## Solutions to Exercise Problems — Class-4 — Summer-2025

## Electric Flux

## Incepting Ideas (1)

Steps to follow:
**Given:**

$$\begin{aligned}
E &= 200\,\mathrm{N\,C^{-1}} \\
\theta &= 30^\circ \\
A &= 10\,\mathrm{cm} \times 20\,\mathrm{cm} = 0.10\,\mathrm{m} \times 0.20\,\mathrm{m} = 0.020\,\mathrm{m^2} \\
\Phi_E &= EA\cos\theta \\
&= \left(200\,\mathrm{N\,C^{-1}}\right)\left(0.020\,\mathrm{m^2}\right)\cos(30^\circ) \\
&= \left(200 \times 0.020 \times \frac{\sqrt{3}}{2}\right)\mathrm{N\,m^2\,C^{-1}} \\
&= 4 \cdot \frac{\sqrt{3}}{2}\,\mathrm{N\,m^2\,C^{-1}} \\
&= 2\sqrt{3}\,\mathrm{N\,m^2\,C^{-1}}
\end{aligned}$$

**Explanation:** Flux is positive because the angle between the electric field and the area vector is less than 90°. This means the electric field has a component in the direction of $\hat{n}$ (the outward area vector).

**Flux Sign and Extremes:**

- Positive flux: $0^\circ < \theta < 90^\circ$
- Negative flux: $90^\circ < \theta < 180^\circ$
- Maximum flux: $\theta = 0^\circ \Rightarrow \cos\theta = 1$
- Minimum (most negative) flux: $\theta = 180^\circ \Rightarrow \cos\theta = -1$
- Zero flux: $\theta = 90^\circ \Rightarrow \cos\theta = 0$

## Incepting Ideas (2)

Steps to follow:
**Part (a): Uniform Electric Field (left to right)**

**Given:** Cylinder of diameter $2R$, placed in a uniform electric field $E$ pointing left to right.
The cylinder has two flat circular ends (caps), each of area

$$A = \pi R^2$$

Electric field is parallel to the axis of the cylinder. The curved side has no flux since $\vec{E} \perp d\vec{A}$ there.
**Flux through left end:**

$$\Phi_{\text{left}} = EA\cos(180^\circ) = -EA = -E\pi R^2$$

**Flux through right end:**

$$\Phi_{\text{right}} = EA\cos(0^\circ) = EA = E\pi R^2$$

**Net flux:**

$$\begin{aligned}\Phi_{\text{net}} &= \Phi_{\text{left}} + \Phi_{\text{right}} \\ &= -E\pi R^2 + E\pi R^2 = 0\end{aligned}$$

**Part (b): Field points away from center, symmetrically**
In this case, the field on the left half of the cylinder points left, and on the right half it points right — both away from the center. The field is still uniform in magnitude.
**Flux through left end:**

$$\Phi_{\text{left}} = EA\cos(0^\circ) = EA = E\pi R^2$$

**Flux through right end:**

$$\Phi_{\text{right}} = EA\cos(0^\circ) = EA = E\pi R^2$$

**Net flux:**

$$\begin{aligned}\Phi_{\text{net}} &= \Phi_{\text{left}} + \Phi_{\text{right}} \\ &= E\pi R^2 + E\pi R^2 \\ &= 2E\pi R^2.\end{aligned}$$

## Application of Gauss's Law

## Incepting Ideas (3-Q1)

**Given:**

$$\begin{aligned}q &= 10\,\text{nC} = 10 \times 10^{-9}\,\text{C} \\ r &= 0.500\,\text{m} \\ \varepsilon_0 &= 8.85 \times 10^{-12}\,\text{C}^2\,\text{N}^{-1}\,\text{m}^2\end{aligned}$$

**(a) Surface area of the sphere:**

$$\begin{aligned}A &= 4\pi r^2 = 4\pi(0.500)^2 = 4\pi(0.25) = \pi\,\text{m}^2 \\ &\approx 3.14\,\text{m}^2\end{aligned}$$

**(b) Electric field on the surface:**

$$\begin{aligned}E &= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \\ &= \left(9.0 \times 10^9\,\text{N}\,\text{m}^2\,\text{C}^{-2}\right) \cdot \frac{10 \times 10^{-9}\,\text{C}}{(0.5)^2\,\text{m}^2} \\ &= 9.0 \times 10^9 \cdot \frac{10 \times 10^{-9}}{0.25} \\ &= 360\,\text{N}\,\text{C}^{-1}\end{aligned}$$

**(c) Electric flux through the sphere:**

$$\begin{aligned}\Phi_E &= \frac{q}{\varepsilon_0} = \frac{10 \times 10^{-9}\,\mathrm{C}}{8.85 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^2}} \\ &\approx 1130\,\mathrm{N\,m^2\,C^{-1}}\end{aligned}$$

**(d) If the point charge is moved inside the sphere but off-center:**
The flux *does not change*. Flux depends only on the total enclosed charge, not its position inside the surface.
**(e) Flux through a cube enclosing the sphere:**

The enclosed charge is still $q = 10\,\mathrm{nC}$, so:

$$\Phi_E = \frac{q}{\varepsilon_0} = \frac{10 \times 10^{-9}}{8.85 \times 10^{-12}} \approx 1130\,\mathrm{N\,m^2\,C^{-1}}$$

## Incepting Ideas (3-Q2)

Steps to follow:
**Given:**

$$\Phi_{\text{one side}} = -1.50\,\mathrm{kN\,m^2\,C^{-1}} = -1.50 \times 10^3\,\mathrm{N\,m^2\,C^{-1}}$$

A cube has 6 sides. Since the charge is at the center, the electric flux is uniformly distributed over all faces.
**Total flux:**

$$\begin{aligned}\Phi_{\text{total}} &= 6 \cdot \Phi_{\text{one side}} = 6 \cdot (-1.50 \times 10^3)\,\mathrm{N\,m^2\,C^{-1}} \\ &= -9.00 \times 10^3\,\mathrm{N\,m^2\,C^{-1}}\end{aligned}$$

**Use Gauss's Law:**

$$\begin{aligned}\Phi_{\text{total}} &= \frac{q}{\varepsilon_0} \\ \Rightarrow q &= \Phi_{\text{total}} \cdot \varepsilon_0 \\ q &= (-9.00 \times 10^3)\,\mathrm{N\,m^2\,C^{-1}} \cdot 8.85 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^2} \\ q &= -7.97 \times 10^{-8}\,\mathrm{C} \approx -79.7\,\mathrm{nC}\end{aligned}$$

## Incepting Ideas (4)

Steps to follow:
**Concept:** Electric flux through a closed surface depends only on the total enclosed charge, not on the shape or size of the surface. By Gauss's law:

$$\Phi = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

**Given:**

- Surfaces A, C, D each enclose charge $+q$
- Surfaces B and E each enclose charge $+2q$

**Fluxes:**

$$\Phi_A = \Phi_C = \Phi_D = \frac{q}{\varepsilon_0} \quad \text{and} \quad \Phi_B = \Phi_E = \frac{2q}{\varepsilon_0}$$

**Ranking from largest to smallest:**

$$\Phi_B = \Phi_E > \Phi_A = \Phi_C = \Phi_D$$