Real Analysis (UG1, Monsoon 2022)

Quiz [10 points]

Saturday, 26 November 2022

1 Instructions

- Give clear reasoning. State clearly which theorem or axioms you are using.
- Please read questions carefully before you begin to answer. Turn both side of the question paper.
- Questions that you are unable to solve during in-class quiz will form exercise for you. Try to solve them at your own pace and then discuss with TAs during office hours if you need any clarification.

Question 1 [$3 \times 1 = 3$ points]

Answer any one of the following:

1. For a field $\mathbb{F}$ with multiplication $\cdot$ and addition $+$, and $a, b \in \mathbb{F}$, prove the following:

   (i) $a \cdot (-b) = (-a) \cdot b = -(a \cdot b)$,

   (ii) $-(-a) = a$,

   (iii) $(-a) \cdot (-b) = a \cdot b$.

2. Consider the set of intervals on the real line $P = \{(a, b) : a, b \in \mathbb{R} \text{ and } a < b\}$. Define the containment relation C as follows:

$$(a, b)C(c, d)$$

   if and only if $a \leq c$ and $d \leq b$. Show that the relation C is antisymmetric.

3. Show that between any two distinct real numbers there is a irrational number.

Question 2 [$3.5 \times 2 = 7$ points]

Answer any two of the following:

1. Consider the set $\{0, 1\}$. Then show that $\{0,1\}^X \approx \mathcal{P}(X)$ for every set X. Here $\mathcal{P}(X)$ denotes the power set of X. The notation Y^X represents the set of all functions from [redacted]. $X \longrightarrow Y$

2. Prove that the set of all infinite binary sequences s_i are uncountable. [Note: The infinite binary sequences are infinite sequences s_i consisting of only binary digits, e.g., $\{0, 0, 0, 0, 0, 0, \ldots\}, \{1, 1, 1, 1, 1, 1, \ldots\}, \{1, 1, 0, 0, 0, 1, \ldots\}$.

3. Using mathematical induction prove: If $a \geq -1$, then $(1 + a)^n \geq 1 + na$ for all $n \in \mathbb{N}$.