Numerical Optimization Assignment

Calculate the gradient for the function f(x, y) = x3 − 2x2y +

4y2 at (1,1)

1. Calculate the gradient for the function $f(x, y) = x^3 - 2x^2y + 4y^2$ at the point (1,1).

The gradient of a function $f(x, y)$ is given by the vector of its partial derivatives with respect to each variable:

$$\nabla f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$$

Partial derivative with respect to x:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left(x^3 - 2x^2y + 4y^2 \right)$$

Let's differentiate each term:

- For x^3, the derivative with respect to x is $3x^2$.
- For $-2x^2y$, treat y as a constant, so the derivative is $-4xy$.
- For $4y^2$, the derivative with respect to x is 0 (since it is independent of x).

Thus,

$$\frac{\partial f}{\partial x} = 3x^2 - 4xy$$

Partial derivative with respect to y:

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \left(x^3 - 2x^2y + 4y^2 \right)$$

Let's differentiate each term:

- For x^3, the derivative with respect to y is 0 (since it is independent of y).
- For $-2x^2y$, treat x^2 as a constant, so the derivative is $-2x^2$.
- For $4y^2$, the derivative with respect to y is $8y$.

Thus,

$$\frac{\partial f}{\partial y} = -2x^2 + 8y$$

Now, substituting $x = 1$ and $y = 1$ into the partial derivatives:

$$\left.\frac{\partial f}{\partial x}\right|_{(1,1)} = 3(1)^2 - 4(1)(1) = 3 - 4 = -1$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,1)} = -2(1)^2 + 8(1) = -2 + 8 = 6$$

So, the gradient at $(1, 1)$ is:

$$\nabla f(1,1) = (-1, 6)$$

2. What is the function called?

The function $h_\theta(x) = \theta_0 + \theta_1 x$ is the hypothesis function in the context of linear regression.

The correct answer is:

a) Hypothesis.

3. What does the function represent?

The function $h_\theta(x) = \theta_0 + \theta_1 x$ represents a straight line, which is used in the context of linear regression with a single predictor variable.

The correct answer is:

b) Single variable linear regression.

4. Is vector norm related to the cost function (True/False)? Explain why in brief.

Answer: True.