Problem 1: Is set of odd numbers with binary operations (+), i.e., $\langle O, + \rangle$ an abelian group? If not explain the reasons with necessary notations.

Step 1: Recall group axioms

A set G with a binary operation $*$ is a group if is satisfies:

1. Closure: For all $a, b \in G$, $a * b \in G$.
2. Associativity: $(a * b) * c = a * (b * c)$.
3. Identity element: There exists $e \in G$ such that $a * e = e * a = a$ for all $a \in G$.
4. Inverse element: For each $a \in G$, there exists $a^{-1} \in G$ such that $a + a^{-1} = e$.

Additionally, if $a * b = b * a$ for all $a, b \in G$, then the group is abelian.

Step 2: Take the set of odd integers

Let,

$$O = \{ \ldots, -3, -1, 1, 3, 5, \ldots \}$$

with binary operation + (usual addition).

Step 3: verify group axioms

1. Closure:

Odd + odd = Even

Example: $3+5=8$ (not odd)

Thus, closure fails.

2. Associativity:

Addition of integers is associative:

$(a+b)+c = a+(b+c)$

3. Identity element:

The additive identity in $(\mathbb{Z},+)$ is 0.

But 0 is not odd, so O has no identity element.

4. Inverse element:

For an odd integer a, its inverse under addition is $-a$.

Example: If $3 \in O$, inverse is -3, which is also odd.

Since closure and identity fail, the set of odd integers with + is not a group. Therefore, it cannot be an abelian group either.

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