Real Analysis(H2) (MA4.101a)

IIIT-H, Semester Monsoon 22, End Semester Examination

Date: 11th February 2023, Full Marks 40, Duration: 90 minutes

1. For a first order linear equation

$$x\frac{dy}{dx} = x^2 + 3y,$$

[4]

[5]

[6]

where x > 0, calculate the integrating factor and then solve the equation.

2. For a second order linear inhomogeneous ordinary differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = 3x^2,$$

find the complementary function and particular integral, and then write down the general solution. [8]

3. In which quadrants of the complex plane is the function f(z) = |x| - i|y| analytic? [3]

4. For a function

$$f(z) = \frac{\sin z}{(z^2 - 1)^2}$$

- (a) Determine the order of the pole, and residue at z = 1
- (b) Compute the following integral along curve C defined as |z-1|=1/2 oriented counterclockwise [6]

$$\oint_C f(z)dz.$$

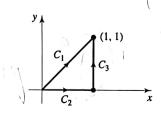
Clarification: You can keep your answers in terms of sine and cosine functions.

5. Compute the Laurent series for a function

$$f(z) = \frac{z+1}{z^3(z^2+1)},$$

around z = 0, and show the region of convergence.

6. Find the value of $\int_0^{i+1} z^2 dz$ along the following contours i) straight line from 0 to 1+i, ii) a line from 0 to 1, and then from 1 to 1+i. The contours are sketched in the figure below [8]



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