

Topics for today

- Boolean Logic
- Sets

Boolean Logic

- is a branch of mathematics
- results can be true or false
- $1 = T = \text{true}$
- $0 = F = \text{false}$

Truth Tables

NOT = ①
! , ~

NOT $\perp = F$ $p = 1$
!P = F

$2^n = \text{no. of operands}$

1 **AND** \wedge

p	q/	$p \wedge q$
T	T	T = 1
T	F	F = 0
F	T	F = 0
F	F	F = 0

4 **\Rightarrow** If... then

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

PQR
FFF
TFF
FTF
FFT
TTF
TFT

2 **OR** \vee

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

5 **\Leftrightarrow** If & only if

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$p \rightarrow q$
 $q \rightarrow p$

TTT

3 **XOR** \oplus

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

NOT XOR $\rightarrow \Leftrightarrow$

Truth Tables - Practice

→ $\text{!P AND (P} \overset{2}{\rightarrow} \text{Q)}$ =

A AND B

P	Q	!P	P → Q	!P AND (P → Q)
T	T	F	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

→ $(\text{P AND Q}) \text{ XOR } (\text{P OR Q})$

A XOR B

P	Q	P AND Q	P OR Q	(P AND Q) XOR (P OR Q)
T	T	T	T	F
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F

Truth tables. Practice!

Not P Q

$(\neg p \wedge q) \Rightarrow (r \vee \neg t)$ is false when:

$A = T \Rightarrow$

$B = F$

→

↓

↓
T

$\neg p$

$P=F, Q=T, R=F, T=F$

$q = T$

$\neg p = T$

$p = F$

$\neg t = F$

$t = T$

~~~~~

Bitwise operations.

$$\begin{array}{r} 00011101 \\ \text{OR } 01101101 \\ \hline 01111101 \end{array}$$

$$\begin{array}{r} 00110110 \\ \text{AND } 00101010 \\ \hline 00100110 \end{array}$$

$$\begin{array}{r} 00101010 \\ \text{XOR } 01011010 \\ \hline 01110000 \end{array}$$

# Bitwise operations.

19 AND 13

```
19 = 10011
13 = 01101
-----
00001 = 1
```

13 >> 2

```
1101
  11
  11 = 3
```

9 << 3

```
1001000 = 72
  100
  11
```

13 >> 2 XOR 9 << 3

```
3 XOR 72
0000011
1001000
-----
1001011
```

→ 75

```
  1111
- 1000
-----
  1111
```



# Sets

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# Sets

- A set is a well-defined collection of **distinct** objects
- Represented using capital letters (S, A, B, U)
- The objects are called **elements/members**
- **No duplicates** in sets
- Examples:
  - $A = \{1, 10, 12, 15\}$
  - $B = \{5, 99, 2, 67, 12\} = \{2, 5, 12, 67, 99\}$
  - $C = \{\}$  or  $\Phi$  (phi - greek) or  $\emptyset$  (miniscule - Danish/Norwegian)

## Sets (contd...)

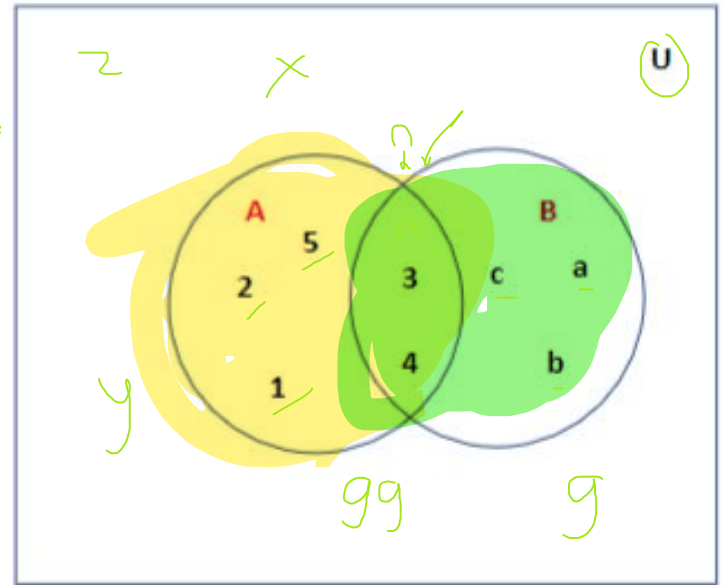
Universal set "U" is a set which consists of all the elements of the relevant sets.

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{3, 4, a, b, c\}$$

$$U = \{1, 2, 3, 4, 5, a, b, c\}$$

$A - B$   
 $B - A$



# Sets. Union.

The union of sets refers to the combination of all elements from the sets

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

example

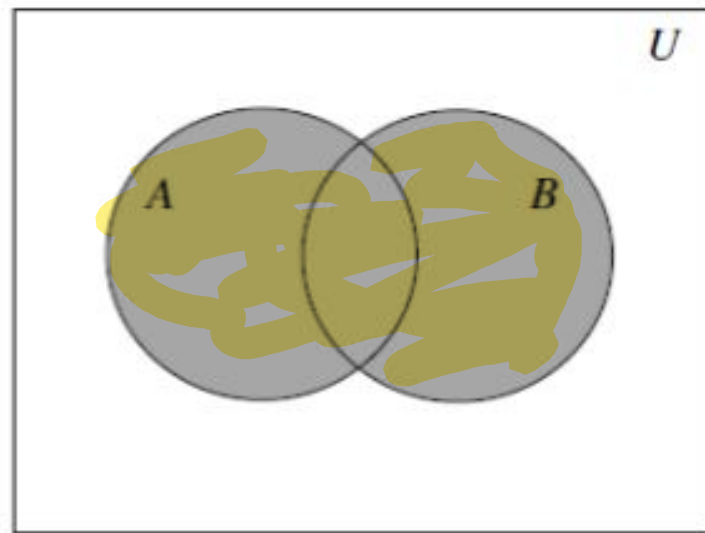
$$A \cup B = \{x\}$$

$$A = \{1, 2, 3, 4\}$$

$$B = \{3, 4, 5, 6\}$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

Venn-Diagram



# Sets. Intersection.

The intersection of sets refers to the common elements between the given sets

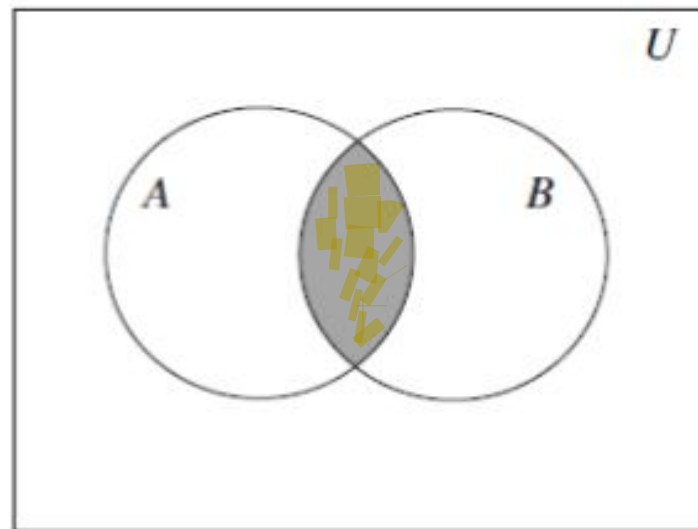
$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

example

$$A = \{1, 2, 3, 4\}$$

$$B = \{3, 4, 5, 6\}$$

$$A \cap B = \{3, 4\}$$



# Sets. Relative complement.

The relative complement also referred to as set difference of a set with respect to another set

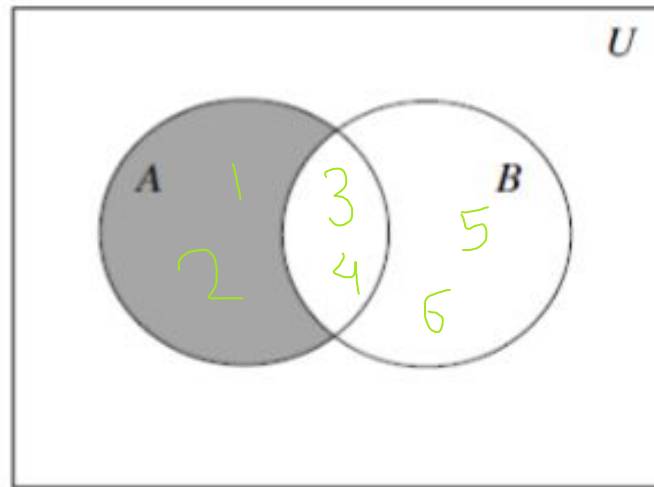
$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

example

$$A = \{1, 2, 3, 4\}$$

$$B = \{3, 4, 5, 6\}$$

$$A - B = \{1, 2\}$$



# Sets. Complement.

$$\boxed{A'} = U - A$$

↑      ↑

The complement of a set is the set of all elements in universal set  $U$  that is not in a given set.

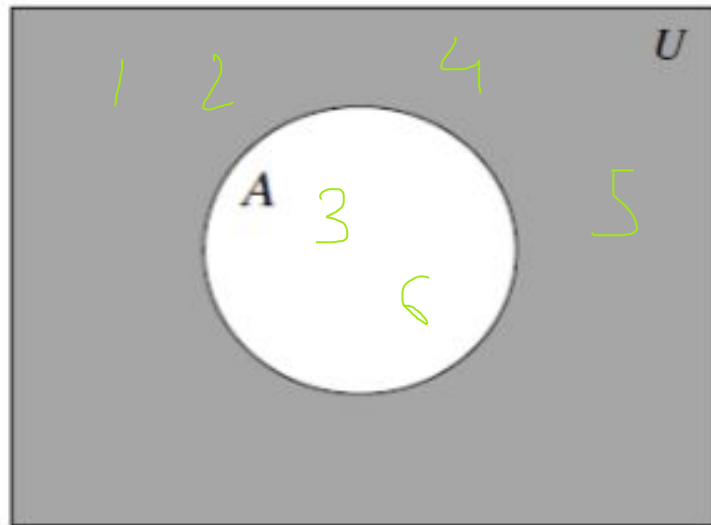
$$\underline{A'} = \underline{U - A} = \{x : x \in U \text{ and } x \notin A\}$$

example

$$U = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{3, 6\}$$

$$A' = \underline{\{1, 2, 4, 5\}}$$



# Sets. Practice!

$$N = \{1, 2, 3, 4, 5, \dots\}$$

| | | | |

What are the elements of the set expressed as:

$$A = \{x \mid x \in N, x < 6, x \bmod 2 == 1\}$$

$$N = \{1, 2, 3, 4, 5, 6, \dots\}$$

$$A = \{1, 3, 5\}$$



# Sets. Practice!

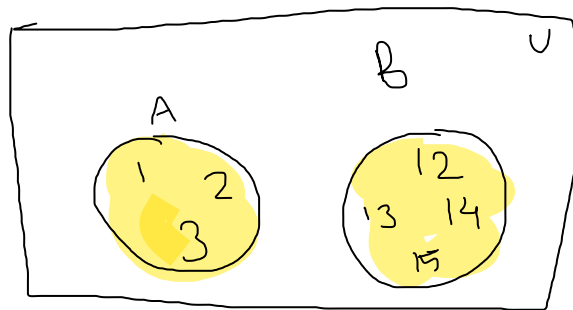
$$A = \{x \mid x \in \mathbb{N}, x < 4\} \quad A = \{1, 2, 3\}$$

$$B = \{x \mid x \in \mathbb{N}, x > 11, x \leq 15\} \quad B = \{12, 13, 14, 15\}$$

What is  $A \cup B$ ?

$$A \cup B = \{1, 2, 3, 12, 13, 14, 15\}$$

$$A - B = \{1, 2, 3\}$$



# Sets. Practice!

What is  $A - B$ ?

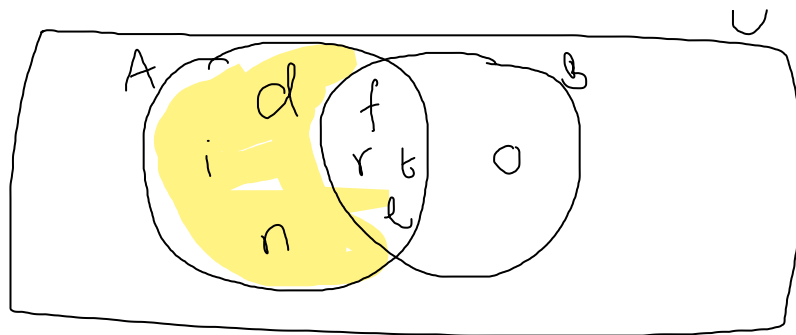
Let  $A$  = set of characters from “different”

Let  $B$  = set of characters from “effort”

$$A - B = \{d, i, n\}$$

$$A = \{d, i, f, r, e, n, t\}$$

$$B = \{e, f, o, r, t\}$$



THE END

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