Intermediate Micro: Midterm 1 Review

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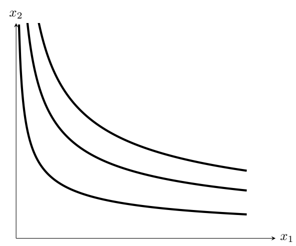
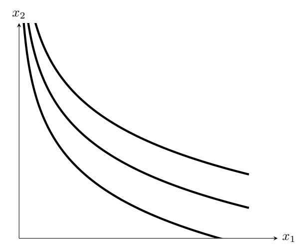
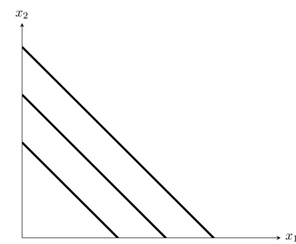
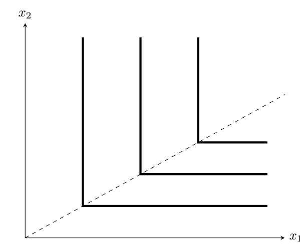
1 Summary

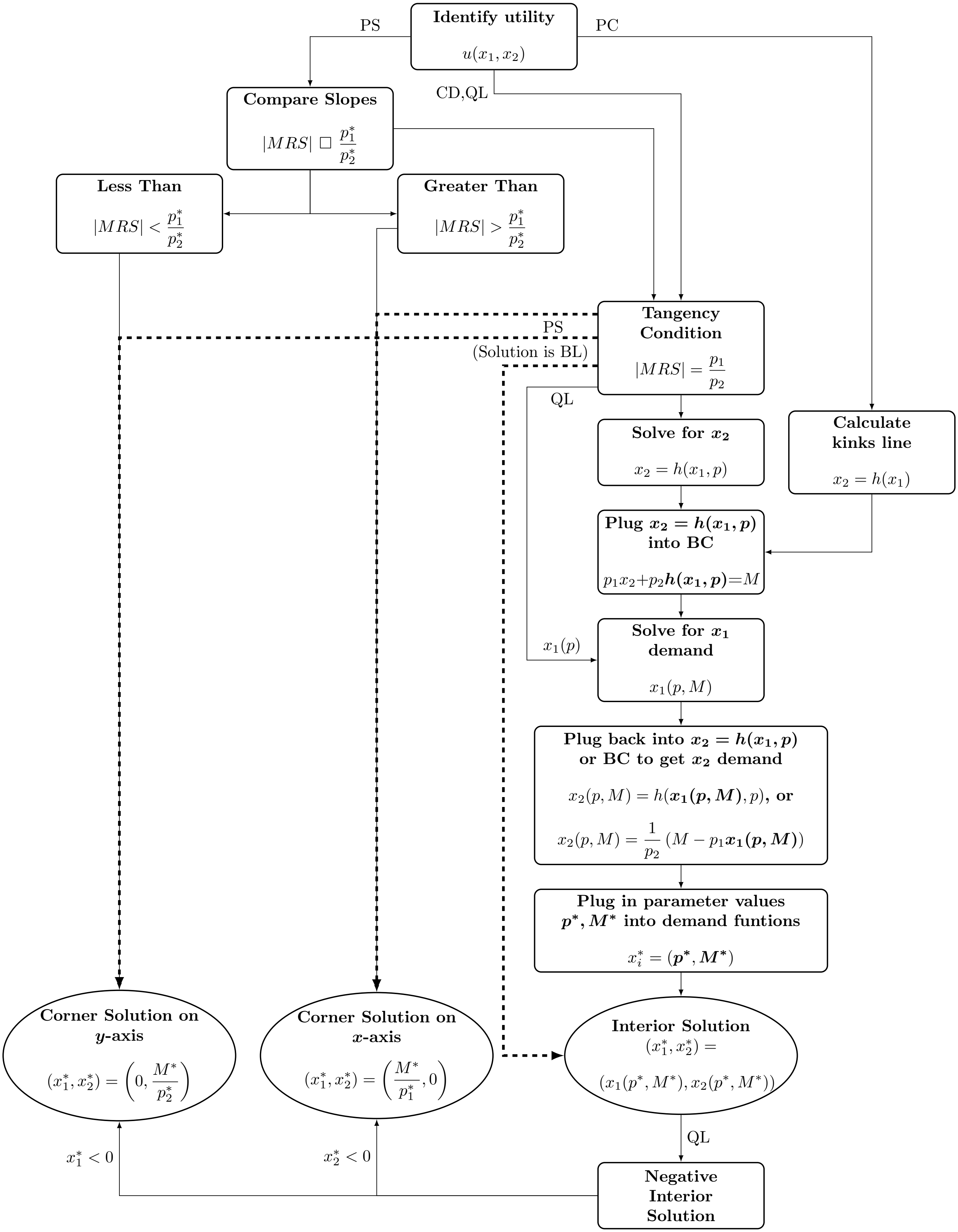
1.1 Exam Tips

- Before you start answering a question, identify what type of utility function it is, and what are the parameters (p_1,p_2,M). Think about the properties of this type of utility functions (e.g. shape of indifference curves)
- When drawing diagrams, make sure that you label your axis and all intercepts of a line. You don't need to use a ruler to measure everything out perfectly, just put clear markings that give a sense of the scale.
- Try to anticipate the result with economic intuition before mathematically solving for it (e.g. if income increases, what should generally happen to demand? Are there exceptions?)
- Never do the full Lagrangian, just start at the tangency condition. You should remember the formulas for Cobb-Douglas and Perfect Complements so you can check whether you've done it correctly.
- It's almost always better to solve for a demand function first and then plug in the parameters: keep the parameters as p_1,p_2,M; solve the optimization problem; at the end, plug in the actual parameter values given in the problem. This helps catch math mistakes because you can verify the demand function using economic intuition. Importantly, if a later question asks to change one of the parameter values, you just plug in the new values into the demand function (instead of re-solving everything).
- If you have time, plug the solutions back into the budget constraint to check they are correct.
- Make life easier for your grader - they want to give you points! Write neatly, in a logical order, and explain what you're doing at each step

1.2 Preferences and Utility

- **You should know:** preference properties (completeness, transitivity, monotonicity, convexity), preference symbols ($\succsim$,$\succ$,$\sim$), MRS, utility representation, and how to draw/interpret indifference curves
- **Plotting Indifference Curves:** Write as $u = u(x_1, x_2)$. Then solve for x_2 to get $x_2 = f(x_1, u)$ (express x_2 as a function of x_1 and u). Plug in different values of u, each giving a different indifference curve.
- **Monotonic Transformation:** A monotonic transformation is a function $f(x)$ which has a strictly positive slope, i.e. $f'(x) > 0$.
  - Two utility functions $u(x_1, x_2)$ and $v(x_1, x_2)$ represent the same preferences if there is a monotonic transformation between them, i.e. $v(x_1, x_2) = f(u(x_1, x_2))$. They should also have the same MRS.
- **Convexity:** A convex function $f(x)$ has a positive second derivative, i.e. $f''(x) \geq 0$ (for strictly convex, we need $f''(x) > 0$)
  - Convex preferences give us convex indifference curves (*not* convex utility functions). To check convexity, express the indifference curves as $x_2 = f(x_1, u)$ and then check the second derivative with respect to x_1: $\partial^2_{x_1} f(x_1, u)$. Equivalently, since MRS is slope, check $\partial_{x_1} MRS$

	Cobb-Douglas	Quasi-Linear	Perfect Substitutes	Perfect Complements
General Form	$x_1^\alpha x_2^\beta$	$f(x_1) + x_2$ $[f(\cdot)$ is non-linear$]$	$\frac{1}{\alpha}x_1 + \frac{1}{\beta}x_2$ **or** $\beta x_1 + \alpha x_2$	$\min\left\{\frac{1}{\alpha}x_1, \frac{1}{\beta}x_2\right\}$ **or** $\min\{\beta x_1, \alpha x_2\}$
Preferences	Always a mixer. α, β = relative care for each good	Generally a mixer. x_2 is numeraire	Non-mixer: α units of $x_1 \sim \beta$ units of x_2	Fixed mixer: α units of x_1 with β units of x_2
IC Graphs				
IC Properties	Vanilla ICs. Never touches the axis	Standard ICs. Touches x-axis, may touch y-axis. ICs parallel along y-axis	Linear. Touches the axis. ICs parallel	L-shaped. Never touches the axis Kinks Line: $x_2 = \dfrac{\beta}{\alpha}x_1$
MRS	$\dfrac{\alpha x_2}{\beta x_1}$	Independent of x_2: $f'(x_1)$	Constant: $\dfrac{\beta}{\alpha}$	Undefined (at kink)
Interior Solution	Always $x_1^* = \dfrac{\alpha}{\alpha+\beta} \cdot \dfrac{M}{p_1}$ $x_2^* = \dfrac{\beta}{\alpha+\beta} \cdot \dfrac{M}{p_2}$	Usually $x_1^* \rightarrow$Invert tangency condition $x_2^* = \dfrac{1}{p_2}\left(M - p_1 x_1^*\right)$	Only if: $\lvert MRS\rvert = \dfrac{p_1}{p_2}$ Then: entire budget line is optimal (incl. corners)	Always, at intersection of kinks line and budget line $x_1^* = \dfrac{\alpha M}{\alpha p_1 + \beta p_2}$ $x_2^* = \dfrac{\beta M}{\alpha p_1 + \beta p_2}$
Corner Solution	Never	If a good is too expensive and interior solution < 0 Always a corner on x-axis, sometimes on y-axis	$\lvert MRS\rvert < \dfrac{p_1}{p_2} \implies \left(0, \frac{M}{p_2}\right)$ $\lvert MRS\rvert > \dfrac{p_1}{p_2} \implies \left(\frac{M}{p_1}, 0\right)$	Never
Other Price Effects	Demand for good i independent of price of j	Substitutes	Substitutes	Complements
Income Effects	Demand increases with income, in fixed proportions. Expenditure on $x_1 = \frac{\alpha}{\alpha+\beta}$ share of income	At an interior, x_1^* is independent of income At an interior, as income increases, lower share spent on x_1	Demand increases with income Income does not affect mixing composition	Demand increases with income Income does not affect mixing composition



Notation: *CD=Cobb-Douglas* *QL=Quasi-Linear* *PS=Perfect Substitutes* *PC=Perfect Complements* *BL=Budget Line*

$x_2 = h(x_1, p)$ is x_2 expressed as a function of x_1 and prices, where $h(\cdot)$ is some function you will solve for. p stands for both prices p_1 and p_2, but both are not necessarily in the function. The actual prices and income in the problem are p_1^*, p_2^*, M^*. For good i, $x_i(p, M)$ is the demand function and x_i^* is the optimal quantity. For PC, $h(x_1, p) = h(x_1)$ (independent of both prices). For QL, $x_1(p, M) = x_1(p)$ (independent of income) and it must be plugged into the BC (i.e. no $h(x_1, p)$ for QL). For PS, if tangency condition is satisfied, all points on the budget line are solutions, including the corners.

2 Exercises

Q1: Aggregating Preferences

This question will show that even though everyone can have reasonable preferences, trying to put them together to make a choice for a whole group can be difficult.

A TA is trying to organize a review session for the three people in the class (D, E, and F). The students are given three possible slots as options: x, y, z. The students preferences are as follows:

$$D : z \succ x,\ z \succ y,\ x \succ y$$
$$E : x \succ y,\ y \succ z,\ x \succ z$$
$$F : z \succ x,\ y \succ x,\ y \succ z$$

1. Do all the students have transitive preferences?
2. Let's create a new preference relation $\succsim_G$ which will represent a group's preference. In particular, we will define it as follows. For any two options a and b, let $a \succsim_G b$ mean that at least 50% of the group's members prefer a over b.

   Give the definitions for the induced preferences $a \succ_G b$ and $a \sim_G b$. (Note: this doesn't have to do with this particular group in the example, we can define these preferences for <u>any</u> group of people)
3. Between the two options listed, determine the group's preferences using $\succsim_G$, i.e. which option does the group weakly prefer or strictly prefer - or are they indifferent between them?
   (a) x and y
   (b) y and z
   (c) x and z
4. Show that the preference relation $\succsim_G$ is not transitive.[1]
5. To solve the transitivity problem, student D suggests using utility functions since comparing numbers using the $\geq$ relation is always transitive.
   (a) Student D says that the following utilities represent the students' preferences

   $$D : u_D(x) = 5,\ u_D(y) = 4,\ u_D(z) = 6$$
   $$E : u_E(x) = 4,\ u_E(y) = 2,\ u_E(z) = 1$$
   $$F : u_F(x) = 1,\ u_F(y) = 6,\ u_F(z) = 2$$

   D claims that all we have to do is take the option that gives the highest total utility. In other words, find the option a that gives the highest group utility u_G, where $u_G(a) = u_D(a) + u_E(a) + u_F(c)$.

   According to this system, which option should be chosen? Do you agree with L's proposal?

[1] For those of you interested in political economy, this is the Condorcet paradox.

(b) Student E likes D's idea, but proposes the following utility representation to standardize the utility of one option (x) so that it is equal to 1 for everyone:

$$D : u_D(x) = 1,\ u_D(y) = 0.5,\ u_D(z) = 2$$
$$E : u_E(x) = 1,\ u_E(y) = 0.5,\ u_E(z) = 0.25$$
$$F : u_F(x) = 1,\ u_F(y) = 2,\ u_F(z) = 1.5$$

According to this system, which option should be chosen? Would this work better than D's suggested utility representation?

6. The TA suggests another way to solve the issue (while also supplementing the paltry grad student stipend). The TA unilaterally decides to hold the review session at slot x, and then asks the students what is the maximum amount they are willing to pay in order to get it switched to another time (i.e. y or z). The students respond in the following way: (where $WTP_x(a)$ represents how many dollars the student is willing to pay to get option a over option x)

$$D : WTP_x(y) = -10,\ WTP_x(z) = 20$$
$$E : WTP_x(y) = -2,\ WTP_x(z) = -30$$
$$F : WTP_x(y) = 40,\ WTP_x(z) = -5$$

   (a) Interpret what a negative willingness to pay means. According to this system, which option should be chosen?

   (b) Do you agree with the TA that this solves the issue? What does this approach capture that is missed in D and E's suggestions?

Q2: Time Constraints

Harry, Ron, and Hermoine have to study for two classes, Charms and Potions . They all face the same time constraints, but they have quite different preferences. Practicing a charm (c) takes half an hour, while practicing how to make a potion (p) takes 2 hours. They've all set aside 8 hours per day to study these two classes.

1. Write out their time budget constraints and plot on a graph with charms on the x-axis and potions on the y-axis (just do this once, since it's the same for all three people)

2. In class, they go through 5 new charms per day and 1 new potion per day. Hermoine wants to make sure she doesn't fall behind in either class, so she wants to make sure she is learning in those proportions too.

   (a) Write a utility function representing Hermoine's preferences

   (b) Solve for Hermoine's optimal bundle of charms and potions (c,p)

3. Harry is a bland character, so he has Cobb-Douglas preferences in the form $u(c, p) = c^{\alpha}p^{1-\alpha}$. Harry is very fortunate because his optimal bundle (c,p) happens to be exactly the same as Hermoine's optimal bundle! This means he can just study with Hermoine and ~~copy her homework~~ collaborate with her.

   (a) What is Harry's value for α? Draw Harry and Hermoine's indifference curve and optimal bundle in the diagram you drew for (1)

(b) Harry has found a useful textbook that's cut down the time it takes for him to learn how to make a potion by half. What is his new optimal bundle of (c,p)? Can he still study charms with Hermoine?

4. Ron is indifferent between the two classes. He's just as happy learning one charm as he is learning one potion.

   (a) What is a possible utility representation for Ron's preferences?

   (b) Given your answer in (a), solve for Ron's optimal bundle of charms and potions (c,p)

5. Ron's mother is trying to make her son care more about school. She says that she will give him \$15 (Muggle money) for every potion he learns or for every 3 charms he learns, whichever is higher. She is a kind mother, and so will also pay Ron proportionally for decimal amounts (while still following the 3:1 ratio). For example, if he learns 4 charms and 1 potion, he will earn \$20. If he learns 2.5 potions and 7 charms, he will earn \$37.50

   (a) This incentive structure has such a profound impact on Ron that it changes his preferences. Write a utility function to represent his new preferences? (hint: think of the money he earns as his utility)

   (b) While his mother's scheme has changed his preferences, has it also changed his behavior? In particular, is Ron learning more charms and potions than compared to before? (hint: don't use calculus; try to solve this using a graph and intuition)

Q3: Government Intervention

The prime minister of a small island nation is concerned that a new fast food restaurant has caused the islanders to start not eating enough vegetables. She's called a meeting of her policy advisors to discuss possible solutions. The island has 200 people, all of whom have the same preferences and budget constraints. They have a daily income of \$50, that they spend on servings of vegetables (v) which have a price of $p_v = 2$ each and fast food meals (f) which have a price of $p_f = 8$. The islander's preferences can be represented by the following utility function: $u(v, f) = \sqrt{v} + f$. This makes everything easier because we can do the analysis based on one individual ("the typical islander") and scale up accordingly.

1. Before the restaurant opened, the islanders only ate vegetables. What was the utility of the typical islander before the restaurant opened?

2. What is the typical islander's optimal bundle of (v, f) now that the restaurant has opened? Has the restaurant made the islanders happier?

3. Draw the typical islander's budget set. Indicate the optimal bundle and draw the indifference curve going through the optimal bundle.

4. The prime minister's aim is to get everyone eating 5 servings of vegetables per day. One policy advisor suggests that the government should simply make vegetables cheaper by subsidizing the price of vegetables.

   (a) What subsidy would get the typical islanders to consume $v^* = 5$? (assume that p_f and M stay fixed)

   (b) What would their new optimal bundle and utility be under this system?

   (c) How much would the subsidy cost the government?

5. Another advisor says that the government shouldn't favor one industry over another. Instead, the advisor suggests that the government give each person an extra tax rebate of \$10.

(a) Would this get the typical islander to consume $v^* = 5$? Why or why not?

(b) What would their new optimal bundle and utility be under this system?

(c) How much would the rebate cost the government?

6. Her last policy advisor has a more extreme suggestion that would achieve the government's aim at zero cost: just force everyone (by law) to purchase *at least* 5 servings of vegetables.

   (a) Draw the the typical islander's new budget set under this system (make sure to shade the entire feasible set)

   (b) What would their new optimal bundle and utility be under this system?

Q4: Same Budget, Different Utilities

Let's compare four consumers (A, B, C, and D), who each have different preferences but all face the same budget constraint. Their utility representations are as follows:

$$\begin{aligned} A &: u_A(x_1, x_2) = x_1^3 x_2 \\ B &: u_B(x_1, x_2) = \frac{1}{4}x_1 + 4x_2 \\ C &: u_C(x_1, x_2) = \min\{5x_1, 3x_2\} \\ D &: u_D(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2} \end{aligned}$$

And for all four, their budget constraint is:

$$2x_1 + x_2 \leq 10$$

1. Find an equivalent budget constraint where $p_1 = 10$

2. We know that utility representations are not unique.

   (a) Find another utility representation for A's preferences

   (b) For B, any utility representation must have an MRS of 1. True or False? (and explain why)

   (c) Find another utility representation for C's preferences

   (d) Does the utility function $v(x_1, x_2) = x_1^2 + x_2^2$ also represent D's preferences? (explain why)

3. Calculate the optimal bundle for each person

4. Is there a p_1 and p_2 that would make A and B choose the same optimal bundle?

5. Suppose that p_1 increased to 3. Would everyone increase their consumption of x_2? (if not, who wouldn't?)

6. Suppose instead that income increased to 15 (p_1 is back to 2 in this question). Would everyone increase their proportion of income spent on x_2? (if not, who wouldn't?)

7. C is not responsive to decreases in p_1, i.e. if p_1 falls, then C doesn't change their optimal bundle. True or False? (and explain why)

8. Suppose a fifth person E had a utility function $u_E(x_1, x_2) = \min\{Kx_1, x_2\}$, where K is some constant. E chooses the same optimal bundle as A. Calculate the value for K. Is the value unique (e.g. for a different utility representation, can we have K be another value)?

Q5: Hodgepodge

Ali spends a \$20 weekly allowance on two hobbies: miniature xylophones (x) and yo-yos (y). Xylophones have a unit cost of \$5, while yo-yos cost \$2 per unit. Ali's preferences can be represented in the form $u(x, y) = x^3y^7$

1. (a) What is Ali's MRS at the following bundles: (2,3), (5,1), (3,7)

   (b) Suppose Ali has an MRS of -3 at bundle A and an MRS of -5 at bundle B. Does this mean that Ali prefers A to B?

2. Ali's father thinks each person only needs one hobby. He is considering cutting Ali's allowance so that Ali is forced to choose to only one hobby to spend money on. What do you think of the father's plan?

3. Calculate Ali's optimal bundle and represent this on a diagram.

4. After a series of high-profile yo-yo accidents, the government has decided to crack down on yo-yos by putting a 300% tax on yo-yos. What is Ali's new optimal bundle? Represent this change in the same diagram you drew for (3)

5. In a surprise turn of events, the Yo-Yo Party of America (YYPA) wins a landslide in the elections by running on a mandate of repealing the yo-yo tax. However, it turns out the xylophone lobby is too powerful and makes it politically untenable to enact the repeal. The YYPA has decided to simply hand money out to encourage more yo-yo purchases.

   (a) How much would Ali need to receive in order to purchase the old quantity of yo-yos? (the amount you calculated in (3) before the tax)

   (b) After receiving the money from the government, is Ali happier than before the yo-yo crisis? (i.e. as compared to the bundle in (3)).

6. Desperately trying to restore a sense of normalcy, the YYPA has decided to implement extreme wealth redistribution. Their goal is to have everyone return back to the same level happiness as before the yo-yo crisis (i.e. back to the same level of utility as in (3)).

   (a) How much would the government have to give or take away from Ali to achieve this goal? (hint: try to express utility as a function of prices and income)

   (b) Show this change in a diagram.

7. The Yo-Yo Party's regime has caused unprecedented political and financial instability leading to rampant inflation. All prices in the economy have quadrupled (multiplied by 4). Has there been a change to Ali's budget set?

8. Ali has become disillusioned with the YYPA and now considers yo-yos a bad.

   (a) Which of the following could represent Ali's preferences now?

   $$(1)\ u(x, y) = \min\{x, y - 10\} \quad (2)\ u(x, y) = 2x - y \quad (3)\ u(x, y) = 5x$$

   (b) On a new diagram, draw Ali's budget set (keep in mind the change described in (6)), at least 3 indifference curves, and the location of Ali's new optimal bundle. Make sure you draw the indifference curve that passes through the optimal bundle. Use the utility function you chose in (a) to draw the indifference curves.

3 Solutions

Q1: Aggregating Preferences

1. Yes, they are all transitive. Recall that $a \succ b$ means that $a \succsim b$ but not $b \succsim a$.

$$D : z \succsim x,\ x \succsim y \implies z \succsim y \checkmark$$
$$E : x \succsim y,\ y \succsim z \implies x \succsim z \checkmark$$
$$F : y \succsim z,\ z \succsim x \implies y \succsim x \checkmark$$

   In other words, we can create a chain for each person to rank their preferences:

$$D : z \succ x \succ y$$
$$E : x \succ y \succ z$$
$$F : y \succ z \succ x$$

2. Using the same definitions we are used to:
   - $a \succ_G b$ means that at least 50% of people prefer a over b (i.e. $a \succsim_G b$) but strictly less than 50% of people prefer b over a(i.e. not $b \succsim_G a$). This means that $a \succ_G b$ can be defined as: strictly more than 50% of people prefer a over b (an absolute majority)
   - $a \sim_G b$ means that at least 50% of people prefer a over b (i.e. $a \succsim_G b$) and at least 50% of people prefer b over a(i.e. $b \succsim_G a$). This means that $a \sim_G b$ can be defined as: exactly 50% of people prefer a over b (an even split)

3. Since we have three people choosing between two options, we can never have an even split. So we only need to use $\succ_G$
   (a) D and E choose x over y. F chooses y over x. The vote is 2 to 1 in favor of x. Therefore: $x \succ_G y$.
   (b) E and F choose y over z. D chooses z over y. The vote is 2 to 1 in favor of y. Therefore: $y \succ_G z$.
   (c) D and F choose z over x. E chooses x over z. The vote is 2 to 1 in favor of z. Therefore: $z \succ_G x$.

4. Clearly it is not transitive. $x \succ_G y$ implies $x \succsim_G y$ and $y \succ_G z$ implies $y \succsim_G z$. Transitivity says that if we have $x \succsim_G y$ and $y \succsim_G z$ (which we do), then it must be the case that $x \succsim_G z$. However, we have $z \succ_G x$, which means that we cannot have $x \succsim_G z$. Therefore, transitivity is violated.

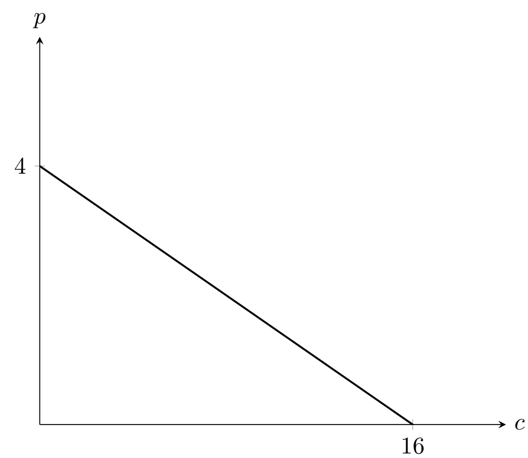
5. (a) Under this system: $u_G(x) = 10$, $u_G(y) = 12$, and $u_G(z) = 9$, so y would be chosen. However, we should absolutely disagree with this approach. Utility representation is not unique, so those numbers don't mean anything. Adding them up is a meaningless comparison (it's comparing apples and oranges, as the saying goes)
   (b) Under this system: $u_G(x) = 3$, $u_G(y) = 3$, and $u_G(z) = 3.75$, so $z$would be chosen. Don't be fooled though - we haven't standardized anything! This is still a meaningless comparison. For example, consider any Cobb-Douglas utility function: $x_1^\alpha x_2^\beta$. Regardless of your value of α and β, the bundle $(1,1)$ will always have a utility of 1 (since $1^\alpha = 1$ and $1^\beta = 1$, for any positive α and β). That doesn't mean that you can compare the utilities! You should notice that there is a problem because two valid utility representations gave us completely different answers: in (a), we chose y, but in (b) we chose z (which in fact had the lowest utility in (a)).

6. (a) A negative willingness to pay means that the student has to be given money in order to choose the alternative option over x. That means that they like x over the alternative. Under this system, we get that the group's willingness to pay for y is $-10 - 2 + 40 = \$28$ and the willingness to pay for z is $20 - 30 - 5 = -15$. This means that the group, as a whole, is willing to pay at most \$28 to switch from x to y, but they have to paid at least \$15 to switch from x to z. This means that option y would be chosen, because that is the most preferred.

   (b) This does solve the issue. While utility values are incomparable, money is! We all agree on the value of money, so this means we can add up their willingness to pay and have a meaningful interpretation. The reason this works over the two is that the WTP captures the magnitude of "happiness" each option gives. Notice that both D and E prefer x over y. However, for D, they have to be paid \$10 to switch to y, while E is happy with just \$2. The preferences only tell us the order, and utilities don't give a meaningful representation of the "happiness distance" between two options. However, money does! (yay money!)[2]

Q2: Time Constraints

1. The budget constraint would be: $\frac{1}{2}c + 2p \leq 8$. Graphing this gives us:



2. (a) This is perfect complements with a ratio of 5 to 1: $u(c,p) = \min\left\{\frac{1}{5}c, p\right\}$ and $u(c,p) = \min\{c, 5p\}$ are examples of possible functions you could write.

   (b) Calculate the kinks line: $\frac{1}{5}c = p$.
   Plug the equation of the kinks line into the budget line to get c^*:

$$\begin{aligned} \frac{1}{2}c + 2 \cdot \frac{1}{5}c &= 8 \\ 5c + 4c &= 80 \\ c^* &= \frac{80}{9} \approx 8.89 \end{aligned}$$

   Plug back into kinks line to get p^*:

$$p^* = \frac{1}{5}c^* = \frac{1}{5} \times \frac{80}{9} = \frac{16}{9} \approx 1.78$$

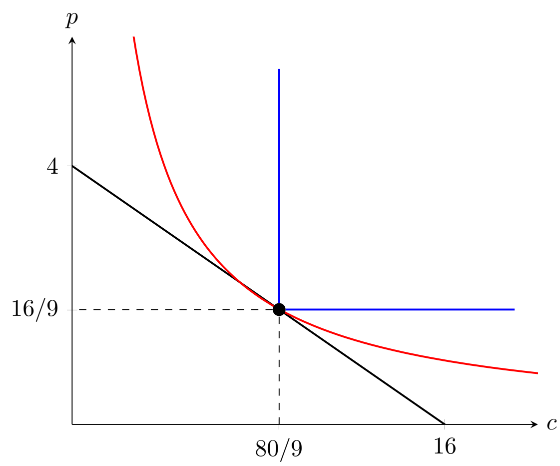
[2] One limitation though is that we haven't considered how much each person can afford. Therefore, this approach gives too much power to the wealthier students (boo money!)

3. (a) Here we have $\beta = 1 - \alpha$, so $\alpha + \beta = 1$. The Cobb-Douglas demand here would be:

$$c = \frac{\alpha}{\alpha+\beta} \cdot \frac{M}{p_c} = \alpha \frac{8}{0.5} = 16\alpha$$
$$p = \frac{\beta}{\alpha+\beta} \cdot \frac{M}{p_p} = (1-\alpha)\frac{8}{2} = 4(1-\alpha)$$

So to get the same values as Hermoine, we need $16\alpha = \frac{80}{9} \implies \alpha = \frac{80}{16\times 9} = \frac{5}{9} \approx 0.56$. But notice this means that $1 - \alpha = \frac{4}{9}$, and this gives us an optimal value of $p = 4 \times \frac{4}{9} = \frac{16}{9}$. So it all works out! (we weren't lucky, this must be true because they have the same budget line).

The diagram would be as follows (Harry in red, Hermoine in blue)



(b) Harry's new budget constraint would be: $\frac{1}{2}c + p \leq 8$, since the cost of potions has been halved (i.e. $p_p = \frac{1}{2} \times 2 = 1$).
However, since this is Cobb-Douglas, we know that c^* is not going to change, so he can still study with Hermoine. You don't actually need to calculate anything to answer that part of the question question. The only change comes from p^*, which we can see if we plug in the new values into the demand function:

$$p^* = (1-\alpha)\frac{8}{1} = \frac{4}{9} \times 8 = \frac{32}{9} \approx 3.56$$

So his new optimal bundle is $(\frac{80}{9}, \frac{32}{9})$.

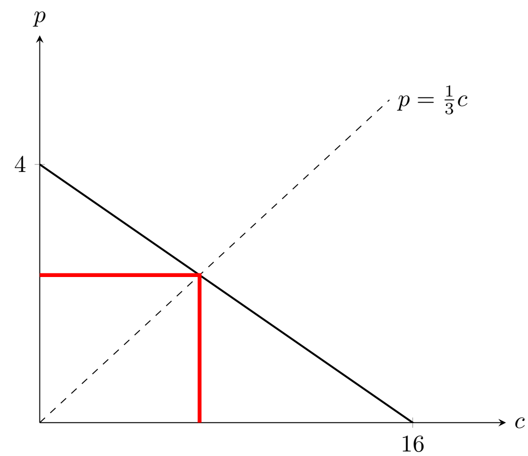
4. (a) This suggests perfect substitutes in a 1 to 1 ratio: $u(c,p) = c + p$

(b) $|MRS| = \frac{1}{1} = 1$. The slope of the budget constraint is $\frac{p_c}{p_p} = \frac{0.5}{2} = \frac{1}{4}$. Since $1 > \frac{1}{4}$, Ron spends all his time on charms (x-corner solution). His optimal bundle is $(c^*, p^*) = (16, 0)$.

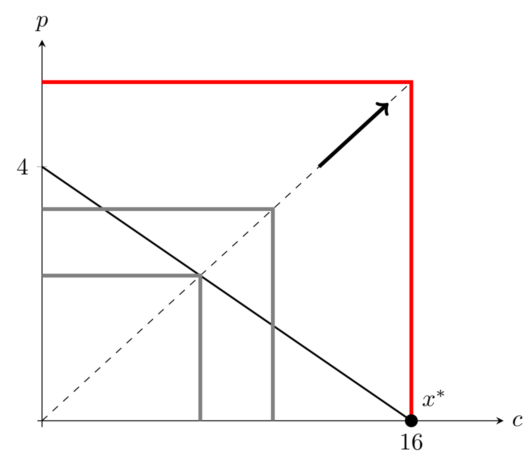
5. (a) This is almost like perfect substitutes. Ron could either focus on 3 charms or 1 potion, so the ratio is 3 to 1. However, he isn't trying to minimize, instead he wants the maximum. This means the utility function is $u(c,p) = \max\left\{\frac{1}{3}c, p\right\}$ or, equivalently, $u(c,p) = \max\{c, 3p\}$.

- Notice we can also make this map onto the monetary reward by scaling it up to $u(c,p) = 15 \times \max\left\{\frac{1}{3}c, p\right\}$. Let's check the examples given. First, $u(4,1) = 15 \times \max\left\{\frac{4}{3}, 1\right\} = 15 \times \frac{4}{3} = 20$. Second, $u(7, 2.5) = 15 \times \max\left\{\frac{7}{3}, 2.5\right\} = 15 \times 2.5 = 37.5$.

(b) Even though this is an unfamiliar utility function, we use the same intuition as usual: the optimal bundle is on the highest indifference curve in the budget set. You may first want to think of this like perfect substitutes and draw the kinks line. This occurs where $p = \frac{1}{3}c$, as plotted below in the dotted line. Then you might get tempted to find a tangent, again as shown below in the diagram (we saw "max" indifference curves in Homework 2, so this shouldn't be too unfamiliar to you).



But this isn't perfect substitutes! So don't get fooled. There are still lots of places in the budget set that have bundles with higher utility. You have to keep moving out along the budget line until there are no more "gaps" in the budget set. In this case, we need to keep moving until we hit a corner.



We know it has to be this corner because the kink for the indifference curve going through (16,0) occurs at $\frac{16}{3} = 5\frac{1}{3} > 4$. So this point gives higher utility than the other corner at $(4, 0)$. Therefore, his mother's scheme hasn't changed his optimal bundle at all!

Q3: Government Intervention

1. Since the islanders only ate vegetables, they must have spent all their money on it. This means they purchased $v = \frac{M}{p_v} = \frac{50}{2} = 25$ units of vegetables. This would have given them a utility of $u(25, 0) = \sqrt{25} + 0 = 5$

2. This is a quasi-linear utility, which means that we can solve for the demand of v directly from the tangency

condition. The tangency condition is:

$$\begin{aligned}
|MRS| = \frac{\frac{1}{2}v^{-\frac{1}{2}}}{1} &= \frac{p_v}{p_f} \\
\frac{1}{2\sqrt{v}} &= \frac{p_v}{p_f} \\
\sqrt{v} &= \frac{p_f}{2p_v} \\
\therefore v(p, M) &= \left(\frac{p_f}{2p_v}\right)^2
\end{aligned}$$

Then to solve for the demand of f, we plug this into the budget constraint:

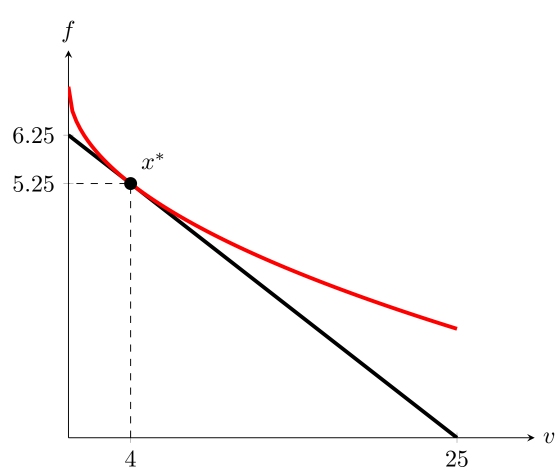
$$\begin{aligned}
p_v v + p_f f &= M \\
f(p, M) &= \frac{1}{p_f}\left(M - p_v v(p, M)\right) \\
&= \frac{M}{p_f} - \frac{p_v}{p_f}\left(\frac{p_f}{2p_v}\right)^2 \\
&= \frac{M}{p_f} - \frac{p_f}{4p_v}
\end{aligned}$$

Note, this is all for an interior solution - remember that quasi-linear can have corner solutions! Let's plug in the values of the parameters (note it's easier to get f^* from the budget constraint):

$$\begin{aligned}
v^* &= \left(\frac{8}{2 \times 2}\right)^2 = 2^2 = 4 \\
f^* &= \frac{1}{p_f}(M - p_v v^*) \\
&= \frac{1}{8}\left(50 - 2 \times 4\right) = \frac{42}{8} = 5.25
\end{aligned}$$

Their utility from this bundle is $u(4, 5.25) = \sqrt{4} + 5.25 = 7.25$. This gives them a higher utility than before, so the islanders are happier.

3. The graph is shown below:



4. (a) To get $v^* = 5$, and given we know $p_f = 8$, then we just need to solve for p_v from the demand function:

$$\begin{aligned} v &= \left(\frac{p_f}{2p_v}\right)^2 \\ 5 &= \left(\frac{8}{2p_v}\right)^2 \\ \sqrt{5} &= \frac{4}{p_v} \\ p_v &= \frac{4}{\sqrt{5}} \approx 1.79 \end{aligned}$$

This means that the government would need to put a subsidy of $2 - 1.79 = \$0.21$ per serving of vegetables.

(b) The new optimal f would become:

$$f^* = \frac{1}{8}\left(50 - 5 \times \frac{4}{\sqrt{5}}\right) = \frac{1}{8}\left(50 - 4\sqrt{5}\right) \approx \frac{41.06}{8} \approx 5.13$$

This gives the islanders a utility of: $u(5, 5.13) = \sqrt{5} + 5.13 \approx 7.37$, which is higher than their current status. So they would be happier.

(c) The government would give \$0.21 per serving, and there are 200 islanders who each purchase 5 servings of vegetables. The subsidy would cost them $\$0.21 \times 200 \times 5 = \$210$.

5. (a) This is quasi-linear utility, where f is the numeraire. That means that for an interior solution, the optimal quantity of v is independent of income. Therefore, the \$10 would not work - in fact, no amount of money will get islanders to increase their consumption to \$5

(b) Consumers put the extra money all towards fast food. Their new income is $50 + 10 = 60$, while we still have $p_v = 2$ and $v^* = 4$. This means that the new f^* is:

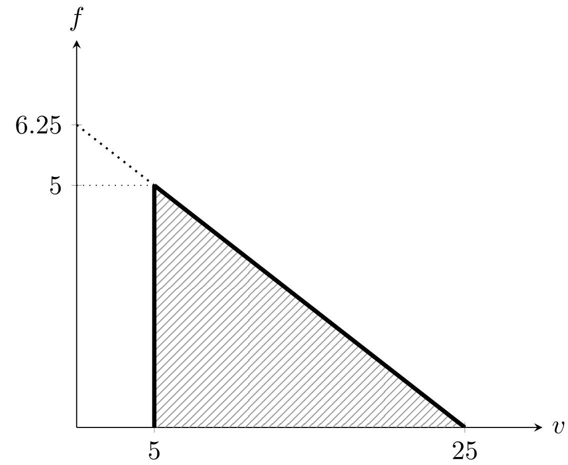
$$f^* = \frac{1}{8}(60 - 2 \times 4) = \frac{52}{8} = 6.5$$

The consumers are obviously happier now, with a utility of $u(4, 6.5) = \sqrt{4} + 6.5 = 8.5$. But the government has totally failed in their mission to increase vegetable consumption

(c) The government would give \$10 per person, and there are 200 islanders. The rebate would cost them $\$10 \times 200 = \$2000$.

6. (a) The graph is shown below. The part of the consumer's budget set from $v = 0$ to $v = 5$ is gone now (the dotted line in the graph). So the budget set now is the thick line and the shaded area.

(b) This hasn't changed their preferences. They still want to be at $(v^*, f^*) = (4, 5.25)$ as you calculated in (2). However, this point is no longer achievable. So they have to get to the point on the budget line that it is closest to their optimal bundle. In this case they choose $(5, 5)$ where the budget line cuts off.



Q4: Same Budget, Different Utilities

1. Multiply everything by 5: $10p_1 + 5x_2 \leq 50$

2. (a) Anything that keeps the exponents in the same ratio, e.g. $x_1^{0.75}x_2^{0.25}$

   (b) False. The MRS for u_B is $\frac{1/4}{4} = \frac{1}{16}$. Any other representation must also have this MRS.

   (c) Any monotone transformation of each component of the min function, e.g. $\min\{10x_1 + 3, 6x_2 + 3\}$

   (d) No. Check the MRS. MRS for u_D:

$$MRS^{u_D} = \frac{MU_1}{MU_2} = \frac{\frac{1}{2\sqrt{x_1}}}{\frac{1}{2\sqrt{x_2}}} = \frac{2\sqrt{x_2}}{2\sqrt{x_1}} = \sqrt{\frac{x_2}{x_1}}$$

   The MRS for v however is:

$$MRS^v = \frac{2x_1}{2x_2} = \frac{x_1}{x_2}$$

   It's also not a monotone transformation. You might think $v(x_1, x_2) = u(x_1, x_2)^4$, but that won't be true in general:

$$\begin{aligned} u(x_1, x_2)^4 &= (\sqrt{x_1} + \sqrt{x_2})^4 \\ &= \left((\sqrt{x_1} + \sqrt{x_2})^2\right)^2 \\ &= (x_1 + 2x_1x_2 + x_2)^2 \\ &\neq x_1^2 + x_2^2 = v(x_1, x_2) \end{aligned}$$

3. Let's calculate for each person:

   (a) This is Cobb-Douglas, so let's go through all the steps but we should already anticipate the results.

$$\begin{aligned} |MRS| &= \frac{p_1}{p_2} \\ \frac{3x_2}{x_1} &= \frac{p_1}{p_2} \\ \therefore x_2 &= \frac{p_1}{3p_2}x_1 \end{aligned}$$

Plug into BC and get demands:

$$\begin{aligned} p_1x_1 + p_2\left(\frac{p_1}{3p_2}x_1\right) &= M \\ p_1x_1 + \frac{p_1}{3}x_1 &= M \\ x_1\left(3p_1 + p_1\right) &= 3M \\ \therefore x_1(p, M) &= \frac{3M}{4p_1} \end{aligned}$$

$$\therefore x_2(p, M) = \frac{p_1}{3p_2} \cdot x_1(p, M) = \frac{M}{4p_2}$$

This exactly fits the Cobb-Douglas formula, so we know we're right.

Now we plug in the parameter values of $p_1 = 2, p_2 = 1, M = 10$

$$x_1^* = \frac{30}{8} = 3.75,\ x_2^* = \frac{10}{4} = 2.5$$

(b) This is perfect substitutes, so we just compare the MRS to the budget constraint slope:

$$|MRS| = \frac{1/4}{4} = \frac{1}{16} < 2 = \frac{p_1}{p_2}$$

Since it's less than the slope, the consumer is on the y-corner, i.e. the optimal bundle is:

$$x_1^* = 0,\ x_2^* = \frac{10}{1} = 10$$

(c) This is perfect complements. So we first find the kinks line:

$$\begin{aligned} 5x_1 &= 3x_2 \\ \therefore x_2 &= \frac{5}{3}x_1 \end{aligned}$$

Then plug into the budget constraint:

$$\begin{aligned} p_1x_1 + p_2 \cdot \frac{5}{3}x_1 &= M \\ x_1\left(3p_1 + 5p_2\right) &= 3M \\ \therefore x_1(p, M) &= \frac{3M}{3p_1 + 5p_2} \end{aligned}$$

$$\begin{aligned} \therefore x_2(p, M) =& \frac{5}{3} \cdot x_1(p, M) \\ =& \frac{5M}{3p_1 + 5p_2} \end{aligned}$$

Again, if you remember the general formula you can confirm that these are correct. Plugging in the parameter values gets us:

$$x_1^* = \frac{30}{6+5} = \frac{30}{11},\ x_2^* = \frac{50}{6+5} = \frac{50}{11}$$

(d) This one is not a type we've seen before, but we still approach it in the same way. First, we go to the tangency condition (we already solved for the MRS in 2(d)):

$$
\begin{aligned}
|MRS| &= \frac{p_1}{p_2} \\
\sqrt{\frac{x_2}{x_1}} &= \frac{p_1}{p_2} \\
\frac{x_2}{x_1} &= \left(\frac{p_1}{p_2}\right)^2 \\
\therefore x_2 &= \left(\frac{p_1}{p_2}\right)^2 x_1
\end{aligned}
$$

Plug into BC and get demands:

$$
\begin{aligned}
p_1x_1 + p_2\left(\left(\frac{p_1}{p_2}\right)^2 x_1\right) &= M \\
p_1x_1 + \frac{p_1^2}{p_2}x_1 &= M \\
x_1\left(p_1p_2 + p_1^2\right) &= Mp_2 \\
\therefore x_1(p, M) &= \frac{Mp_2}{p_1(p_1+p_2)}
\end{aligned}
$$

$$\therefore x_2(p, M) = \frac{p_1^2}{p_2^2} \cdot x_1(p, M) = \frac{Mp_1}{p_2(p_1+p_2)}$$

Finally, we plug in the parameter values:

$$x_1^* = \frac{10}{2(2+1)} = \frac{5}{3},\ x_2^* = \frac{20}{1(2+1)} = \frac{20}{3}$$

4. A has Cobb-Douglas utilities, which means they always have an interior solution. B has perfect substitutes utility, which means the only way they would have an interior solution is the budget line equals their MRS of 1/16. Then everything on the budget line is an optimal solution for B, including the choice made by A. So it is possible, as long as $p_1/p_2 = 1/16$, e.g. $p_1 = 1$ and $p_2 = 16$.

5. A has Cobb-Douglas, and the demand of x_2 does not depend on p_1, so they wouldn't change.

   For B, even if $p_1 = 3$, we still have $MRS < \frac{p_1}{p_2} = 3$, so they would stay on their corner buying only x_2.

   For C, because the goods are perfect complements, an increase in the price of x_1 means less of both goods are purchased. So they would not increase their x_2 quantity.

   Finally, for D we can start by looking at the demand function. It's a little tricky because p_1 appears in both the numerator and denominator so it's a little unclear without doing some calculus. However, we can also simply plug in $p_1 = 3$ into the demand function and we get $x_2^* = \frac{30}{1(3+1)} = \frac{30}{4} = 7.5 > \frac{20}{3}$.

   So, only D increases their quantity demand of x_2.

6. A has Cobb-Douglas, which always spends in equal proportion. So they wouldn't change.

The price ratio hasn't changed, so B will still consume 100% of their income on x_2. So no change here either.

C will increase consumption - you can see this in the demand function. Now they purchase: $x_2^* = \frac{5M}{3p_1+5p_2} = \frac{75}{6+5} = \frac{75}{11}$. The question is not if they purchase more, but rather whether they spend more as a proportion of their income. The expenditure on x_2 (i.e. $p_2 x_2^*$) as a fraction of income M is:

$$\text{Before:} \frac{1 \times \frac{50}{11}}{10} = \frac{5}{11}$$
$$\text{After:} \frac{1 \times \frac{75}{11}}{15} = \frac{5}{11}$$

It's exactly the same! Just like Cobb-Douglas, they buy more, but in the same proportion as before.[3]

For D, let's calculate the new quantity demanded for x_2: $\frac{Mp_1}{p_2(p_1+p_2)} = \frac{30}{1(2+1)} = 10$. The expenditure on x_2 (i.e. $p_2 x_2^*$) as a fraction of income M is:

$$\text{Before:} \frac{1 \times \frac{20}{3}}{10} = \frac{2}{3}$$
$$\text{After:} \frac{1 \times 10}{15} = \frac{2}{3}$$

Again, there is no increase. This is to be expected. Look at the utility function for D, both x_1 and x_2 enter in the exact same way. There is nothing special about x_2 over x_1, in terms of utility. So there should be no reason that they should just start spending more on x_2 just because they have more income.

The answer to this question is that no one increases their proportion of income spent on x_2.

7. False. If p_1 falls so low (i.e. below $\frac{1}{16}$), then we will have $|MRS| = \frac{1}{16} > \frac{p_1}{p_2}$. Then the consumer will switch from only buying x_2 to only buying x_1 (that's a ∞% change - I would call that very responsive!).

8. We calculated that $A's$ optimal bundle is $(3.75, 2.5)$. For this to be E's optimal bundle, this point has to be on $E's$ kink line, which is given by $x_2 = Kx_1$. Plugging in this point, we get $K = \frac{x_2}{x_1} = \frac{2.5}{3.75} = \frac{5/2}{15/4} = \frac{2}{3}$. Yes, it is unique. While we can scale the utility function, because we must have 1 as the coefficient to x_2 in the utility representation given, then K has to be $\frac{2}{3}$ otherwise we will not represent the same preferences.

Q5: Hodgepodge

1. (a) This is Cobb-Douglas with $\alpha = 3$ and $\beta = 7$, so the MRS is:

$$|MRS| = \frac{\alpha x_2}{\beta x_1} = \frac{3x_2}{7x_1}$$

Evaluating the MRS at each bundle gives us:

$$MRS(2,3) = -\frac{3 \times 3}{7 \times 2} = -\frac{9}{14} \qquad MRS(5,1) = -\frac{3 \times 1}{7 \times 5} = -\frac{3}{35} \qquad MRS(3,7) = -\frac{3 \times 7}{7 \times 3} = -1$$

[3] Unlike Cobb-Douglas, for perfect complements, a price change generally causes a change in the share of income spent on each good

(b) Not at all. MRS tells us the slope of the indifference curve, and for Cobb-Douglas, the slope of each indifference curve goes from $-\infty$ to 0. Therefore, you can be on a higher indifference curve but have a larger (in absolute value) slope. For example, consider the points $A = (3, 21)$ and $B = (3, 35)$. Clearly, $B \succ A$ but also note that $MRS(A) = -3$ and $MRS(B) = -5$.

2. Cobb-Douglas always purchases both goods no matter how low the income gets. So her father's evil plan won't work

3. Let's take all the usual steps:

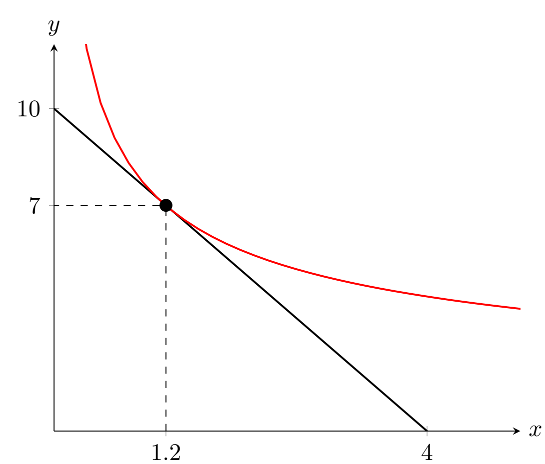
$$|MRS| = \frac{3y}{7x} = \frac{p_x}{p_y}$$
$$\therefore y = \frac{7p_x}{3p_y}x$$

$$p_x x + p_y\left(\frac{7p_x}{3p_y}x\right) = M$$
$$x\,(3p_x + 7p_x) = 3M$$
$$\therefore x(p, M) = \frac{3M}{10p_x}$$
$$\therefore y(p, M) = \frac{7M}{10p_y}$$

Plugging in the parameter values:

$$x^* = \frac{60}{50} = 1.2,\; y^* = \frac{140}{20} = 7$$

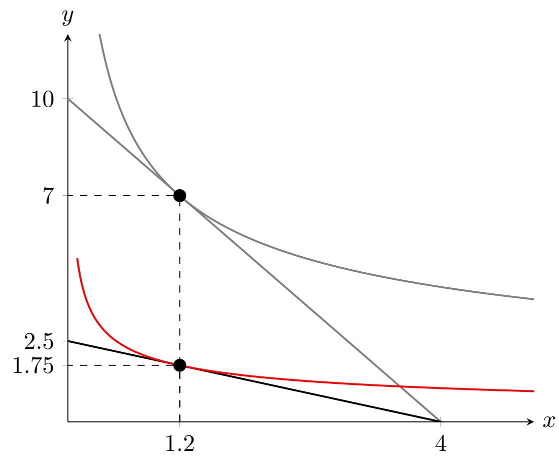
Draw the graph:



4. A tax of 300% means adding a tax of $t = 3 \times p_y = 6$. This means the new price of yo-yos is $p_y + t = 8$. The new optimal bundle is:

$$x^* = \frac{60}{50} = 1.2,\; y^* = \frac{140}{80} = 1.75$$

And the graph becomes:



5. (a) For Ali to have $y = 7$ with $p_y = 8$, we need income to be:

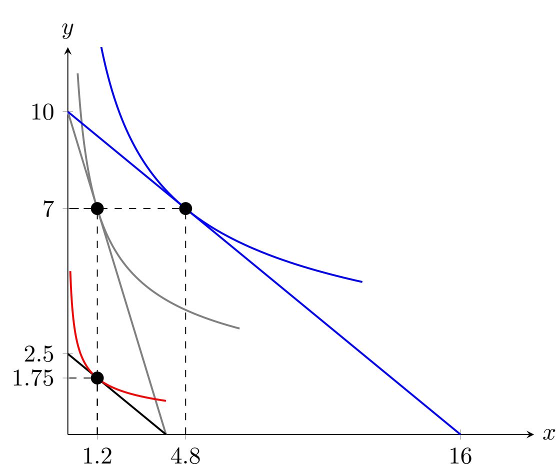
$$y = \frac{7M}{10p_y}$$
$$7 = \frac{7M}{80}$$
$$M = 80$$

Therefore, Ali would need to receive an extra \$60 per week to purchase the old quantity of yo-yos.

(b) With $M = 80$, Ali purchases $y = 7$ and $x = \frac{3M}{10p_x} = \frac{240}{50} = 4.8$. Ali must be happier because the bundles are the same in the y component and the new bundle has more x_1. To also see this, let's call the old utility as u_0 and the new utility (after the tax and income distribution) as u_1. We want to see if $u_1 > u_0$:

$$u_1 > u_0$$
$$\left(\frac{240}{50}\right)^3 7^7 > \left(\frac{60}{50}\right)^3 7^7$$
$$240 > 60\checkmark$$

This inequality holds true, Ali is happier now. We can also see this on the graph (the u_1 world is in blue). So the government has "over-shot" in some sense. They've given enough money to be able to buy 7 yo-yos, but with less money could have gotten Ali back to the old level of utility.



6. (a) To get utility as a function of prices and income, we just plug in the demand functions into the utility function (this is called "indirect utility"). Let's call it $\tilde{u}(p, M)$. This gives us:

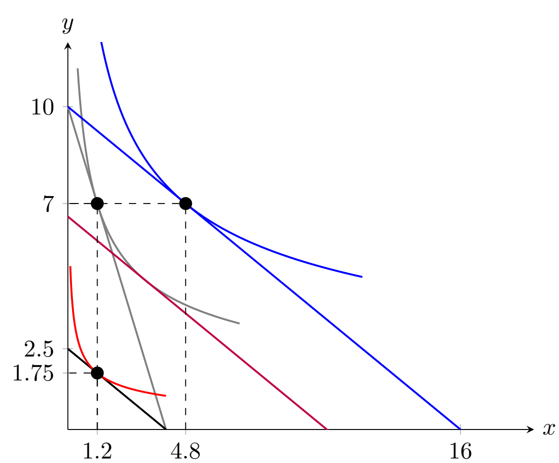
$$\begin{aligned}\tilde{u}(p, M) =& u(x(p, M), y(p, M)) \\ =& \left(\frac{3M}{10p_x}\right)^3 \left(\frac{7M}{10p_y}\right)^7\end{aligned}$$

Then to make sure that Ali is as happy as before the crisis, we need $\tilde{u}(p, M) = u_0 = \left(\frac{60}{50}\right)^3 7^7$. We know that prices are still the same, so we have $p_x = 5$ and $p_y = 8$. We plug in all these values and then we can solve for M:

$$\begin{aligned}\tilde{u}(p, M) =& u(x(p, M), y(p, M)) \\ \left(\frac{60}{50}\right)^3 7^7 =& \left(\frac{3M}{50}\right)^3 \left(\frac{7M}{80}\right)^7 \\ 20^3 \left(\frac{3}{50}\right)^3 7^7 =& \left(\frac{3}{50}\right)^3 7^7 \left(\frac{1}{80}\right)^7 M^{10} \\ 20^3 =& \left(\frac{1}{80}\right)^7 M^{10} \\ \therefore M =& 80^{0.7} \times 20^{0.3} \\ =& 4^{0.7} \times 20^{0.7} \times 20^{0.3} \\ =& 4^{0.7} \times 20 \\ \approx& 52.78\end{aligned}$$

This means that the government has to take away from Ali $80 - 52.78 = \$27.22$ to bring utility back to the same level as before. Based on our analysis in 5(b), we should have expected this result.

(b) Representing this in the same graph as before (where purple is the budget line after the income redistribution). The key things to note is that the purple line has the same slope as the blue budget line (after the rebate) but is tangent to the gray indifference curve (the pre-crisis utility)



7. Yes. Even if prices have quadrupled, we've said nothing about income (so assume it's the same). So now Ali's budget set has shrunk down because the real purchasing power of income has fallen.

8. (a) Only $u(x, y) = 2x - y$. The first one is still perfect complements (with kinks at $y = x + 10$) and the third one has y as a neutral (i.e. vertical indifference curves).

   (b) The new parameter values are $p_x = 10$, $p_y = 16$, and $M = 80$. To plot an indifference curve, re-arrange to have y on the left hand side: $y = 2x - u$. Now we just plug in different values of u, e.g. I'll first try $u = 2$, $u = 5$, and $u = 10$. This gives us the following diagram. Notice the direction of the indifference curves. Utility increases as we move in a southeast direction. Since y is a bad, Ali would definitely prefer to consume $y = 0$ over any strictly positive amounts. Therefore, the optimal bundle will be at the x-corner $(8, 0)$, which gives Ali a utility of 16.