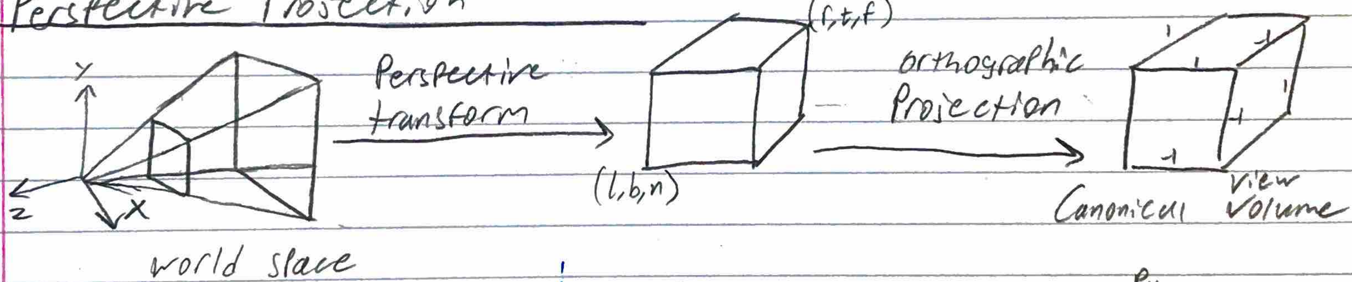
Orthographic Projection

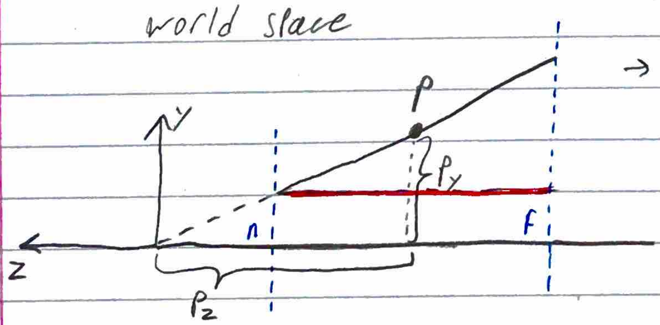


$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{f-n} & -\frac{f+n}{f-n} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

→ Note: no rotations

Perspective Projection





→ for all points along the line, $\frac{P_y}{P_z}$ is same

$P'_y = \frac{P_y}{P_z} n$, $P'_x = \frac{P_x}{P_z} n$ → Since the two lines meet @ "n"

$$\hookrightarrow \begin{bmatrix} P'_x \\ P'_y \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \end{bmatrix} \frac{n}{P_z}$$

$$\text{So, } \begin{bmatrix} P'_x \\ P'_y \\ P'_z \\ 1 \end{bmatrix} = \begin{bmatrix} nP_x/P_z \\ nP_y/P_z \\ ? \\ 1 \end{bmatrix} \equiv \begin{bmatrix} nP_x \\ nP_y \\ ? \\ P_z \end{bmatrix}$$

→ using the homogeneous Property

if $P_z = n$; $P'_z = (P_z(n+f) - fn)/P_z$
$= n + f - fn/P_z$
$= n + f - f \boxed{= n}$

$$\begin{bmatrix} P'_x \\ P'_y \\ P'_z \\ 1 \end{bmatrix} \equiv \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \\ 1 \end{bmatrix}$$

if $P_z = f$; $P'_z = (n+f) - n$
$= f$

↳ so still preserve z values