Topics for today

- Number Bases
- Binary and hexadecimal
- Conversion between bases
- Hex colors and rgb colors
- Counting in different bases
- Bit shifting
- Addition of binary numbers

Base

- Number of digits that the counting system or number system uses
- For example, we (humans) use decimal number system which has 10 digits (0-9), so, the base of decimal number system is 10
- All other numbers are combinations of these 10 digits.
- When we run out of digits in one place, we move to the next place.
- ..., 6, 7, 8, 9, 10, 11, ... 19, 20, 21, ... 29, ... 99, 100, 101, ...

Binary

- Base 2 number system.
- Only 2 digits 0 and 1
- Used by electronic system at the most basic level
- 1 electricity present. 0 no electricity
- 0, 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, ...
- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Place values - decimal

- Result of the multiplication of digit and 'it's place' in the number system.
- In decimal, every place will have some powers of 10.
- \bullet 7345 = 7 * 1000 + 3 * 100 + 4 * 10 + 5 * 1
- \bullet = 7 * 10³ + 3 * 10² + 4 * 10¹ + 5 * 10⁰
- So, the place value of '3' in '7345' will be 3 * 100 = 300

Place values - binary

- Result of the multiplication of digit and 'it's place' in the number system.
- In binary, every place will have some powers of 2.
- 1011
- 1 * 2³ 0 * 2² 1 * 2¹ 1 * 2⁰
- So, the place value of leftmost '1' in '1011' will be $1*2^3 = 8$ (in decimal)

Converting from binary to decimal

- Calculate place value of each digit and add them
- 11001₂ = ?₁₀
- \bullet = 1 * 2⁴ + 1 * 2³ + 0 * 2² + 0 * 2¹ + 1 * 2⁰
- \bullet = 1 * 16 + 1 * 8 + 0 * 4 + 0 * 2 + 1 * 1
- = 25₁₀
- Note: In binary, even numbers end in 0 and odd numbers end in 1

Converting from decimal to binary

- Divide by base (2) and track the reminders from below
- 23₁₀ = ?₂
- 23 / 2 = 11 R = 1
- 11/2 = 5 R = 1
- 5/2 = 2 R = 1
- 2/2 = 1 R = 0
- So, now from below, starting with the quotient '1'
- = 10111₂

Hexadecimal

- Base 16 number system.
- 16 digits- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
- Decimal- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
- Compact representation for binary numbers
- Group of 4 binary digits to each hex digits
- 0, 1, 2, ... 9, A, ... F, 10, 11, 12, ... 19, 1A, 1B, ... 1F, 20
- 0, 1, 2, 9, 10,...15, 16, 17, 18,... 25, 26, 27,...31, 32

Converting from hexadecimal to decimal

- Calculate place value of each digit and add them
- Sometimes, prefixed by '0x'. '0x12' is hexadecimal 12, not decimal
- $C3F_{16} = 0xC3F = ?_{10}$ $= C * 16^2 + 3 * 16^1 + F * 16^0$
- \bullet = C * 256 + 3 * 16 + F * 1
- = 12 * 256 + 3 * 16 + 15 * 1
- = 3135₁₀

Converting from decimal to hexadecimal

- Divide by base (16) and track the reminders from below
- 1354₁₀ = ?₁₆
- 1354 / 16 = 84 R = 10 = A
- \bullet 84 / 16 = 5 R = 4
- So, now from below, starting with the quotient '5'
- \bullet = 54A₁₆

Hexadecimal colour

- hash (#) followed by 6 hex digits. #RRGGBB
- Convert to rgb format
- #1F256A
- $1F_{16} = decimal = 31_{10}$
- 25_{16} = decimal = 37_{10}
- $6A_{16} = decimal = 106_{10}$
- \bullet = rgb(31, 37, 106)

Hex and binary

groups of 4 binary digits represent one hex digit

Hex	0	1	2	3	4	5	6	7	8	9	Α	В	С	D	Е	F
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

- So, 4 binary digits can be converted directly to 1 hex digit
- And 1 hex digit can be converted directly to 4 binary digits

Hex and binary

- 10101110010101₂ = ?₁₆
- 0010 1011 1001 0101
- 2 B 9 5
- \bullet = 2B95₁₆ = 0x2B95

• $F12C_{16} = ?_2$

- 1111 0001 0010 1100
- = 1111 0001 0010 1100₂

Convert from one base to another

- Convert to decimal and then into the target base
- 872₉ = ?₁₂
- 872₉ to decimal
- $8 * 9^{2} + 7 * 9^{1} + 2 * 9^{0} = 8 * 81 + 7 * 9 + 2 * 1 = 648 + 63 + 2 = 713_{10}$
- 713_{10}^{10} to base 12
- 713 / 12 = 59 R = 5
- 59/12 = 4 R = 11 = B
- \bullet = 4B5₁₂

Counting

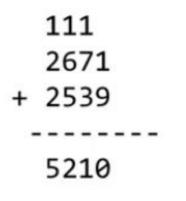
- Decimal 0, 1, 2,...9, 10, 11, ...99, 100, 101, ...
- Hexadecimal 0, 1, 2,...9, A, B,...F, 10, 11,...
- Base 13 0, 1, 2,...9,A, B, C, 10, 11,...19, 1A, 1B, 1C, 20
- Base 4 0, 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23, 30, 31, 32, 33, 100, 101, ...

Bit Shifting

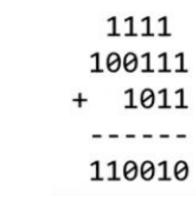
- Shifting of the digits in binary (used in registers)
- Right Shift: 11010 >> 2 (Drop the right-most 2 digits)
- \bullet = 110₂ = 6₁₀
- Left Shift: 100 << 1 (Add 1 zero to the end)
- \bullet = 1000₂ = 8₁₀
- If you have any number in other base, convert to binary and do the shifting
- 45₁₀ >> 3 (Convert to binary)
- = 101101₂ >> 3
- \bullet = 101₂ = $\overline{5}_{10}$

Addition of binary numbers

 Let's revise the decimal
Now, turn for binary addition



addition



Demo