Lecture 8-2 SEIF SLAM

- Sparsification in the information mtx is removing direct links to landmarks that were active but have become passive
- Sparsification happens on every iteration

↳ Effect: - robot's pose is only linked to active landmarks
  - landmarks have links to only nearby landmarks

Corrected Mean Computing

mean needed in motion model, measurement model, sparsification step

↳ already have μ_{t-1}, need corrected μ_t from $\bar{\mu}_t$

- This is costly: $\mu = \Omega^{-1} \xi$

- thus, SEIF SLAM approximates the corrected mean

↳ Compute a few dimensions of the mean in an approximated way

↳ treat as an optimization problem to find:

↳ only need robot & active landmark poses

$$\hat{\mu} = \operatorname{argmax} p(\mu)$$
$$= \operatorname{argmax} \exp\left(-\tfrac{1}{2}\mu^T \Omega \mu + \xi^T \mu\right)$$

↳ Seeks to find value that maximizes the probability density func.

Approximating Mean

- Can be done in many ways
- Can be efficient given only a few dims are needed (Robot & landmark poses)

↳ more on this later in the course

$$\boxed{P(a|c)P(c) = P(a,c)} \text{ so, } P(a,b,c) = P(a|b,c)P(b,c) = P(a|b,c)P(b|c)P(c)$$

Sparsification

- removing direct connections between robot's pose & non-active landmarks (off-diag.)
  ↳ assuming conditional independence between them

Sparsification Generally;

- replace the distribution → $P(a,b,c)$ by an approx. $\tilde{P}$ so that a & b are independent given c

$$\tilde{P}(a|b,c) = P(a|c)$$
$$\tilde{P}(b|a,c) = P(b|c)$$

- approximation by assuming Conditional Independence;

$$
\begin{aligned}
P(a,b,c) &= P(a|b,c)P(b|c)P(c) \\
&\simeq P(a|c)P(b|c)P(c) \\
&= P(a|c)\frac{P(c)}{P(c)}P(b|c)P(c) \\
&= \frac{P(a,c)P(b,c)}{P(c)}
\end{aligned}
$$

approximation

→ a = robot pose
→ $b = m_1$, $c = m_2$ → (or everything else)
this is what we will do in the context of the information matrix

- quick notation; $\underset{\text{all landmarks}}{m} = \underset{\text{active}}{m^+} + \underset{\text{active to passive}}{m^0} + \underset{\text{Passive}}{m^-}$

Sparsification;

$P(x_t, m | z_{1:t}, u_{1:t}) = P(x_t, m^+, m^0, m^- | z_{1:t}, u_{1:t})$ → Still actually there → ignore z & u for writing sake

$$
\begin{aligned}
P(x_t, m) &= P(x_t, m^+, m^0, m^-) \\
&= P(x_t | m^+, m^0, m^-)P(m^+, m^0, m^-) \\
&= P(x_t | m^+, m^0, m^- = 0)P(m^+, m^0, m^-) \\
&\simeq P(x_t | m^+, m^- = 0)P(m^+, m^0, m^-) \\
\tilde{P}(x_t, m) &= \frac{P(x_t, m^+ | m^- = 0)}{P(m^+ | m^- = 0)} P(m^+, m^0, m^-)
\end{aligned}
$$

(m^0 disappears)

→ ignoring non-active landmarks (= 0)
→ Sparsification; assume conditional independence of the robot's pose from landmarks that become passive

So, ⟶

☆ Q. why is $\tilde{\xi} = \tilde{\Omega}_t \mu_t$
slower than $\tilde{\xi} = \xi + (\tilde{\Omega}_t - \Omega_t)\mu_t$?

Information Mtx Update

- sparsifying direct links between joint pose & m^- results in;

$$\boxed{\tilde{p}(x_t, m \mid z_{1:t}, u_{1:t}) \simeq \frac{p(x_t, m^+ \mid m^- = 0, z_{1:t}, u_{1:t})}{p(m^+ \mid m^- = 0, z_{1:t}, u_{1:t})} \, p(m^0, m^+, m^- \mid z_{1:t}, u_{1:t})}$$

↳ put Z & U back in

(each of these corresponds to one of the 3 beliefs from)

- now replace Ω, ξ by approximated values

↳ express $\tilde{\Omega}$ as a sum of 3 matrices ; $\boxed{\tilde{\Omega} = \Omega_t^1 - \Omega_t^2 + \Omega_t^3}$

- Conditioning Ω_t on $m^- = 0$ yields Ω_t^0 : $p(x_t, m^+, m^0 \mid m^- = 0)$
- marginalizing m^0 from Ω_t^0 yields Ω_t^1 : $\int p(x_t, m^+, m^0 \mid m^- = 0)\, dm^0$
- marginalizing x, m^0 from Ω_t^0 yields Ω_t^2 : ...
- marginalizing x from Ω_t yields Ω_t^3 : ...
- generate sparsified information mtx ; $\tilde{\Omega}_t = \Omega_t^1 - \Omega_t^2 + \Omega_t^3$

- now compute info. vector directly ; $\tilde{\xi} = \tilde{\Omega}_t \mu_t$
- this is very efficient cuz the difference here is a very small # of non-zero elements, just getting rid of 1 or 2 active landmarks to be passive

$$\begin{aligned}
\tilde{\xi} &= \tilde{\Omega}_t \mu_t \\
&= (\Omega_t - \Omega_t + \tilde{\Omega}_t)\mu_t \\
&= \Omega_t \mu_t + (\tilde{\Omega}_t - \Omega_t)\mu_t \\
&= \xi_t + (\tilde{\Omega}_t - \Omega_t)\mu_t
\end{aligned}$$

SEIF_sparsification(ξ_t, Ω_t, μ_t):

define F_{m_0}, F_{x,m_0}, F_x as projection matrices to m_0, $\{x, m_0\}$, x respectively

$$\Omega_t^0 = F_{x,m^+,m^0} F_{x,m^+,m^0}^T \Omega_t F_{x,m^+,m^0} F_{x,m^+,m^0}^T$$

$$\begin{aligned}
\tilde{\Omega}_t = \Omega_t &- \Omega_t^0 F_{m_0} (F_{m_0}^T \Omega_t^0 F_{m_0})^{-1} F_{m_0}^T \Omega_t^0 \\
&+ \Omega_t^0 F_{x,m_0} (F_{x,m_0}^T \Omega_t^0 F_{x,m_0})^{-1} F_{x,m_0}^T \Omega_t^0 \\
&- \Omega_t F_x (F_x^T \Omega_t F_x)^{-1} F_x^T \Omega_t
\end{aligned}$$

$$\tilde{\xi}_t = \xi_t + (\tilde{\Omega}_t - \Omega_t)\mu_t$$

return $\tilde{\xi}_t, \tilde{\Omega}_t$

SEIF SLAM Summary

- roughly constant time (vs. quadratic for EKF)
- Linear memory (vs. quadrat. for EKF)
- less accurate than EKF (sparsification, mean recovery estimation)

↳ worth it in practice?

↳ 7-10 active landmarks seems to be a good compromise