Intermediate Micro: Recitation 10
Midterm Review

Motaz Al-Chanati

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1 Exercises

Consider a consumer with the utility function $u(x_1, x_2) = x_1 + 2\ln x_2$. The price of both goods is 5 and the consumer's income is 40. For each parameter change, only consider that change for that question.

1. Derive the consumer's demand function for both goods (hint: don't forget about checking for corner solutions)

2. Draw the x_1 demand curve at the given parameter values. Draw it with price on the y-axis and quantity on the x-axis - and make sure to label the intercepts! Plot the point on the demand curve representing the current parameter values.

3. Suppose income increased to 80. Draw this change in the demand curve diagram you drew in (2), including the new quantity and price. Has consumer surplus increased or decreased? Shade the change in net consumer surplus on the diagram.

4. Draw the Engel curve for x_2. Plot the point on the Engel curve representing the current parameter values.

5. On separate diagrams, draw the following:

   (a) The p_1 price offer curve

   (b) The p_2 price offer curve

   (c) The income expansion path (i.e. income offer curve)

   For each, draw an arrow indicating how the direction of the offer curve as the parameter of interest increases (e.g. for (a), in which direction do we move along the price offer curve as p_1 goes from a low value to a high value?).

6. Suppose that p_1 increases to 10.

(a) On the same diagram, draw the budget lines, indifference curves, and optimal bundles before and after the change. Include the p_1 price offer curve (the same as you draw in 5(a)).

(b) Decompose the effect of the price change into the substitution and income effect (for both goods)

(c) Show the decomposition in your diagram from (a). Make sure to label the substitution and income effects (for both goods)

7. Suppose that p_2 decreases to 2. Calculate the CV of this change and represent it in a diagram.

8. Suppose that p_1 increases to 25. Calculate the EV of this change and represent it in a diagram.

9. At an interior solution, calculate the following elasticities:

   (a) Own-price elasticity of x_1. At the current parameter values, is x_1 demand elastic or inelastic?

   (b) Cross-price elasticity of x_2. At the current parameter values, is x_2 demand elastic or inelastic?

   (c) The income elasticities of both goods. Which good is more income elastic?

10. True or False: If p_1 decreases, the proportionate increase in x_1 will always be larger than the proportionate decrease in x_2 (i.e. in terms of absolute percentage change). For simplicity, only consider this at an interior solution.

2 Solutions

Question 1

Start with the tangency condition:

$$
\begin{aligned}
|MRS| = \frac{1}{\frac{2}{x_2}} &= \frac{p_1}{p_2} \\
\frac{x_2}{2} &= \frac{p_1}{p_2} \\
\therefore x_2 &= \frac{2p_1}{p_2}
\end{aligned}
$$

Next, we plug this into the budget constraint:

$$
\begin{aligned}
p_1x_1 + p_2x_2 &= M \\
p_1x_1 + p_2\left(\frac{2p_1}{p_2}\right) &= M \\
p_1x_2 + 2p_1 &= M \\
\therefore x_1 &= \frac{M}{p_1} - 2
\end{aligned}
$$

We're not done. We need to check for corner solutions. In particular, we should see whether the demand functions can ever be negative. Notice that this can never happen with x_2 since both $p_1 > 0$ and $p_2 > 0$. So let's check for x_1:

$$
\begin{aligned}
x_1 = \frac{M}{p_1} - 2 &< 0 \\
\frac{M}{p_1} &< 2 \\
M &< 2p_1 \\
p_1 &> \frac{M}{2}
\end{aligned}
$$

This gives us the condition for a corner solution where $x_1 = 0$ and all income is spent on x_2. Therefore, the demand functions are:

$$
x_1(p, M) = \begin{cases} \frac{M}{p_1} - 2 & \text{if } p_1 \leq \frac{M}{2} \\ 0 & \text{if } p_1 > \frac{M}{2} \end{cases} \qquad x_2(p, M) = \begin{cases} \frac{2p_1}{p_2} & \text{if } p_1 \leq \frac{M}{2} \\ \frac{M}{p_2} & \text{if } p_1 > \frac{M}{2} \end{cases}
$$

Question 2

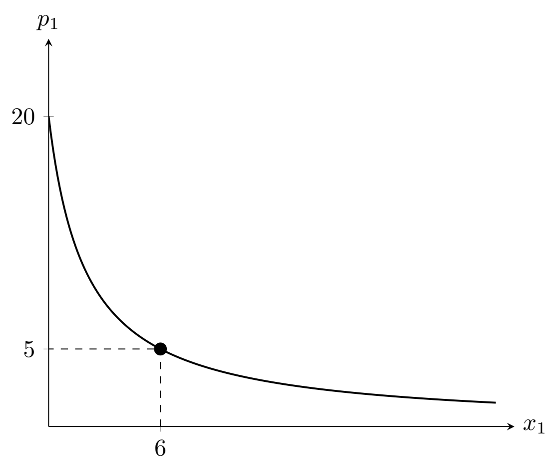
At the given parameter values, the x_1 demand curve is: (i.e. plug in $M = 40$)

$$
x_1(p_1) = \frac{40}{p_1} - 2
$$

To plot it, we should use the inverse demand curve. Solving for p gives us this:

$$x_1 + 2 = \frac{40}{p_1}$$
$$p_1(x_1) = \frac{40}{x_1 + 2}$$

Let's plot this. Looking at the inverse demand function, we can see that the y-intercept occurs at $p_1(0) = \frac{40}{2} = 20$. However, there is no x-intercept (it's an asymptote).



At the current price of $p_1 = 5$, we can see that the quantity demanded is $x_1(5) = \frac{40}{5} - 2 = 8 - 2 = 6$. Therefore, we plot the point $(6, 5)$.

Question 3

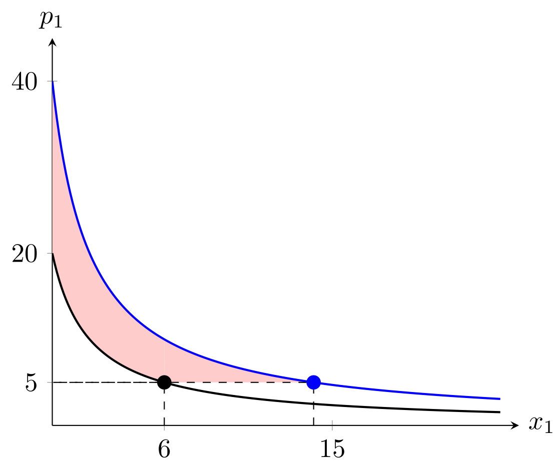
If $M = 80$, then the demand curve becomes:

$$x_1(p_1) = \frac{80}{p_1} - 2$$

And the inverse demand curve becomes:

$$p_1(x_1) = \frac{80}{x_1 + 2}$$

The price hasn't changed, so the new optimal quantity would be: $x_1(5) = \frac{80}{5} - 2 = 16 - 2 = 14$. So, the diagram becomes as follows:



The red shaded area shows the increase in net consumer surplus.

Question 4

The x_2 demand curve is:

$$x_2(p, M) = \begin{cases} \frac{2p_1}{p_2} & \text{if } p_1 \leq \frac{M}{2} \\ \frac{M}{p_2} & \text{if } p_1 > \frac{M}{2} \end{cases}$$

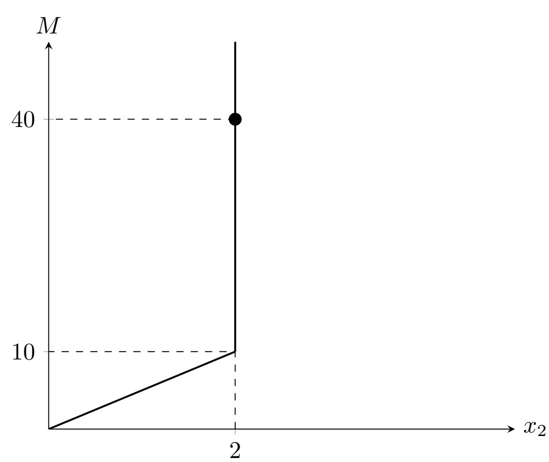
Let's plug in the values for prices, but we have to keep income as a variable. This gives us:

$$x_2(M) = \begin{cases} 2 & \text{if } 5 \leq \frac{M}{2} \implies M \geq 10 \\ \frac{M}{5} & \text{if } 5 > \frac{M}{2} \implies M < 10 \end{cases}$$

Notice that at an interior solution, x_2 doesn't depend on income. So we know that for that $M \geq 10$, we have a vertical line where $x_2 = 2$. However, if $M < 10$, then the Engel curve becomes:

$$x_2(M) = \frac{M}{5} \Longleftrightarrow M(x_2) = 5x_2$$

We are now ready to plot this:



Question 5

(a) p_1 Price Offer Curve

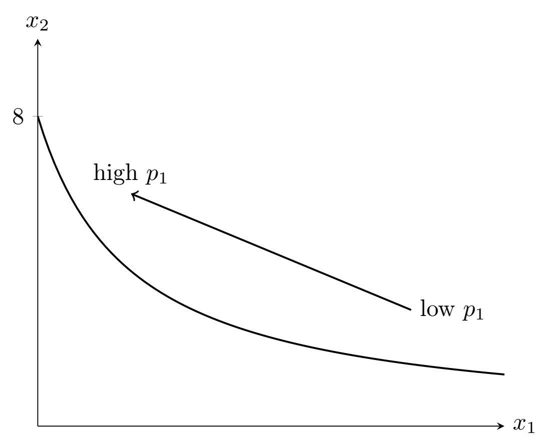
To get the price offer curve, we want to get x_2 as a function of x_1. Since this is the p_1 price offer curve, we want to remove p_1 from the relationship. From the tangency condition, we get that:

$$x_2 = \frac{2p_1}{p_2}$$

We have to get rid of the p_1, which we can do by substituting in the inverse demand curve: $p_1 = \frac{40}{x_1+2}$. For p_2, we can substitute in the current value of 5. This gives us:

$$\begin{aligned} x_2 &= \frac{2}{5} \cdot \left(\frac{40}{x_1+2}\right) \\ &= \frac{16}{x_1+2} \end{aligned}$$

We can now plot this curve:



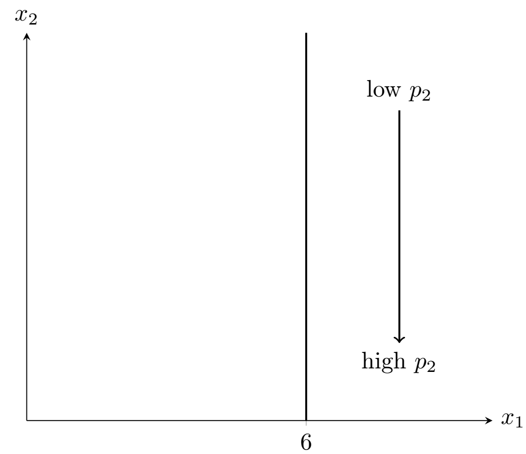
(b) p_2 Price Offer Curve

As before, we want x_2 as a function of x_1. Starting again from the tangency condition:

$$x_2 = \frac{2p_1}{p_2}$$

We now need to get rid of p_2. The usual trick is to substitute an inverse demand function (i.e. p_2 as a function of x_2). However, x_1 is not a function of p_2! We need to take a different approach.

Notice that this means that no matter what p_2 is, x_1 will always be the same value. This suggests that the p_2 price offer curve is actually a vertical line! In fact, given that $p_1 = 5$ and $M = 40$, then we must have that $x_1 = \frac{40}{5} - 2 = 6$. This means the p_2 price offer curve looks as follows:



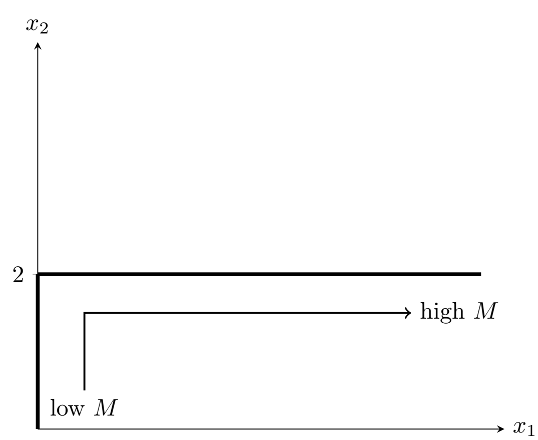
(c) Income Offer Curve

Again, we start from the tangency condition:

$$x_2 = \frac{2p_1}{p_2}$$

Here, there is no M. This means that the income expansion path is a horizontal line, since the optimal x_2 doesn't change as income changes. In fact, at the given parameter values of $p_1 = p_2 = 5$, we have $x_2 = \frac{2(5)}{5} = 2$.

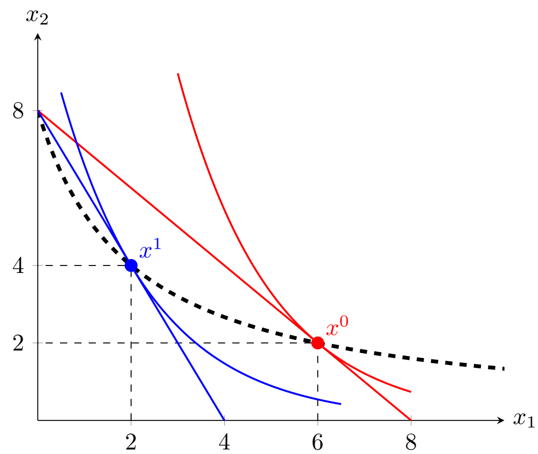
However, there is an exception. We know from the demand functions that if income is too low, then we get to a corner solution. In particular, if $M < 2p_1 = 10$, then we have a corner solution. When this happens, then we have $x_1 = 0$ and $x_2 = \frac{M}{5}$. Notice that at $M = 10$, we have $x_1 = 0$ and $x_2 = 2$.



Question 6

(a) Diagram

Before the price change, we had $p_1 = 5$, $p_2 = 5$, and $M = 40$. This gave us an optimal bundle of $(x_1, x_2) = (6, 2)$. If $p_1 = 10$, then both quantities will change. We get $x_1 = \frac{40}{10} - 2 = 4 - 2 = 2$ and $x_2 = \frac{2(10)}{5} = 4$. The diagram looks as follows:



(b) Decomposition

To decompose this, we need to first find the income which makes the old optimal bundle affordable at the new prices:

$$
\begin{aligned}
p_1^1 x_1^0 + p_2^1 x_2^0 &= M^s \\
10 \cdot 6 + 5 \cdot 2 &= M^s \\
60 + 10 &= M^s \\
\therefore M^s &= 70
\end{aligned}
$$

We now plug this back into the demand function:

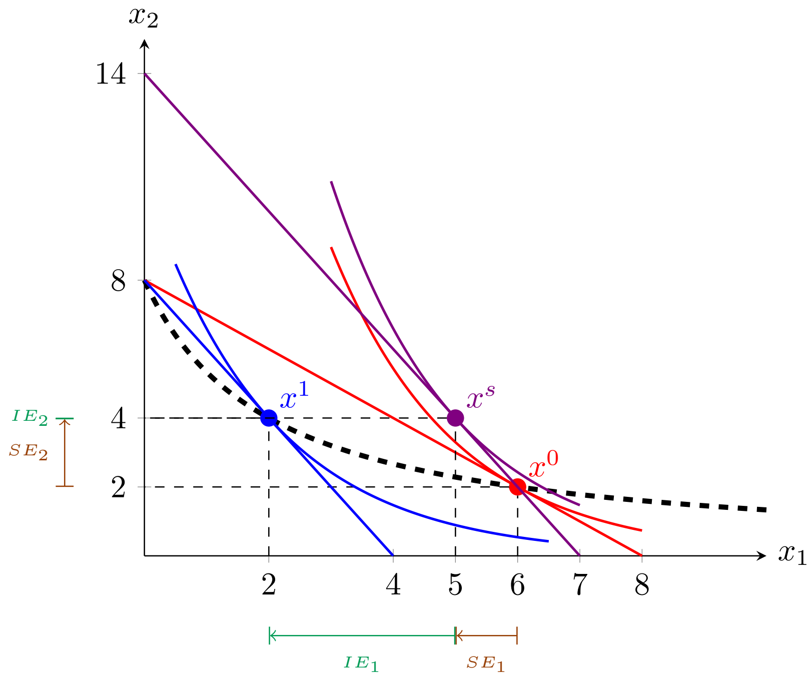
$$
\begin{aligned}
x_1(10, 5, 70) &= \frac{70}{10} - 2 = 5 \\
x_2(10, 5, 70) &= \frac{2 \cdot 10}{5} = 4
\end{aligned}
$$

Therefore, the decomposition is:

$$
\begin{aligned}
SE_1 &= 5 - 6 = -1 & \qquad SE_2 &= 4 - 2 = 2 \\
IE_1 &= 2 - 5 = -3 & \qquad IE_2 &= 4 - 4 = 0
\end{aligned}
$$

(c) Diagram

For the diagram, we need to draw the Slutsky budget line (purple). This has the same slope as the new budget line (blue) and passes through the old optimal bundle (red). The diagram looks as follows



Question 7

If p_2 decreases to 2, then our new optimal bundle is:

$$x_1(5, 2, 40) = \frac{40}{5} - 2 = 6$$
$$x_2(5, 2, 40) = \frac{2 \cdot 5}{2} = 5$$

The CV is calculated at the new prices with the old utility. The old utility is evaluated at the point $x^0 = (6, 2)$:

$$u(6, 2) = 6 + 2 \ln 2$$

The demand at the new prices (and with an income M^c that we need to solve for) is:

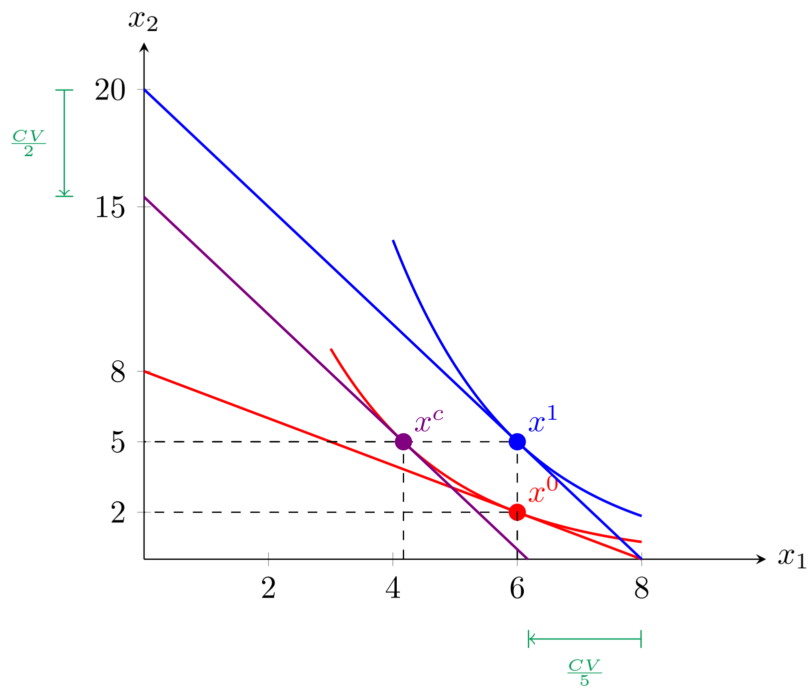
$$x_1^c = x_1(5, 2, M^c) = \frac{M^c}{5} - 2$$
$$x_2^c = x_2(5, 2, M^c) = \frac{2 \cdot 5}{2} = 5$$

We know that the utility at this point equals the old utility. This gives us:

$$u(x_1^c, x_2^c) = u(x_1^0, x_2^0)$$

$$\frac{M^c}{5} - 2 + 2\ln 5 = 6 + 2\ln 2$$
$$\frac{M^c}{5} = 8 + 2\ln\frac{2}{5}$$
$$M^c = 40 + 10\ln 0.4$$
$$\approx 30.84$$

Therefore, the CV is: $M^c - M^1 \approx 30.84 - 40 = -10.84$. The diagram looks as follows:



Question 8

If p_1 increases to 25, then our new optimal bundle becomes: (note we are in a corner solution since $p_1 = 25 > 20 = \frac{M}{2}$)

$$x_1(25, 5, 40) = 0$$
$$x_2(25, 5, 40) = \frac{40}{5} = 8$$

The EV is calculated at the old prices with the new utility. The new utility is evaluated at the point $x^1 = (0, 8)$:

$$u(0, 8) = 2\ln 8$$

The demand at the old prices (and with an income M^e that we need to solve for) is:

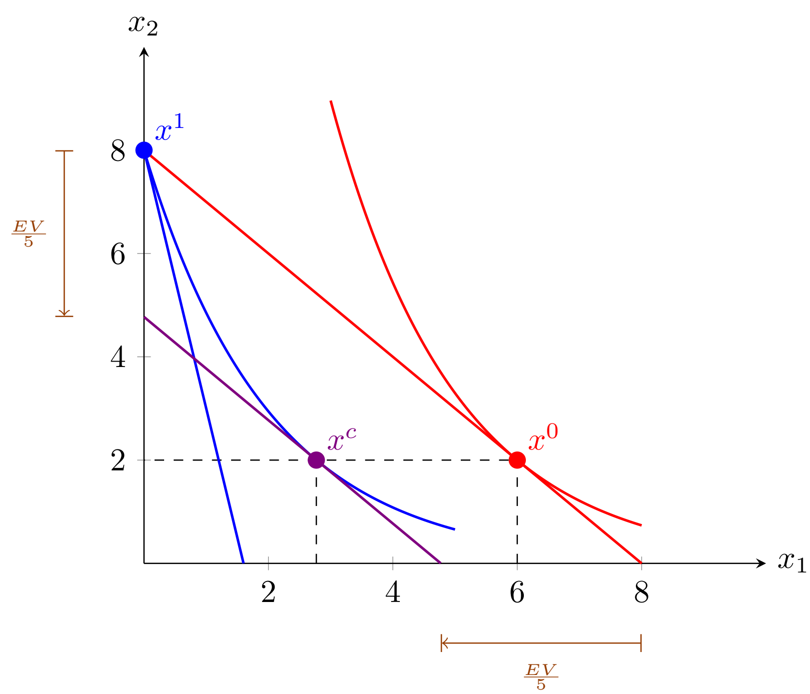
$$x_1^e = x_1(5, 5, M^e) = \frac{M^e}{5} - 2$$

$$x_2^e = x_2(5, 5, M^e) = \frac{2 \cdot 5}{5} = 2$$

We know that the utility at this point equals the new utility. This gives us:

$$\begin{aligned}
u(x_1^e, x_2^e) &= u(x_1^1, x_2^1) \\
\frac{M^e}{5} - 2 + 2\ln 2 &= 2\ln 8 \\
\frac{M^e}{5} &= 2 + 2\ln 8 - 2\ln 2 \\
&= 2 + 2\ln 4 \\
M^e &= 10 + 10\ln 4 \\
&\approx 23.86
\end{aligned}$$

Therefore, the EV is: $M^e - M^0 \approx 23.86 - 40 = -16.14$. The diagram looks as follows:



Question 9

(a) Own-Price Elasticity

The own-price elasticity of x_1 would be:

$$\begin{aligned}
\varepsilon_{p_1}^1 &= \frac{\partial x_1(p, M)}{\partial p_1} \times \frac{p_1}{x_1} \\
&= \frac{\partial}{\partial p_1}\left(\frac{M}{p_1} - 2\right) \times \frac{p_1}{x_1}
\end{aligned}$$

$$= \left(-\frac{M}{p_1^2}\right) \times \frac{p_1}{x_1}$$
$$= -\frac{M}{p_1 x_1}$$

However, we want this as a function of p_1 so let's get rid of the x_1 by plugging in the demand function:

$$\varepsilon^1_{p_1} = -\frac{M}{p_1\left(\frac{M}{p_1} - 2\right)}$$
$$= -\frac{M}{M - 2p_1}$$

At the current parameter values, we have $p_1 = 5, M = 40$, so the own-price elasticity is:

$$\varepsilon^1_{p_1} = -\frac{40}{40 - 2 \cdot 5} = -\frac{40}{30} = -\frac{4}{3}$$

Since this is greater than 1 (in absolute value), it is *elastic*.

(b) Cross-Price Elasticity

The cross-price elasticity of x_2 would be:

$$\varepsilon^2_{p_1} = \frac{\partial x_2(p, M)}{\partial p_1} \times \frac{p_1}{x_2}$$
$$= \frac{\partial}{\partial p_1}\left(\frac{2p_1}{p_2}\right) \times \frac{p_1}{x_2}$$
$$= \left(\frac{2}{p_2}\right) \times \frac{p_1}{x_2}$$
$$= \frac{2p_1}{p_2 x_2}$$

However, we want this as a function of p_1 so let's get rid of the x_2 by plugging in the demand function:

$$\varepsilon^2_{p_1} = \frac{2p_1}{p_2\left(\frac{2p_1}{p_2}\right)}$$
$$= \frac{2p_1}{2p_1}$$
$$= 1$$

Therefore, at all parameter values (including the current ones), we have a unit-elastic cross-price elasticity.

(c) Income Elasticity

The income elasticity of x_1 would be:

$$\varepsilon^1_M = \frac{\partial x_1(p, M)}{\partial M} \times \frac{M}{x_1}$$

$$
\begin{aligned}
&= \frac{\partial}{\partial M}\left(\frac{M}{p_1} - 2\right) \times \frac{M}{x_1} \\
&= \left(\frac{1}{p_1}\right) \times \frac{M}{x_1} \\
&= \frac{M}{p_1 x_1}
\end{aligned}
$$

However, we want this as a function of M so let's get rid of the x_1 by plugging in the demand function:

$$
\begin{aligned}
\varepsilon_M^1 &= \frac{M}{p_1\left(\frac{M}{p_1} - 2\right)} \\
&= \frac{M}{M - 2p_1}
\end{aligned}
$$

At the current parameter values, we have $p_1 = 5, M = 40$, so the income elasticity is:

$$
\varepsilon_M^1 = \frac{40}{40 - 2 \cdot 5} = \frac{40}{30} = \frac{4}{3}
$$

The income elasticity of x_2 would be:

$$
\begin{aligned}
\varepsilon_M^2 &= \frac{\partial x_2(p, M)}{\partial p_1} \times \frac{M}{x_2} \\
&= \frac{\partial}{\partial M}\left(\frac{2p_1}{p_2}\right) \times \frac{M}{x_2} \\
&= 0 \times \frac{M}{x_2} \\
&= 0
\end{aligned}
$$

Unsurprisingly, since income doesn't affect x_2 demand (at an interior solution), then we have an elasticity of zero (i.e. perfectly inelastic). Clearly, x_1 is more income elastic.

Question 10

To answer this, we use the elasticities we just calculated. We want to measure the responsiveness of demand to a change in p_1. So we need the own-price elasticity of x_1 and the cross-price elasticity of x_2:

$$
\begin{aligned}
\varepsilon_{p_1}^1 &= -\frac{M}{M - 2p_1} \\
\varepsilon_{p_1}^2 &= 1
\end{aligned}
$$

The elasticities tell us how each quantity demanded changes (as a percentage change) in response to a 1% increase in p_1. So if we had a 1% decrease in p_1, then the responses would:

$$
\begin{aligned}
\%\Delta x_1 &= -\varepsilon_{p_1}^1 = \frac{M}{M - 2p_1} \\
\%\Delta x_2 &= -\varepsilon_{p_1}^2 = -1
\end{aligned}
$$

What this question is basically asking is this: Is $|\%\Delta x_1| > |\%\Delta x_2|$ at all parameter values? Or equivalently, is $\left|\varepsilon^1_{p_1}\right| > \left|\varepsilon^2_{p_1}\right|$ at all parameter values? Let's check:

$$\begin{aligned}
\left|\varepsilon^1_{p_1}\right| &> \left|\varepsilon^2_{p_1}\right| \\
\frac{M}{M-2p_1} &> 1 \\
M &> M - 2p_1 \\
2p_1 &> 0 \\
\therefore p_1 &> 0
\end{aligned}$$

So this statement is true as long as $p_1 > 0$, but that's always the case! Therefore, we always have an *elastic* own-price elasticity of x_1 and a *unit elastic* cross-price elasticity of x_2. Hence x_1 will always respond proportionately more to a change in p_1 than x_2 will. So the statement is true.