

Assignment 2

Problem 4.34: Standard Deviation of X

Given the probability distribution:

x	-2	3	5
$f(x)$	0.3	0.2	0.5

Step 1: Calculate the Expected Value (Mean), μ_X

The expected value μ_X is calculated using the formula:

$$\mu_X = \sum x_i \cdot f(x_i)$$

Substitute the values:

$$\mu_X = (-2 \times 0.3) + (3 \times 0.2) + (5 \times 0.5)$$

$$\mu_X = -0.6 + 0.6 + 2.5 = 2.5$$

So, the expected value (mean) μ_X is **2.5**.

Step 2: Calculate the Variance, σ^2_X

The variance σ^2_X is calculated using the formula:

$$\sigma^2_X = \sum (x_i - \mu_X)^2 \cdot f(x_i)$$

Substitute the values:

$$\sigma^2_X = [(-2 - 2.5)^2 \cdot 0.3] + [(3 - 2.5)^2 \cdot 0.2] + [(5 - 2.5)^2 \cdot 0.5]$$

$$\sigma^2_X = [(-4.5)^2 \cdot 0.3] + [(0.5)^2 \cdot 0.2] + [(2.5)^2 \cdot 0.5]$$

$$\sigma^2_X = (20.25 \cdot 0.3) + (0.25 \cdot 0.2) + (6.25 \cdot 0.5)$$

$$\sigma^2_X = 6.075 + 0.05 + 3.125 = 9.25$$

So, the variance σ^2_X is 9.25.

Step 3: Calculate the Standard Deviation, σ_X

The standard deviation σ_X is the square root of the variance: $\sigma_X = \sqrt{9.25} \sim 3.041$

So, the standard deviation σ_X is approximately 3.041.

Problem 4.10: Find μ_X and μ_Y

Given the joint distribution:

$f(x, y)$		y		
		1	2	3
x	1	0.10	0.05	0.02
	2	0.10	0.35	0.05
	3	0.03	0.10	0.20

Step 1: Calculate the Marginal Distribution $f_X(x)$ and $f_Y(y)$

The marginal distribution $f_X(x)$ is obtained by summing the joint probabilities across all values of y for each x :

$$f_X(1) = 0.10 + 0.05 + 0.02 = 0.17$$

$$f_X(2) = 0.10 + 0.35 + 0.05 = 0.50$$

$$f_X(3) = 0.03 + 0.10 + 0.20 = 0.33$$

So, the marginal distribution $f_X(x)$ is:

x	1	2	3
$f_X(x)$	0.17	0.50	0.33

Similarly, the marginal distribution $f_Y(y)$ is obtained by summing the joint probabilities across all values of x for each y :

$$f_Y(1) = 0.1 + 0.1 + 0.03 = 0.23$$

$$f_Y(2) = 0.05 + 0.35 + 0.1 = 0.5$$

$$f_Y(3) = 0.02 + 0.05 + 0.2 = 0.27$$

So, the marginal distribution $f_Y(y)$ is:

y	1	2	3
$f_Y(y)$	0.23	0.50	0.27

Step 2: Calculate the Expected Values μ_X and μ_Y

For μ_X :

$$\mu_X = \sum_x x \cdot f_X(x)$$

Substitute the values:

$$\mu_X = (1 \cdot 0.17) + (2 \cdot 0.5) + (3 \cdot 0.33)$$

$$\mu X = 0.17 + 1 + 0.99 = 2.16$$

So, μX is 2.16.

For μY :

Substitute the values:

$$\mu Y = (1 \times 0.23) + (2 \times 0.50) + (3 \times 0.27)$$

$$\mu Y = 0.23 + 1.00 + 0.81 = 2.04$$

So, μY is 2.04.