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Routing policies and COI-based storage policies in picker-to-part systems

F. CARON†*, G. MARCHET† and A. PEREGO‡

The paper evaluates and compares the expected travel distance for different routing strategies – namely traversal and return policies — in low-level picker-to-part systems. Items are assigned to storage locations on the basis of the ratio of the required space to the order frequency (cube-per-order index or COI). The focus is on narrow-aisle systems, in which the distance travelled crossing the aisle from one side to the other is negligible compared to the distance travelled along the centreline of the aisle. For both routing policies, an efficient COI-based stock location assignment strategy is first developed. Second, analytical models are derived which relate the expected travel distance required to fill an order to the main system parameters (i.e. the COI-based ABC curve; the number of picks in a tour; the number, length and width of aisles). Simulation results confirming the accuracy of the analytical models are presented. Finally, preference regions as a function of the number of picks in a tour and differently skewed COI-based ABC curves are given for traversal and return policies.

1. Introduction

Solutions aimed at enhancing order picking efficiency often play a decisive role in reducing warehouse operating costs and improving customer service associated with picking activity (Frazelle 1989, Malton 1991). It is widely recognized that dramatic improvements are obtainable by optimizing the main operating policies: *batching*, i.e. grouping of customer orders in a warehouse order; *routing*, i.e. the sequence in which the items on a warehouse order are retrieved; and *storage*, i.e. where items are stored in the warehouse.

We focus on low-level picker-to-part systems in which items are typically stored on racks (or bin shelving) and in a tour the picker either walks or rides between locations picking all the items specified on the pick list (which is a list of locations to be visited in a tour). Given their inherent flexibility, these systems are extensively used in industrial applications. Assuming that the batching problem has already been solved, i.e. considering a given pick list, we restrict our investigation to routing and storage policies. The pick list may correspond to one customer order (order picking) or several customer orders (batch picking). In the remainder of this paper, the term 'order' indicates a 'warehouse order', i.e. a pick list which may derive from the application of batching policies. Even though a number of papers have concentrated on the improvement of picking efficiency through the optimisation of either routing or storage policy, little work has been done on the results which stem from the integrated optimization of these two basic operating policies.

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The objective of the paper is to compare different routing strategies, namely traversal and return policies, and associated storage policies based on the cube-per-order index (COI), i.e. the ratio of the required storage space to the order frequency of the item (Heskett 1963, Kallina and Lynn 1976), which has proven to be an efficient assignment methodology (Gibson and Sharp 1992, Kallina and Lynn 1976, Jarvis and McDowell 1991).

Improving order picking efficiency when dealing with routing and storage policies generally means minimizing the total expected order picking time, which includes three main components (Frazelle 1989):

(1) Travel time incurred in picking an order which corresponds to the length of a picking tour starting and ending at an input/output (I/O) point. Travel time can be seen as the sum of the 'within aisle' travel time, i.e. time spent travelling within stocking aisles, and the 'across aisles' travel time, i.e. time spent moving along cross-aisles from one stocking aisle to another.
(2) Processing time at the location points, i.e. the time required to execute tasks such as searching for pick locations, extracting items, documenting picking activity.
(3) Administrative time incurred at the start and finish of a picking tour, i.e. the time spent on administrative and start-up tasks, e.g. collecting or depositing a pick device (cart, roll cage, etc.), obtaining a pick list, etc.

Assuming that administrative and processing times are independent of the storage and routing policies and that travel time is a monotone increasing function of travel distance, the objective of minimizing the expected order picking time is equivalent to minimizing the expected distance travelled in a tour (Ratliff and Rosenthal 1983, Goetschalckx and Ratliff 1988, Frazelle 1989, Rana 1990, Jarvis and McDowell 1991, Hall 1993). To this end, analytical models will be presented which assess the expected travel distance required to fill an order for different routing policies and associated storage policies. The application of these models as a function of the main system parameters, i.e. the COI-based ABC curve, the number of locations visited in a tour and the warehouse layout, allows preference regions for the respective routing strategies to be identified.

The main features of the picker-to-part system referenced by the analytical models are:

- Horizontal travel system, i.e. the picker moves only along the aisle floor (low-level system with maximum location height less than two metres);
- Rectangular layout, with two sections, picking aisles running parallel to the warehouse front-end where the I/O point is located and an even number of aisles per section (as illustrated in Fig. 1). The alternative layout with picking aisles running perpendicular to the warehouse front-end has not been considered, since it yields worst performance in terms of 'across aisles' travel distance.
- Narrow-aisle system, i.e. travel distances are measured along the aisle centre-line;
- COI-based storage policy and random storage as a reference for comparison;
- Pick locations are assumed to be subject to independent demand.

The remainder of the paper is divided into six sections. Section 2 reviews the main alternatives with regards routing and storage policies; § 3 describes the general framework and main assumptions of the analytical models. Sections 4 and 5 describe the

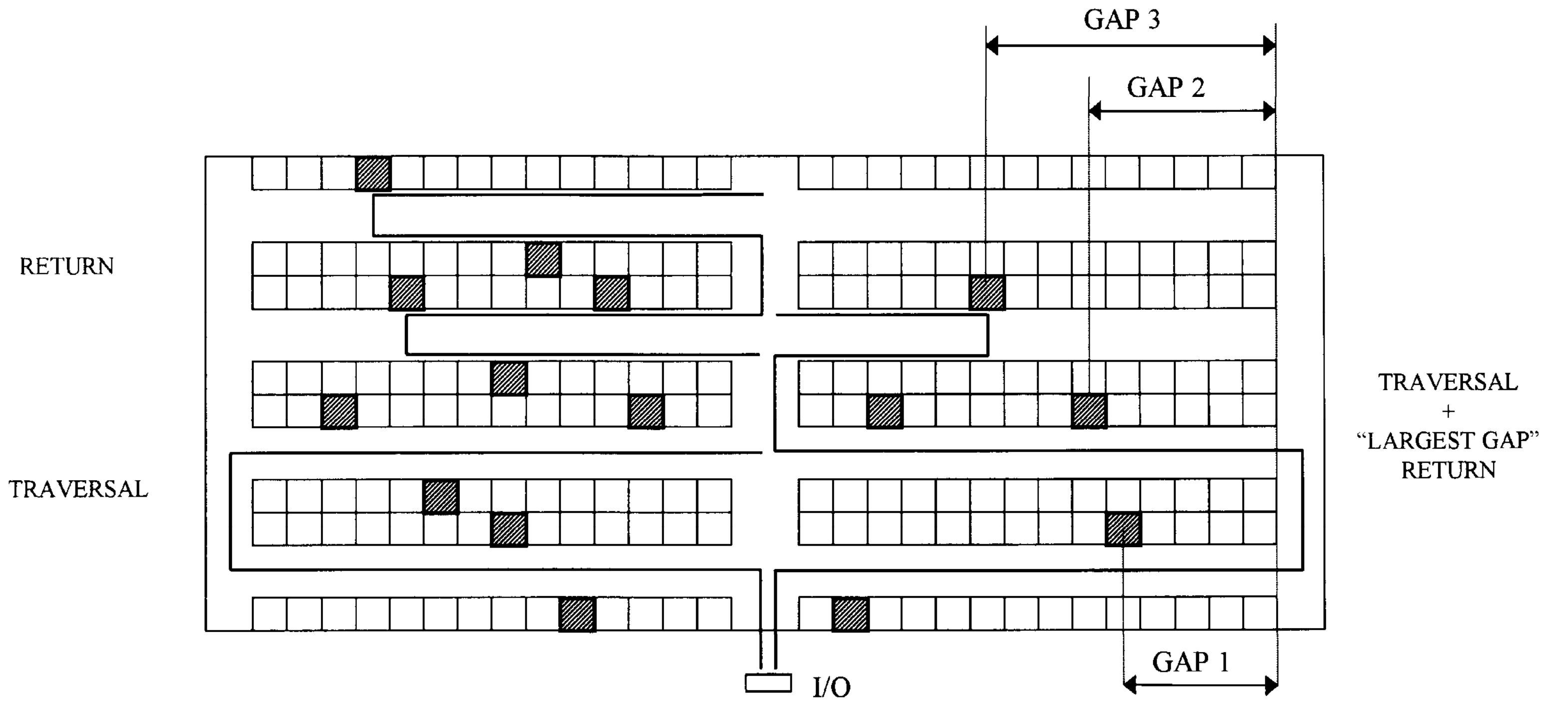

Figure 1. Layout and routing policies: traversal, return and traversal corrected by the largest gap (i.e. the picking tour requires the visit of 3 aisles in the right section; the third aisle has the largest gap and is selected for a return travel).

analytical travel models for the return and the traversal policies, respectively. In § 6, the results of the analytical models are compared with simulation results to test their accuracy. In the last section, the analytical models are used to define preference regions for traversal and return policies, so supporting the choice of the best picking strategy in terms of the main system parameters.

2. Literature review on routing and storage policies

The *routing problem* is a variant of the well-known and difficult to solve travelling salesman problem. Fortunately, in low-level picker-to-part systems the problem is simplified, since travel is only along aisles (horizontal travel systems with aisle-based metric, see Gibson and Sharp 1992). Different routing policies have been proposed for horizontal travel systems (Ratliff and Rosenthal 1983, Goetschalckx and Ratliff 1988, Hall 1993) (see Fig. 1; the circles indicate the locations of items to be picked):

- Traversal policy, i.e. the picker enters at one end of an aisle containing at least one pick and exits at the other end (see Goetschalckx and Ratliff 1988);
- Return policy, i.e. the picker enters and exits at the same end of the aisle (see Goetschalckx and Ratliff 1988);
- Largest gap return policy, i.e. the picker follows a return policy whereby the portion of the aisle which is not traversed is chosen so as to be the 'largest gap' (see Hall 1993). The largest gap may be between the central cross aisle and the first pick location, between the last pick location and the lateral cross aisle or between a pair of adjacent locations within the aisle.

Even though the largest gap policy may be useful in improving the basic traversal and return policies, choosing between the basic approaches may provide near-optimal routes and avoids the unnecessary complexity inherent in hybrid policies (Hall 1993).

Of the proposed *storage policies*—e.g. based on popularity, demand, size, merchandise type, hazard, etc.—the most effective in reducing picking travel distance with respect to random storage seem to be those based on COI (Kallina and Lynn 1976, Gibson and Sharp 1992) and correlated assignment (Frazelle 1989).

COI-based storage means assigning items with a low ratio of the required storage space to the order frequency to the locations nearest to the I/O point. The adoption of COI-based storage policies is generally a more 'information intensive' approach than random storage, since order and storage data must be processed in order to rank and assign items by decreasing COI. The availability of low cost computer systems operating on large data-bases makes the above requirement negligible, especially if the dramatic improvements in picking efficiency which stem from the adoption of advanced stock location assignment policies are taken into account. However, COI-based storage really requires item locations to be constantly reviewed in order to maintain storage strictly based on the ratio of required space to order frequency which is always changing in a highly dynamic environment. Since in practice only periodic reviews are possible, the COI-based ABC curve is less skewed than the theoretical curve, based on strict COI data, except in a short period following an assignment review. A similar conclusion applies if only a limited number of COI-based classes and corresponding storage classes are distinguished.

In addition, while random storage policies generate uniformly distributed activity over the picking area, the COI-based storage policies tend to concentrate picking operations in the areas dedicated to items with low COI (Kallina and Lynn 1976).

Obviously, congestion is not a problem if there is only one picker in the system (Jarvis and McDowell 1991). This applies not only to small warehouses, but also to large warehouses partitioned into zones with only one picker in each zone and a balanced work-load across the zones. In such systems, each operator picks a part of an order which is then consolidated with the other parts into a complete order. It should be noted that in picker-to-part systems COI-based storage policies may be used in conjunction with *correlated assignment policies* (Frazelle 1989). In this latter configuration, items with correlated demand are incorporated into the same cluster, and the clusters are assigned to the same column or to adjacent locations on the basis of their average COI values (Frazelle 1989). A potential disadvantage, which must be evaluated case by case, in adopting this approach is that the COI-based ABC curve for clusters tends to be less skewed than the ABC curve for single items.

The problem of evaluating expected travel distances for low-level picker-to-part systems has been studied by various authors in contexts assuming random storage (Kunder and Gudehus 1975, Ratliff and Rosenthal 1983, Goetschalckx and Ratliff 1988, Hall 1993). In the case of narrow-aisle warehouses and random assignment, the traversal policy always performs better than the basic return policy (Goetschalckx and Ratliff 1988, Hall 1993). However, in the case of COI-based storage, there are very few reports on the evaluation and selection of routing policies (e.g. Jarvis and McDowell 1991 who focus on traversal policies to provide a popularity-based method of assigning items in order to minimize the expected travel distance) and further investigation is needed.

3. Framework and assumptions of the travel models

In low-level picking systems, the time required to pick items is substantially independent of the location height. Therefore, only horizontal travel is considered when seeking to minimize the travel time incurred in picking an order. In addition, in narrow-aisle warehouses the distance travelled crossing the aisle from one side to the other is, in general, negligible compared to the distance travelled along the centreline of the aisle, if the number of picks per aisle is not very large (Rana 1990, Hall 1993). As a result, minimizing the total expected order picking time in low-level, narrow-aisle picker-to-part systems is equivalent to minimizing the expected horizontal distance, measured along aisle centrelines, required to fill an order.

Travel models for low-level picker-to-part systems aim to relate the expected horizontal travel distance to the main system parameters, such as the number of picks in a tour, the COI-based ABC curve and the warehouse layout (i.e. number, length and width of stocking and cross-aisles).

For both traversal and return policies, an efficient COI-based storage policy has been developed that assigns items with the lowest COI values to the locations which, in line with the routing policy, are closest to the I/O point.

For the return policy, this means assigning items with low COI values to the locations closest to the enter/exit point of each aisle (see Fig. 2). In fact, this strategy reduces the probability of visiting locations farthest from the enter/exit point of the aisle, thus reducing the expected 'within aisles' travel distance with respect to random storage.

For the traversal policy, the COI-based assignment strategy aims to reduce the number of aisles entered, while the allocation of items within a single aisle is irrelevant. In fact, all aisles which store at least one item to be picked have to be traversed completely, irrespective of the actual location of the item in the aisle. To this

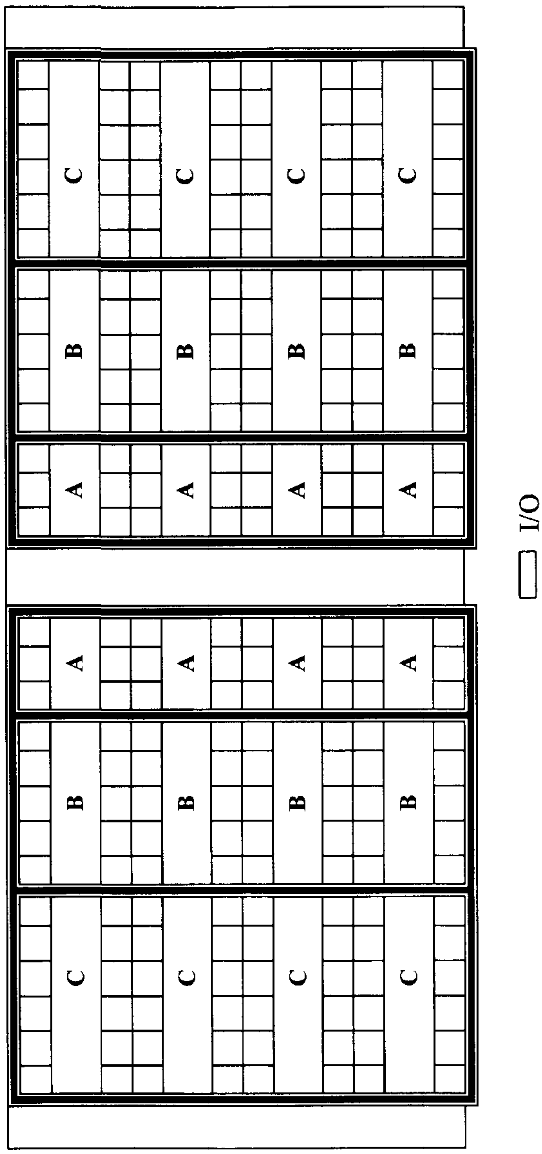

Figure 2. Partition of the warehouse in areas dedicated to low (A), medium (B) and high (C) COI items for the return policy.

end, items with the lowest COI values are assigned to the aisles closest to the warehouse I/O point, following the pattern illustrated in Fig. 3.

The COI-based ABC curve is obtained by ranking the items in ascending COI order and provides the fraction of location visits corresponding to a generic partition of the warehouse (for example, a given aisle).

An analytical function which well describes the COI-based ABC curve has the form:

$$F(x) = \frac{(1+s)\cdot x}{s+x} \quad F(x) \geq 0 \text{ and } x \leq 1, \; s \geq 0 \text{ and } s + x \neq 0 \tag{1}$$

where:

x indicates the ratio of required storage space to total storage space, corresponding to the items whose order frequency represents a fraction $F(x)$ of total warehouse activity;

s is the shape factor. It takes large values when the ABC curve is hardly skewed, for instance, s takes the following values: 0·33; 0·20; 0·12; 0·07 for 50/20, 60/20, 70/20 and 80/20 COI based ABC curves, respectively. (For large s, $F(x)$ approximates to a straight line, i.e. uniformly distributed activity over the picking locations).

Function (1) has been chosen since it depends on a single parameter and is therefore simple from the analytical point of view and second, its derivative always takes finite values even for x tending to zero.

The basic traversal policy will be slightly modified to incorporate return travel in the aisle with the largest gap each time the tour requires that an odd number of aisles in a section are visited (see Fig. 1). In the latter case, an exclusively traversal policy would cause the picker to end the tour in a lateral cross-aisle of the warehouse, thus needing to traverse an additional aisle to return to the central cross-aisle. The average return travel when adopting the 'largest gap' return policy is assumed to be half the stocking aisle.

For both travel models, the length of picking tours is computed assuming P as the reference point (see Fig. 4), as travel between P and the I/O point does not change with the different routing policies.

Table 1 and Fig. 4 provide the notation used.

4. Travel model for return policy

With return policy, the COI-based storage policy aims to reduce the expected travel distance 'within aisles' by assigning the lowest COI items to the locations closest to the enter/exit point of each aisle (Fig. 2). A representative sample of the total set of items is assigned to each aisle and consequently, the COI-based ABC curve within each aisle may be assumed to be the same for all aisles and identical to the ABC curve which applies to the total set of items.

The total expected travel distance (L^r) is the sum of the 'within aisles' component (L_I^r) and the 'across aisles' component (L_E^r):

$$L^r = L_I^r + L_E^r. \tag{2}$$

The expected travel distance 'within aisles' (L_I^r) depends on the expected number of aisles to be visited and the expected value of the farthest pick location within each aisle.

Since each aisle is equally likely to be visited, the expected number of visited aisles (v) has the form:

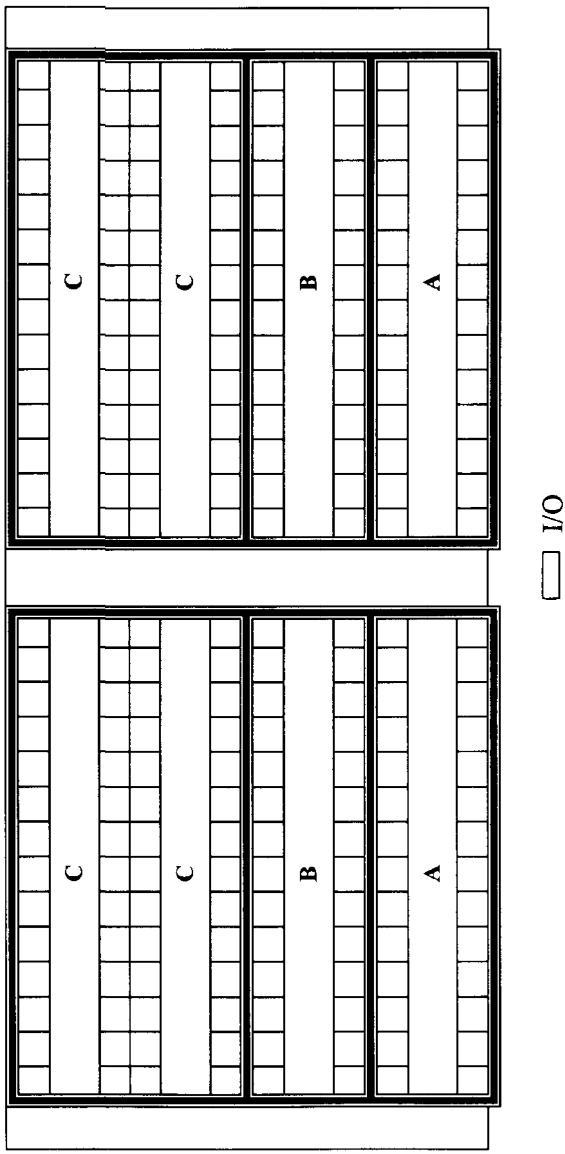

Figure 3. Partition of the warehouse in areas dedicated to low (A), medium (B) and high (C) COI items for the traversal policy.

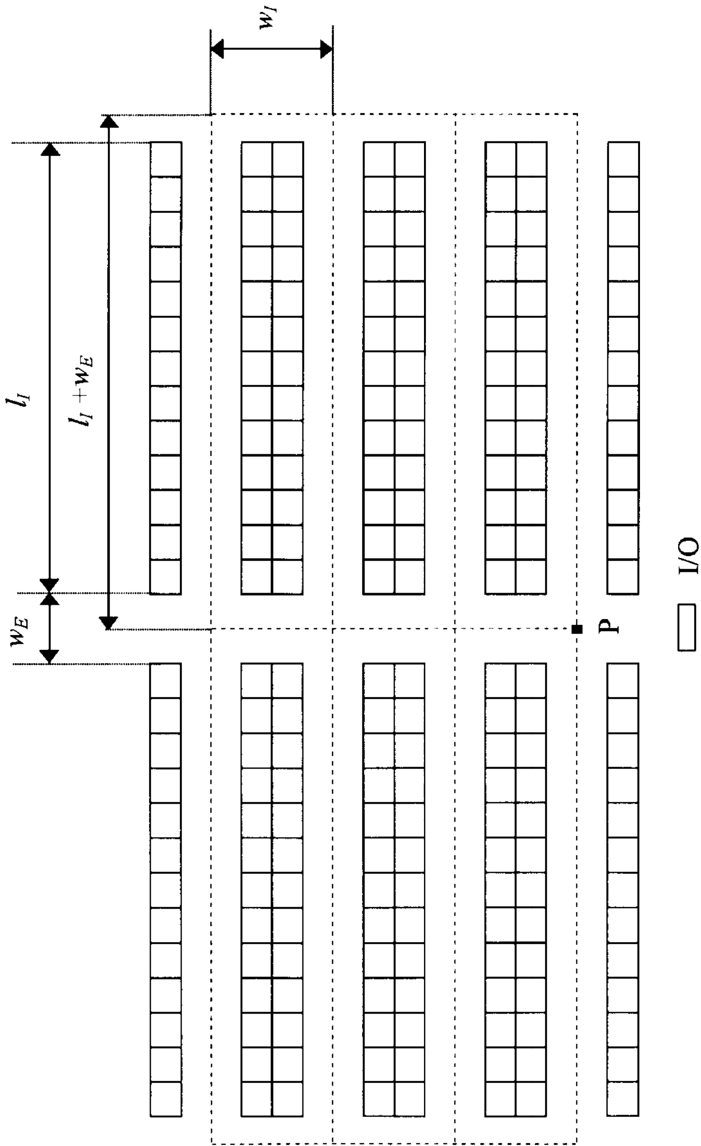

Figure 4. Warehouse layout.

a	number of stocking aisles
N	number of picks in a tour (required to fill a pick list)
v	expected number of visited aisles (when activity is uniformly distributed over aisles)
n	expected average number of picks in visited aisles ($n = N/v$)
l_I	length of stocking aisles [m]
w_I	width of stocking aisles (inclusive of shelves' width) [m]
w_E	width of cross aisles [m]
L^r	expected total travel distance for the return policy [m]
L_I^r	expected within aisles travel distance for the return policy [m]
L_E^r	expected across aisles travel distance for the return policy [m]
L^t	expected travel distance for the traversal policy [m]
L_I^t	expected within aisles travel distance for the traversal policy [m]
L_E^r	expected across aisles travel distance for the traversal policy [m]
$R(n)$	ratio of the expected value of the farthest pick location to the length of a stocking aisle
$F(x)$	COI-based ABC curve
x	ratio of required storage to total storage space
s	ABC curve shape factor

Table 1. Summary of notation.

$$v = a \cdot \left[1 - \left(1 - \frac{1}{a}\right)^{N}\right] \tag{3}$$

where:

a indicates the number of stocking aisles;

N indicates the number of picks in a tour;

$[1 - (1/a)]^N$ indicates the probability that N independent picks are not in a specific aisle.

As a consequence, the expected average number of picks per visited aisle (n) has the form:

$$n = \frac{N}{v}. \tag{4}$$

Assuming n picks per visited aisle and a continuous model of the aisle, the pick locations represent a random sample of n values extracted from a random variable with given cumulative distribution (uniform for random storage and described by an ABC curve for COI-based assignment). The farthest pick location in each aisle is thus a random variable distributed as the largest order statistic. If items are assigned to aisle locations on the basis of COI and the COI-based ABC curve has the previously described form—see equation (1)—then the expected value of the farthest pick location in each aisle corresponds to a fraction $R(n)$ of the aisle length (l_I) which equals the expected value of the largest order statistic:

$$R(n) = \int_0^1 \frac{dF^n(x)}{dx} \cdot x \cdot dx = \int_0^1 n \cdot \left[\frac{(1+s)\cdot x}{s+x}\right]^{n-1} \cdot \frac{s\cdot(1+s)}{(s+x)^2} \cdot x \cdot dx \tag{5}$$

The expected value of the farthest pick location is then used to derive the expected travel distance 'within aisles' (L_I^r):

$$L_I^r = v \cdot (w_E + 2 \cdot l_I \cdot R(n)), \tag{6}$$

where:

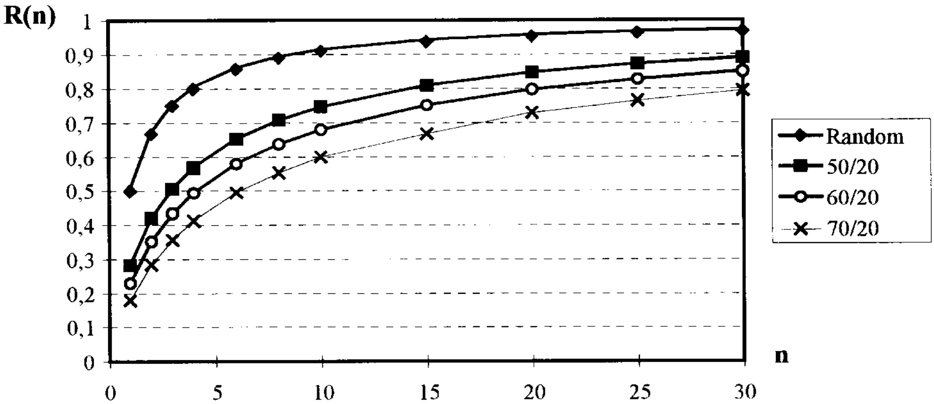

Figure 5. Largest order statistics function ($R(n)$, corresponding to the ratio of the expected value of the farthest pick location to the length of a stocking aisle) for different ABC curves.

the travel distance—there and back—between the centreline of the central cross-aisle and the beginning of the stocking aisle equals w_E;

$2 \cdot l_I \cdot R(n)$ indicates the expected travel distance—there and back–within the stocking aisle.

Figure 5 reports parameter $R(n)$ as a function of the average number of picks per aisle (n) and of the COI-based ABC curve relating to the generic aisle.

With random assignment policies, the 'within aisles' component has a simplified expression, in that the parameter $R(n)$ equals $[n/(n+1)]$

It should be noted that equation (6), which is based on the average number of picks per aisle, tends to overestimate the expected travel distance 'within aisles' (L_I^r). Consider, for instance, a COI-based 60/20 curve. Assuming 6 picks in 2 aisles, the expected travel distance 'within aisles' equals ($2 \cdot 0{\cdot}434 l_I = 0{\cdot}868 l_I$) if picks are equally distributed over the aisles ($n = 3$), while it takes a lower value ($0{\cdot}494 l_I + 0{\cdot}352 l_I = 0{\cdot}846 l_I$) if 4 picks are in one aisle and 2 in the other. The over-estimation deriving from the use of the average number of picks per aisle (n) rather than the exact distribution in equation (6) has a maximum for values of n around 2 and becomes negligible for large values of n. The simulation experiments support this conclusion (see in particular Fig. 8).

The expected travel distance 'across aisles' (L_E^r), i.e. travel along the central cross-aisle, depends on which is the farthest aisle to be visited. L_E^r may be evaluated with the following formula, which assumes that each aisle is equally likely to be visited (distances are measured taking **P** as the reference point, see Fig. 4):

$$L_E^r = 2 \cdot \sum_{j=2}^{a/2} \left(w_I \cdot (j-1) \cdot \left[\left(\frac{2 \cdot j}{a} \right)^N - \left(\frac{2 \cdot j - 2}{a} \right)^N \right] \right) = 2 \cdot w_I \cdot \left[\frac{a}{2} - \sum_{j=1}^{a/2} \left(\frac{2 \cdot j}{a} \right)^N \right], \tag{7}$$

where

$$\left[\left(\frac{2 \cdot j}{a} \right)^N - \left(\frac{2 \cdot j - 2}{a} \right)^N \right]$$

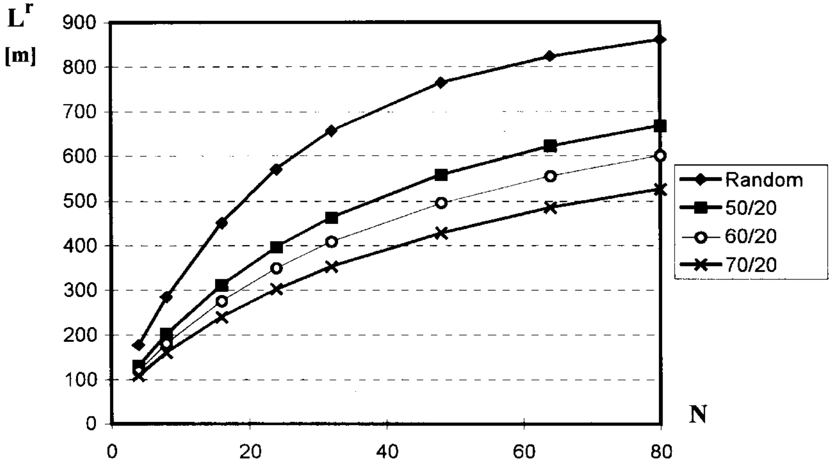

Figure 6. Expected total travel distance for the return policy (number of aisles $a = 16$; length of stocking aisles $l_I = 28$ m; stocking aisle width (inclusive of shelves' width) $w_I = 5$ m; cross aisle width $w_E = 3$ m).

indicates the probability that the farthest aisle to be entered is one of the two aisles which are $w_I \cdot (j - 1)$ from the point P.

For illustrative purposes, the relation for different COI-based ABC curves between the total expected travel distance with return policy, given by equations (2), (6) and (7), and the number of picks in a tour (N) is illustrated in Fig. 6. The expected travel distance for random storage $(L^r(ran))$ can be used as a reference to evaluate the improvement obtained using the COI-based storage policy.

5. Travel model for traversal policy

In the case of traversal policy, COI-based storage aims to reduce the expected number of aisles visited. To this end, the lowest COI items are assigned to the aisles closest to the I/O point, as illustrated in Fig. 3. Furthermore, it is preferable to adopt a warehouse layout with an even number of aisles per section, so as to avoid additional travel when there is a large number of location visits per tour and it is thus necessary to enter all aisles.

When an odd number of aisles is to be visited in a section, traversal policy is not applied to all aisles. It would make no sense to travel at the end of the tour to the end of the current aisle and then traverse an additional aisle—corresponding to the length l_I—in order to return to the central cross aisle. The solution is a routing policy with an even number of completely traversed aisles and return travel in the aisle with the 'largest gap' (i.e. the shortest return distance of the aisles in that section of the warehouse, see Fig. 1). We assume that, on average, the farthest picking location in the 'largest gap' aisle is positioned halfway along the aisle, and thus the expected travel distance—there and back—equals $(l_I + w_E)$.

As with return policy, the total expected travel distance with traversal policy (L^t) is given by the sum of the 'within aisles' (L_I^t) and 'across aisles' (L_E^t) components:

$$L^t = L_I^t + L_E^t \tag{8}$$

The expected travel distance 'within aisles' (L_I^t) can be evaluated as the product of the expected number of visited aisles and the distance $(l_I + w_E)$. The latter is the exact distance for all completely traversed aisles and an estimate for the aisle which may be visited as a result of a return policy. The expected travel distance 'within aisles' thus has the form:

$$L_I^t = (l_I + w_E) \cdot \sum_{i=1}^{a} \left(1 - \left\{ 1 - \left[F\left(\frac{i}{a}\right) - F\left(\frac{i-1}{a}\right) \right] \right\}^N \right), \tag{9}$$

where

$F(\cdot)$ is the COI-based ABC curve;

$$\left[F\left(\frac{i}{a}\right) - F\left(\frac{i-1}{a}\right) \right]$$

indicates the probability that a pick is in the ith aisle, and

$$\left\{ 1 - \left[F\left(\frac{i}{a}\right) - F\left(\frac{i-1}{a}\right) \right] \right\}^N$$

indicates the probability that N independent picks are not in the ith aisle.

Following the same reasoning applied to return policy, the expected travel distance 'across aisles' (L_E^t) equals the distance of the enter/exit point of the farthest aisle from the point P. However, due to the particular COI-based storage associated to traversal policy, the aisles closest to the I/O point are more likely to be visited. L_E^t thus has the form:

$$L_E^t = 2 \cdot \sum_{j=2}^{a/2} \left(w_I \cdot (j-1) \cdot \left\{ \left[F\left(\frac{2 \cdot j}{a}\right) \right]^N - \left[F\left(\frac{2 \cdot j - 2}{a}\right) \right]^N \right\} \right) \tag{10}$$

where

$$\left[F\left(\frac{2 \cdot j}{a}\right) \right]^N - \left[F\left(\frac{2 \cdot j - 2}{a}\right) \right]^N$$

indicates the probability that the farthest aisle entered is one of the two which are $w_I \cdot (j-1)$ from point P.

In the case of random assignment, the 'within aisles' travel component (L_I^t) is simply evaluated as the product of the distance $(l_I + w_E)$ and the expected number of visited aisles (v), while the 'across aisles' component (L_E^t) is the same as that for return policy, since each aisle is equally likely to be visited. Hence, the total expected travel distance with random assignment $(L^t(ran))$ has the form:

$$L^t(ran) = v \cdot (w_E + l_I) + 2 \cdot w_I \cdot \left[\frac{a}{2} - \sum_{j=1}^{a/2} \left(\frac{2 \cdot j}{a} \right)^N \right] \tag{11}$$

$L^t(ran)$ can be used as a reference to evaluate the improvement obtained using the COI-based storage policy.

For illustrative purposes, the relation for different COI-based ABC curves between the total expected travel distance with traversal policy, given by equations (8), (9), and (10), and the number of picks in a tour (N) is illustrated in Fig. 7.

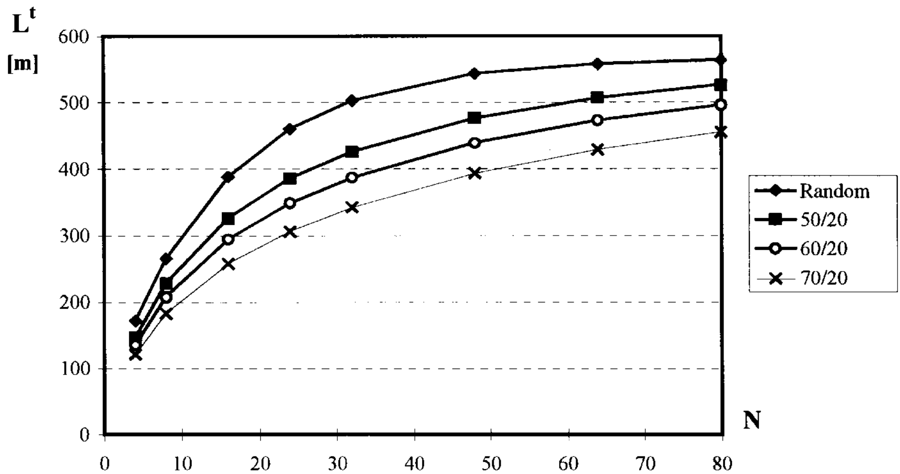

Figure 7. Expected total travel distance for the traversal policy (number of aisles a= 16; length of stocking aisles l_I = 28 m; stocking aisle width (inclusive of shelves' width) w_I = 5 m; cross aisle width w_E = 3 m).

6. Validation of the models by simulation

A simulation model of the picking system referenced by the analytical models described in sections 4 and 5 has been implemented in Visual Basic. A set of simulation experiments was conducted in order to test the accuracy of the analytical models.

Given a certain order size (N) which determines the number of locations to be visited in a tour, the simulation software generates a sample of pick locations for each run on the basis of the routing and associated storage policies. For return policy, the distribution of the N pick locations within the different aisles is random, while positioning within each aisle is described by the COI-based ABC. The opposite rule applies to the traversal policy, whereby the assignment of the pick locations within the aisles follows the COI-based ABC curve and positioning within each aisle is random.

This way of generating the sample of pick locations is fully consistent with the probability framework developed for the analytical models. On the other hand, the calculation of travel distance differs from that adopted for the analytical approach in two respects:

- No assumption is made with regard to the return travel distance when adopting the 'largest gap' return policy, since the simulation model calculates the effective largest gap and executes return travel in the corresponding aisle.
- The 'within aisles' component is calculated from the actual distribution of locations in each aisle rather than on the basis of the average numbers of visited aisles and picks per aisle.

The simulation model calculates the length of each randomly generated tour according to the routing policy chosen. The number of picking tours, and consequently the number of simulation runs, is a compromise between time-efficiency and the required width of the confidence interval for the outcomes. The final outcome

Number of picks in a tour	Simulation experiments: Travel distance [m] Mean	Simulation experiments: Travel distance [m] Std. dev.	Simulation experiments: Confidence interval width (%) (0·01 significance level)	Analytical model: Travel distance [m]	Difference between analytical and simulation results (%)
4	129·8	30·8	1·6	130·5	0·54
8	198·8	39·0	2·0	201·0	1·11
16	302·2	48·8	2·5	311·2	2·98
24	380·5	54·0	2·8	396·0	4·07
32	442·9	56·6	2·9	462·6	4·45
48	534·5	56·8	2·9	558·1	4·42
64	598·7	54·7	2·8	621·7	3·84
80	646·9	52·7	2·7	667·2	3·14

Table 2. Simulation results for the return policy—50/20 rule (number of aisles $a = 16$; length of stocking aisles $l_I = 28$ m; stocking aisle width (inclusive of shelves' width) $w_I = 5$ m; cross aisle width $w_E = 3$ m).

Number of picks in a tour	Simulation experiments: Travel distance [m] Mean	Simulation experiments: Travel distance [m] Std. dev.	Simulation experiments: Confidence interval width (%) (0·01 significance level)	Analytical model: Travel distance [m]	Difference between analytical and simulation results (%)
4	144·2	32·2	1·7	146·0	1·25
8	219·7	39·7	2·0	227·8	3·68
16	316·2	46·4	2·4	326·2	3·16
24	375·5	47·6	2·4	385·5	2·66
32	415·3	46·0	2·4	425·8	2·53
48	468·2	42·9	2·2	476·8	1·84
64	499·4	39·4	2·0	506·8	1·48
80	521·7	32·2	1·8	525·7	0·77

Table 3. Simulation results for the traversal policy–50/20 rule (number of aisles $a = 16$; length of stocking aisles $l_I = 28$ m; stocking aisle width (inclusive of shelves' width) $w_I = 5$ m; cross aisle width $w_E = 3$ m).

reports the mean tour length, standard deviation and width of the confidence interval at the desired level of significance for the chosen system configuration.

The simulation model was run 10 000 times for each set of system parameters examined and the width of the confidence interval for the travel distance was assessed at a significance of 0·01.

For illustrative purposes, Tables 2 and 3 report the results generated for both routing policies by the analytical and simulation models with a 50/20 ABC curve and varying numbers of picks in a tour. The percentage difference between the predicted travel distance and the mean evaluated by the simulation experiments is highlighted (see Figs 8 and 9 for a visual representation of these differences).

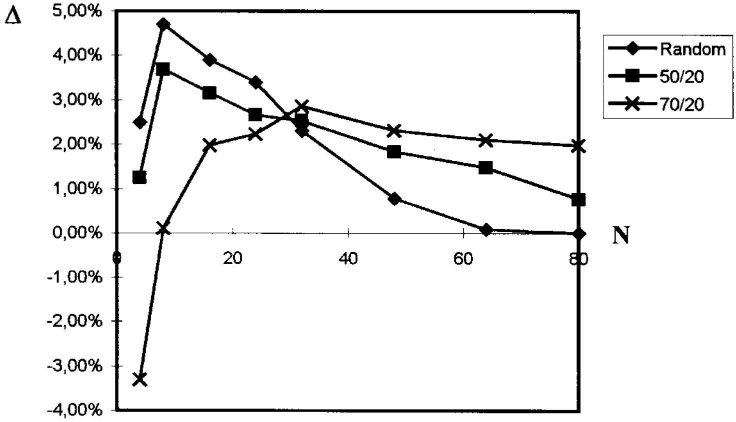

Figure 8. Difference (Δ) between the picking travel distance value predicted analytically and the mean simulation result (10 000 runs) for the return policy (number of aisles a= 16; length of stocking aisles l_I = 28 m; stocking aisle width (inclusive of shelves' width) w_I = 5 m; cross aisle width w_E = 3 m).

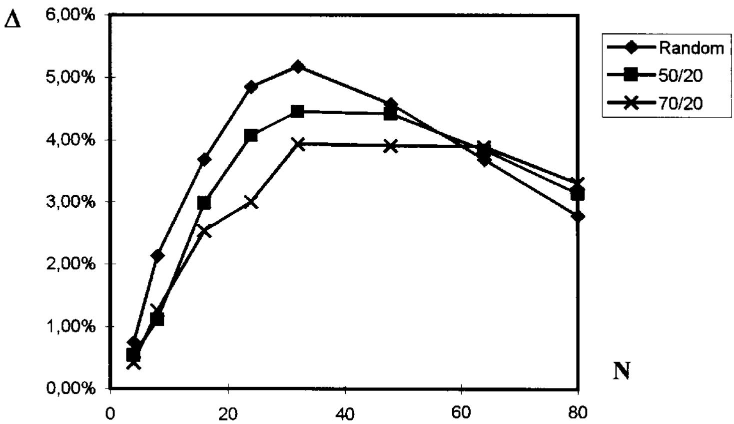

Figure 9. Difference (Δ) between the picking travel distance value predicted analytically and the mean simulation result (10 000 runs) for the traversal policy (number of aisles a= 16; length of stocking aisles l_I = 28 m; stocking aisles width (inclusive of shelves' width) w_I = 5 m; cross aisle width w_E = 3 m).

Some brief conclusions may be drawn from the results of the simulation experiments:

(1) The differences between the value predicted analytically and the mean simulation result are generally less than 5%, confirming that the models provide good evaluations of the expected total travel distances.
(2) The large standard deviation is a result of the great variability in the length of sampled picking tours. Notwithstanding the large number of simulation runs (10 000), the width of the confidence interval at a significance of 0·01 is about 2–3% of the mean total travel distance.

(3) With return policy, the analytical model overestimates the expected travel distance (but differences do not exceed 5%), thus providing a conservative approximation. This bias is due to the evaluation of the 'within aisles' travel on the basis of the average number of picks per aisle. In fact, as previously described, the actual distribution of picks in the warehouse generates shorter picking tours than a uniform distribution of picks in the aisles visited. This bias peaks when the number of picks in a tour is about twice the number of aisles, i.e. the average number of picks per visited aisle is about 2, since this represents the situation in which the approximation of the analytical formula seems to be more prominent.

(4) With traversal policy, the differences between the value predicted by the analytical model and the mean from the simulation result primarily from the assumption about the length of the possible 'largest gap' return travel when an odd number of aisles is visited in a section. The analytical model assumes the average 'largest gap' return travel to be half the aisle length. In the simulation model, the 'largest gap' policy should allow shorter return travel to be chosen. This explains why, in general, the analytical model overestimates the total travel distance with respect to the simulation experiments (see Fig. 9). For a large number of picks in a tour, differences generally tend to be negligible, as (1) the same absolute differences represent a smaller part of the overall picking tour and (2) the probability of entering all aisles, thus reducing the need for a return policy, becomes correspondingly high. Obviously, this trend is more evident in random as opposed to COI-based storage, which tends to concentrate the locations to be visited in the aisles closest to the I/O point. For a low number of picks in a tour and especially for skewed COI-based ABC curves, the picker is likely to visit only a few aisles. Consequently, return travel is highly probable, but there is little choice in the 'largest gap' aisle.

In conclusion, the simulation experiments definitely confirm that the analytical models are satisfactorily accurate in evaluating the expected travel distances, as the main approximations lead to negligible errors.

7. Comparison between return and traversal policies

The aim of this section is to provide some guidelines for choosing the best picking strategy for given warehouse and order parameters in COI-based picking systems. The formulas in §§ 4 and 5 calculate the total expected travel distance for return and traversal policies, respectively, as a function of the main system parameters, i.e. the COI-based ABC curve, the number of picks in a tour and the warehouse layout. The application of these formulas can be translated into tables of relative preference which represent the best routing policy for given values of the system parameters. For illustrative purposes, Tables 4 and 5 report the preference region for the two routing strategies as a function of the COI-based ABC curve and the average number of picks per aisle.

The results obtained show that preference regions are primarily affected by the average number of picks per aisle and the different ABC curves, while they are relatively insensitive to the number of aisles. The traversal policy generally outperforms the basic return policy, except where there is a low number of picks per aisle

Number of picks in a tour (N)	Average number of picks per aisle	Random storage	COI-based ABC curve		
			50/20	60/20	70/20
4	0·25	T/I	R	R	R/I
8	0·5	T/I	R	R	R
16	1	T	R/I	R/I	R/I
24	1·5	T	T/I	T/I	R/I
32	2	T	T/I	T/I	T/I
48	3	T	T	T	T/I
64	4	T	T	T	T
80	5	T	T	T	T

R: return policy dominance,
T: traversal policy dominance,
I: indifference (traversal and return travels differ less than 10%).

Table 4. Dominance region for return and traversal policies (number of aisles a = 16; length of stocking aisles l_I = 28 m; stocking aisle width (inclusive of shelves' width) w_I = 5 m; cross aisle width w_E = 3 m).

Number of picks in a tour (N)	Average number of picks per aisle	Random storage	COI-based ABC curve		
			50/20	60/20	70/20
4	0·5	T/I	R	R	R
8	1	T	R/I	R/I	R/I
16	2	T	T/I	T/I	T/I
24	3	T	T	T	T/I
32	4	T	T	T	T
48	6	T	T	T	T
64	8	T	T	T	T
80	10	T	T	T	T

R: return policy dominance,
T: traversal policy dominance,
I: indifference (traversal and return travels differ less than 10%).

Table 5. Dominance region for return and traversal policies (number of aisles a = 8; length of stocking aisles l_I = 28 m; stocking aisle width (inclusive of shelves' width) w_I = 5 m; cross aisle width w_E = 3 m).

(average less than 1) or skewed COI-based ABC curves. The above conclusions rely on the following facts:

- With random storage, the traversal policy always performs equal to or better than the basic return policy, in accordance with the conclusions of Goetschalckx and Ratliff (1988) and Hall (1993). Indeed, the 'across aisles' component is always shorter for the traversal policy. Moreover, with traversal policy, the expected 'within aisles' travel distance in an aisle is equal to the length of the aisle, irrespective of the number of picks in that aisle, while with return policy this only holds for one pick per aisle. With more than one pick in an aisle, the expected return travel is higher than the aisle length (see the values corresponding to random storage in Fig. 5).

- The advantages stemming from the adoption of COI-based storage with respect to random storage are greater with return policy than with traversal policy, such that the former, which is always outperformed with random storage, becomes preferable for a low number of picks per aisle (see Tables 4 and 5). This is due to a more intensive use with return policy of COI-based information which assigns items to locations within aisles in function of COI ordering. On the other hand, with traversal policy COI-based information is used to create a finite number of classes and allocate these to different aisles, but the allocation in each aisle does not follow COI-based ordering.
- For large values of N, the traversal policy outperforms the return policy, irrespective of the skewness of the COI-based ABC curve, since the increase with return policy in the expected 'within aisles' travel more than compensates the advantages stemming from the efficient COI-based storage assignment within each aisle.

The preference region for the return policy, illustrated in Tables 4 and 5 on the assumption of narrow aisles, would be wider if the distances travelled across the aisle from one side to the other were considered. In fact, this zig-zag travel penalizes only traversal policy (in return policy the picker crosses the aisle from one side to the other only once at the end of travel within an aisle). On the other hand, the advantages of return policy for skewed COI-based ABC curves rely on the assumption that items are frequently re-located to preserve strict storage based on COI values. This may be a major limitation in a highly dynamic environment. Conversely, with traversal policy, item assignment 'within aisles' is not subject to any constraint.

For the same reason, traversal policies may allow correlated and COI-based assignment policies to be integrated. As already mentioned, assignment 'within aisles' is highly flexible, thus allowing items with correlated demand to be assigned to the same column or to adjacent locations.

8. Conclusions

The expected travel distance in narrow-aisle COI-based picker-to-part picking systems using traversal and return policies has been evaluated by means of analytical models taking account of the main system parameters. In particular, the number of picks in a tour and the warehouse layout determine the average number of picks in an aisle, which appears to be a very critical parameter. In turn, the number of picks in a tour depends on the order profile, i.e. the distribution of the number of lines per order and on whether batch picking (i.e. aggregation of several customer orders in one warehouse order) or order picking (i.e. picking of a single customer order in each tour) is used.

Efficient COI-based assignment policies have been designed to exploit the specific features of the two routing policies.

The results obtained from the analytical models have been tested in simulation experiments. Finally, the traversal and the return policies have been compared on the basis of the expected travel distances provided by the analytical formulas.

The choice of the best routing policy appears to depend primarily on the average number of picks per aisle, while the ABC curve seems to play only a secondary role.

In general, for COI-based storage systems, the return policy outperforms the traversal policy only for a low number of average picks per aisle (i.e. <1) and for skewed COI-based ABC curves (for instance 70/20). This strictly holds for narrow-aisle systems. However, if the aisle width is not negligible, the expected value of the traversal travel has to include crossovers from one side of the aisle to the other. As a consequence, in warehouses with aisles of non-negligible width, the preference region of traversal policy may be reduced with respect to return policy.

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