$$\mu = \xi \Omega^{-1}$$

Lecture 7 - Extended Information Filter

Canonical Parameterization

- alternative representation of Gaussians
- described by:
  - information matrix $\Omega = \Sigma^{-1}$
  - information vector $\xi = \Sigma^{-1}\mu = \Omega\mu$

$\Big\}$ → cubic complexity to convert; can convert back and forth between moment space & information space

Dual Representation

Canonical Parameterization:

$$p(x) = \underbrace{\frac{\exp(-\frac{1}{2}\mu^T\xi)}{\det(2\pi\Omega^{-1})^{1/2}}}_{\text{Constant}} \exp\left(-\tfrac{1}{2}x^T\Omega x + x^T\xi\right)$$

Moments Parameterization:

$$p(x) = \det(2\pi\Sigma)^{-1/2} \exp\left(\tfrac{-1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

Marginalization & Conditioning

$$p(\alpha,\beta) = \mathcal{N}\left(\begin{bmatrix}\mu_\alpha \\ \mu_\beta\end{bmatrix}, \begin{bmatrix}\Sigma_{\alpha\alpha} & \Sigma_{\alpha\beta} \\ \Sigma_{\beta\alpha} & \Sigma_{\beta\beta}\end{bmatrix}\right) = \mathcal{N}^{-1}\left(\begin{bmatrix}\eta_\alpha \\ \eta_\beta\end{bmatrix}, \begin{bmatrix}\Lambda_{\alpha\alpha} & \Lambda_{\alpha\beta} \\ \Lambda_{\beta\alpha} & \Lambda_{\beta\beta}\end{bmatrix}\right)$$

	Marginalization $p(\alpha) = \int p(\alpha,\beta)\,d\beta$	Conditioning $p(\alpha \mid \beta) = p(\alpha,\beta)/p(\beta)$
Cov. form	$\mu = \mu_\alpha$	$\mu' = \mu_\alpha + \Sigma_{\alpha\beta}\Sigma_{\beta\beta}^{-1}(\beta - \mu_\beta)$
	$\Sigma = \Sigma_{\alpha\alpha}$	$\Sigma' = \Sigma_{\alpha\alpha} - \Sigma_{\alpha\beta}\Sigma_{\beta\beta}^{-1}\Sigma_{\beta\alpha}$
Info. Form	$\eta = \eta - \Lambda_{\alpha\beta}\Lambda_{\beta\beta}^{-1}\eta_\beta$	$\eta' = \eta_\alpha - \Lambda_{\alpha\beta}\beta$
	$\Lambda = \Lambda_{\alpha\alpha} - \Lambda_{\alpha\beta}\Lambda_{\beta\beta}^{-1}\Lambda_{\beta\alpha}$	$\Lambda' = \Lambda_{\alpha\alpha}$

expensive

trivial

Information Filter Algorithm

1. Information_Filter($\xi_{t-1}, \Omega_{t-1}, u_t, z_t$)

2. $\bar{\Omega}_t = (A_t \Omega_{t-1}^{-1} A_t^T + R_t)^{-1}$
3. $\bar{\xi}_t = \bar{\Omega}_t (A_t \Omega_{t-1}^{-1} \xi_{t-1} + B_t u_t)$

4. $\Omega_t = C_t^T Q_t^{-1} C_t + \bar{\Omega}_t$
5. $\xi_t = C_t^T Q_t^{-1} z_t + \bar{\xi}_t$
6. return ξ_t, Ω_t

Complexity

KF; ↱ can be improved in practice (linear in slam)

- Prediction; $O(n^2)^*$
- Correction; $O(n^2 + k^{2.4})$

IF;

- prediction; $O(n^{2.4})$
- correction; $O(n^2)^*$

Transformation (parameterization); $O(n^{2.4})$

EKF to EIF - Prediction Step

- Problem is the non-lin func requires prev. mean, cant take information form

EKF

$$\bar{\mu}_t = g(u_t, \mu_{t-1})$$
$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$$

EIF

$$\bar{\xi}_t = \bar{\Omega}_t \overbrace{g(u_t, \overbrace{\Omega_{t-1}\xi_{t-1}}^{\mu_t})}^{\mu_{t-1}}$$
$$\bar{\Omega}_t = (G_t \Sigma_{t-1} G_t^T + R_t)^{-1}$$

Swap

=

1 EKF($\xi_{t-1}, \Omega_{t-1}, u_t, z_t$):

2 $\mu_{t-1} = \Omega_{t-1} \xi_{t-1}$

3 $\bar{\Omega}_t = (G_t \Sigma_{t-1} G_t^T + R_t)^{-1}$

4 $\bar{\mu}_t = g(u_t, \mu_{t-1})$

5 $\bar{\xi}_t = \bar{\Omega}_t \bar{\mu}_t$

EIF - Correction Step

- as from KF to IF, sub the moments in the measurement update

$$bel(x) = \eta \exp\Big[\underbrace{-\tfrac{1}{2}(z_t - h(\bar{\mu}_t) - H_t(x_t - \bar{\mu}_t))^T Q_t^{-1} (z_t - h(\bar{\mu}_t) - H_t(x_t - \bar{\mu}_t))}_{\text{observation model}} \underbrace{- \tfrac{1}{2}(x_t - \bar{\mu}_t)^T \bar{\Sigma}_t^{-1} (x_t - \bar{\mu}_t)}_{\text{predicted belief } \overline{bel}}\Big]$$

- this leads to

$$\Omega_t = \bar{\Omega}_t + H_t^T Q_t^{-1} H_t$$
$$\xi_t = \bar{\xi}_t + H_t^T Q_t^{-1} (z_t - h(\bar{\mu}_t) + H_t \bar{\mu}_t)$$

↳ again need predicted mean

Extended_Information_Filter $(\xi_{t-1}, \Omega_{t-1}, u_t, z_t)$:

$$\mu_{t-1} = \Omega_{t-1}^{-1} \xi_{t-1}$$
$$\bar{\mu}_t = g(u_t, \mu_{t-1})$$
$$\bar{\Omega}_t = (G_t \Omega_{t-1}^{-1} G_t^T + R_t)^{-1}$$
$$\bar{\xi}_t = \bar{\Omega}_t \bar{\mu}_t$$

$$\Omega_t = \bar{\Omega}_t + H_t^T Q_t^{-1} H_t$$
$$\xi_t = \bar{\xi}_t + H_t^T Q_t^{-1} (z_t - h(\bar{\mu}_t) + H_t \bar{\mu}_t)$$

Summary

- EIF is EKF in information (canonical) form
- complexities of prediction & correction step differ
- EKF & EIF basically have the same results
- EKF is more popular in practice

- KF : efficient prediction, slow correction
- IF : slow prediction, efficient correction

↳ application determines which is the better choice