Real Analysis(H2) (MA4.101a)

IIIT-H, Semester Monsoon 22, End Semester Examination

Date: 11th February 2023, Full Marks 40, Duration: 90 minutes

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1. For a first order linear equation

$$x\frac{dy}{dx} = x^2 + 3y\,,$$

where $x > 0$, calculate the integrating factor and then solve the equation. [4]

2. For a second order linear inhomogeneous ordinary differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = 3x^2\,,$$

find the complementary function and particular integral, and then write down the general solution. [8]

3. In which quadrants of the complex plane is the function $f(z) = |x| - i|y|$ analytic? [3]

4. For a function

$$f(z) = \frac{\sin z}{(z^2-1)^2}$$

   (a) Determine the order of the pole, and residue at $z = 1$ [5]

   (b) Compute the following integral along curve C defined as $|z-1| = 1/2$ oriented counterclockwise [6]

$$\oint_C f(z)dz\,.$$

Clarification: You can keep your answers in terms of sine and cosine functions.

5. Compute the Laurent series for a function

$$f(z) = \frac{z+1}{z^3(z^2+1)}\,,$$

around $z = 0$, and show the region of convergence. [6]

6. Find the value of $\int_0^{i+1} z^2 dz$ along the following contours *i)* straight line from 0 to $1+i$, *ii)* a line from 0 to 1, and then from 1 to 1+i. The contours are sketched in the figure below [8]