Intermediate Micro: Recitation 8
Substitution and Income Effects

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1 Effects of a Price Change

When prices change, consumers re-optimize and choose a new optimal bundle. In this recitation, we are interested in understanding how the optimal bundle changes in response to a change in price. In particular, we *decompose* the effect a price change into two effects:

- **Substitution effect (SE):** If a price of a good p_i *increases*, then good i becomes relatively more expensive, so you want to *decrease* consumption of good i and *increase* consumption of the other good j (i.e. substitute away from i to j)
- **Income effect (IE):** If the price of a good p_i *increases*, then the overall price level in the economy is higher and real income *falls*. You feel poorer overall and so you will want to *decrease* consumption of normal goods and *increase* consumption of inferior goods

The **total effect (TE)**, i.e. the total change in quantity demanded, is simply the sum of the two individual effects:

$$\textbf{Total Effect} = \textbf{Substitution Effect} + \textbf{Income Effect}$$

Say we have an increase in p_i, what can we say about the change in x_i? From the above description, you can see that the substitution effect on x_i is always negative for an increase in own price (it might be zero, but it can never be strictly positive). However, the sign of the income effect is not clear. It might be negative if x_i is a normal or it might be positive if it is inferior. So we are adding a negative number to a number that may be positive or negative. Therefore, the sign of the total effect is ambiguous. Consider the following examples:

- x_1 and x_2 are normal. Then an increase in p_1 means that for x_1, we have $IE < 0$. We are adding two negative numbers, so the total effect for x_1 is $TE < 0$ (i.e. x_1 decreases)
- x_1 and x_2 are normal and substitutes. Then an increase in p_1 means that for x_2, we have $SE > 0$ (since it is a substitute) and $IE < 0$. Therefore, TE is ambiguous

- x_1 and x_2 are normal and complements. Then an increase in p_1 means that for x_2, we have $SE < 0$ (since it is a complement) and $IE < 0$. Therefore, $TE < 0$
- x_1 is inferior. Then an increase in p_1 means that for x_1, we have $IE > 0$, so TE is ambiguous.
  - If $TE < 0$, then we have an ordinary good. This means that the income effect was smaller than the substitution effect ($IE < |SE|$)
  - If $TE > 0$, then we have a Giffen good. This means that the income effect was larger than the substitution effect ($IE > |SE|$)

The last example in particular is very interesting, because it gives an insight into how such an odd case like a Giffen good can occur. The intuition for a Giffen good is as follows. Suppose the price of the good increases. Since the good is more expensive, you want to substitute to consuming a different good ($SE < 0$). However, the price increase has made you feel so much poorer that you want to reduce your consumption of normal goods and increase your consumption of all inferior goods (including the Giffen good). This effect of "feeling poorer" is so strong that it actually makes you buy *more* of the Giffen good (whose price increase made you feel poorer to begin with!). How could such a thing happen? Well, realistically, it's quite hard to find a true Giffen good - and even if you find a Giffen good, it may not be Giffen at all prices and income levels. Traditionally, we think that a Giffen good occurs (if ever) when the consumer has very low income and the Giffen good makes up a very large share of their income. This could be a reason why the income effect is so strong (i.e. the price change of one good makes the consumer feel so much poorer because it affects such a large part of their usual consumption, and they have very little to substitute to).

2 Decomposition

Hopefully, you see that there is a value in decomposing the effects of a price change. Next, we want to know how to actually do this decomposition. To do this, let's consider what happens when prices change. For this, we will go from what I call the "old world" (with the old prices) to a "new world" (with the new prices). We will also have a thought-experiment and imagine a third "alternate world" (to be explained soon). The notation for this setup will be as follows:

	Old World (0)	**New World (1)**	**Alternate World (a)**
$\boldsymbol{p_1}$	p_1^0	p_1^1	p_1^a
$\boldsymbol{p_2}$	p_2^0	p_2^1	p_2^a
$\boldsymbol{M}$	M^0	M^0	M^a
Optimal Bundle $\boldsymbol{(x_1^*, x_2^*)}$	$x^0 = (x_1^0, x_2^0)$	$x^1 = (x_1^1, x_2^1)$	$x^a = (x_1^a, x_2^a)$
Budget Line	B^0	B^1	B^a
Indifference Curve	I^0	I^1	I^a

Notice that when we go from the old world to the new world, the prices change but the income stays the same. Our total effect is easy to see from this table: (it's just what we are used to)

$$TE_1 = x_1^1 - x_1^0$$
$$TE_2 = x_2^1 - x_2^0$$

So how do we get the decomposition? For this we do the following thought-experiment using our alternate world:

1. Consider an alternate world where there is a substitution effect but no income effect
   - Since SE is determined by relative prices, this means that the **prices** in the alternate world must be the same as in the new world, i.e. $p_1^a = p_1^1$ and $p_2^a = p_2^1$
   - Since there is no IE, your **income** in the alternate world needs to be at a level where you are "*as rich as you were before*" in the old world. This will give you an income level M^a
2. Now that you have all the parameters in the alternate world, you can solve for a new optimal bundle $x^a = (x_1^a, x_2^a)$.
   - The change from the old world to the alternate world is entirely a substitution effect (since we ensured there was no income effect in step 1)
   - The SE is therefore defined as $SE_1 = x_1^a - x_1^0$ and $SE_2 = x_2^a - x_2^0$
3. The change from the alternate world to the new world is parallel shift of the budget line (since we made the prices equal)
   - This change is entirely an income effect (since prices don't change, there is no substitution effect)
   - The IE is therefore defined as $IE_1 = x_1^1 - x_1^a$ and $IE_2 = x_2^1 - x_2^a$

Intuitively, we are basically creating a "middle step" between the change from the old world to the new world. The first change to the middle step will be entirely the substitution effect, and the second change to the last step will be entirely the income effect. It's important to keep in mind that in the *real* world there is no hypothetical alternate world. This is just a thought experiment to decompose the effects, but in reality, everything is happening simultaneously.

However, there is a big part that is still unclear. We said in step 1 that we want to make sure you are "*as rich as you were before*". What does that mean? There are two possible definitions:

- The **Slutsky** definition $(a = s)$: You can still *buy* the old bundle x^0 in the alternate world
  - In other words, x^0 is on your *budget line* in the alternate world (s) : $p_1^s x_1^0 + p_2^s x_2^0 = M^s$
- The **Hicks** definition $(a = h)$: You achieve the same *utility* as the old bundle x^0 in the alternate world
  - In other words, x^0 is on the same *indifference curve* as your optimal bundle in the alternate world (h) : $u(x_1^0, x_2^0) = u(x_1^h, x_2^h)$

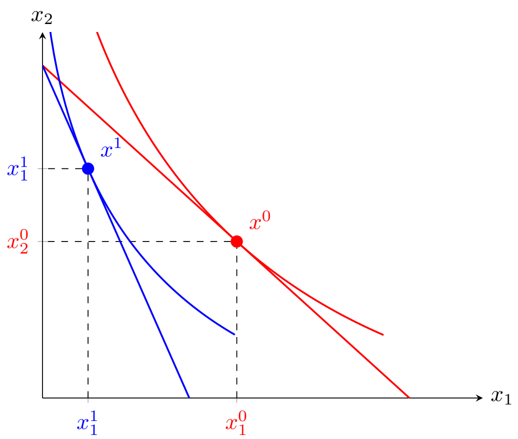
In this course, we will only focus on the Slutsky definition, but I'll explain how to use both approaches.

Slutsky Decomposition

For the graphs, suppose we have p_1 increase, i.e. $p_1^1 > p_1^0$, and everything else stays the same. However, the steps are general and apply for any setting.

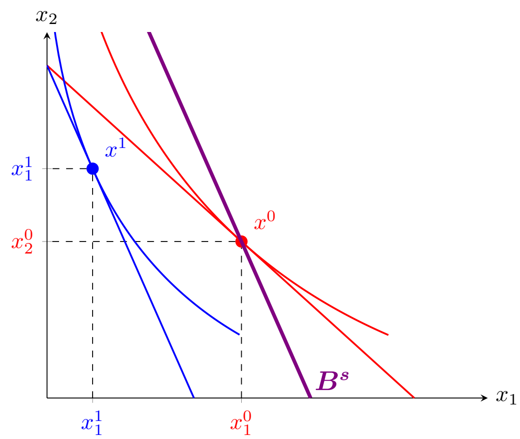
Step 1

- Derive the consumer's demand function $x(p, M)$
- Plug in parameters to get the old bundle x^0 and new bundle x^1
- Plot the graphs with BL B^0 and IC I^0 for the old world and BL B^1 and IC I^1 for the new world



Step 2

- Draw the Slutsky budget line B^s
- This BL has to:
  - have the same slope as B^1, and
  - pass through x^0



Step 3

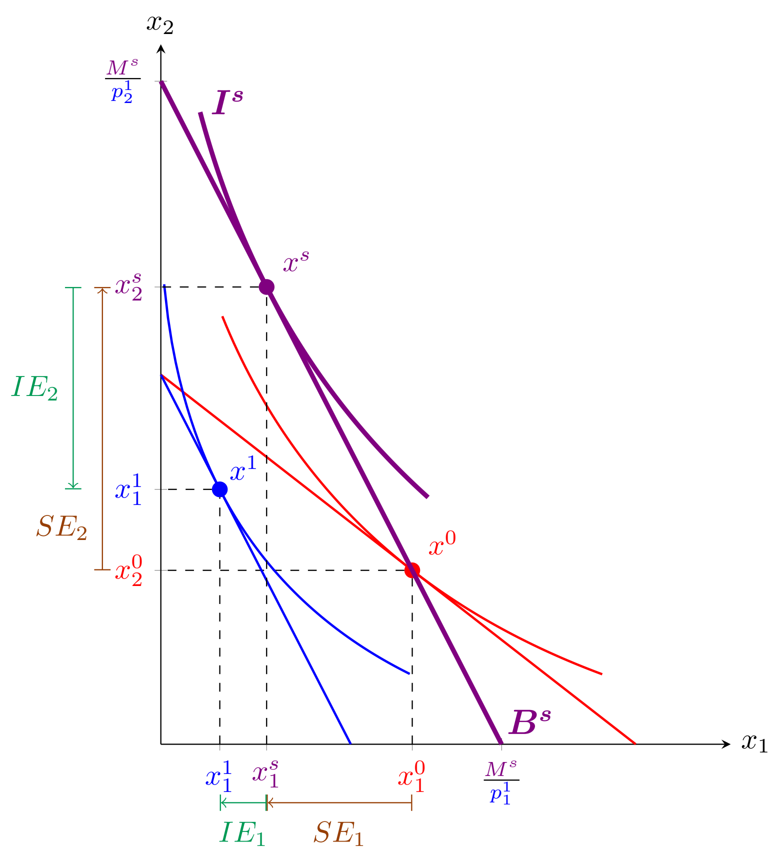
- Calculate the Slutsky income M^s
  - Since x^0 on $B^s \implies p_1^s x_1^0 + p_2^s x_2^0 = M^s$
  - Since $p^s = p^1 \implies p_1^1 x_1^0 + p_2^1 x_2^0 = M^s$
  - Plug in values of old bundle and new prices to get M^s

Step 4

- Find optimal Slutsky bundle x^s
  - Plug in parameters into demand function: $x^s = x(p^s, M^s) = x(p^1, M^s)$
- Plot new IC I^s

Step 5

- Calculate SE: $SE_i = x_i^s - x_i^0$
- Calculate IE: $IE_i = x_i^1 - x_i^s$
- Calculate TE: $TE_i = IE_i + SE_i = x_i^1 - x_i^0$

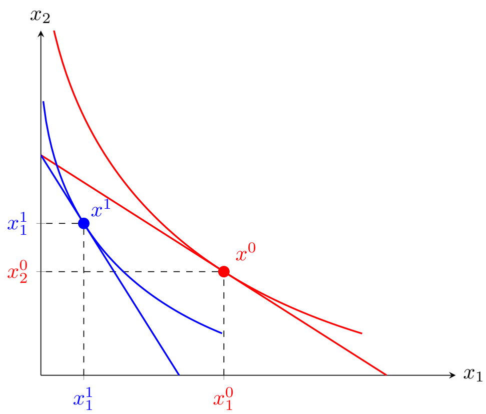


Hicks Decomposition

For the graphs, suppose we have p_1 increase, i.e. $p_1^1 > p_1^0$, and everything else stays the same. However, the steps are general and apply for any setting.

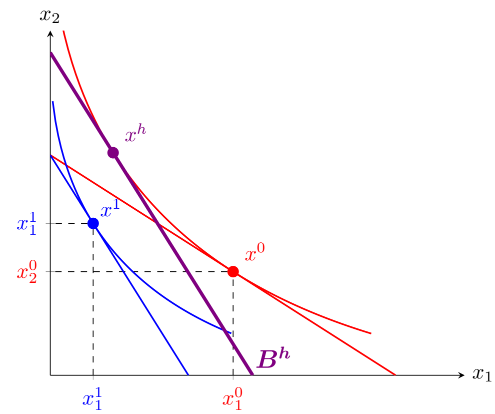
Step 1

- Derive the consumer's demand function $x(p, M)$
- Plug in parameters to get the old bundle x^0 and new bundle x^1
- Plot the graphs with BL B^0 and IC I^0 for the old world and BL B^1 and IC I^1 for the new world



Step 2

- Draw the Hicks budget line B^h
- This BL has to:
  - have the same slope as B^1, and
  - be tangent to I^0 at x^h



Step 3

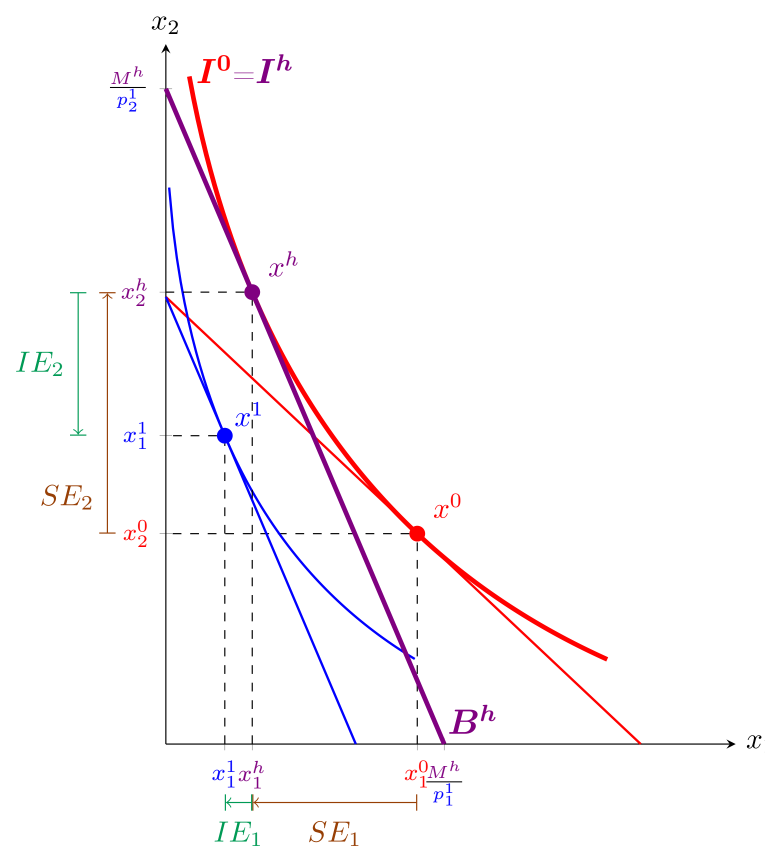
- Calculate the Hicks income M^h
  - Since x^h and x^0 are both on I^0
    $\implies u(x_1^h, x_2^h) = u(x_1^0, x_2^0)$
  - From demand function:
    $x_i^h = x_i(p^h, M^h) = x_i(p^1, M^h)$
  - Can calculate old utility: $u(x_1^0, x_2^0) = u^0$
- Plug in old utility and new prices
  $\implies u(x_1(p^1, M^h), x_2(p^1, M^h)) = u^0$
  - Only unknown is M^h, which you can solve for

Step 4

- Find optimal Hicksian bundle x^h
  - Plug in parameters into demand function:
    $x^h = x(p^h, M^h) = x(p^1, M^h)$

Step 5

- Calculate SE: $SE_i = x_i^h - x_i^0$
- Calculate IE: $IE_i = x_i^1 - x_i^h$
- Calculate TE: $TE_i = IE_i + SE_i = x_i^1 - x_i^0$



3 Examples

For all these examples, I will only use the **Slutsky decomposition method**.

3.1 Cobb-Douglas

Consider the following setup:

- $u(x_1, x_2) = x_1^5 x_2^3$
- $p_1 = 5, p_2 = 2, M = 120$
- Then p_1 decreases to $p_1 = 3$

We want to decompose the total effect of the price change into the income and substitution effect.

Step 1

First, derive the demand function:

$$\begin{aligned} |MRS| = \frac{5x_2}{3x_1} &= \frac{p_1}{p_2} \\ x_2 &= \frac{3p_1}{5p_2} x_1 \end{aligned}$$

$$\begin{aligned} \therefore p_1 x_1 + p_2 \left(\frac{3p_1}{5p_2} x_1 \right) &= M \\ p_1 x_1 \left(1 + \frac{3}{5} \right) &= M \\ x_1(p, M) &= \frac{5M}{8p_1} \\ \implies x_2(p, M) &= \frac{3M}{8p_2} \end{aligned}$$

Then, we calculate the optimal bundles before and after the price change:

Before:	**After** :
$x_1(5, 2, 120) = \frac{5 \times 120}{8 \times 5} = 15$	$x_1(3, 2, 120) = \frac{5 \times 120}{8 \times 3} = 25$
$x_2(5, 2, 120) = \frac{3 \times 120}{8 \times 2} = 22.5$	$x_2(5, 2, 120) = \frac{3 \times 120}{8 \times 2} = 22.5$

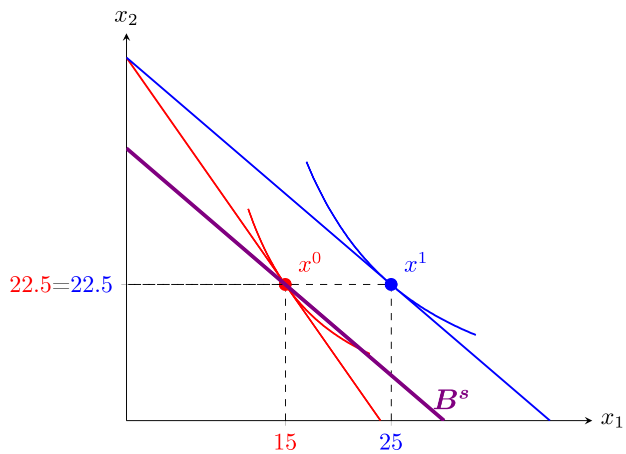
From this, we can see the total effect: $TE_1 = 25 - 15 = 10$ and $TE_2 = 22.5 - 22.5 = 0$. This is what we expect with Cobb-Douglas: as p_1 decreases, demand for x_1 increases but the demand for x_2 is unaffected.

Step 2

Next, we determine the Slutsky budget line. This is the new prices with an income level M^s, which we will need to solve for:

$$3x_1 + 2x_2 = M^s$$

Drawing this in the diagram, we know that this budget line has to pass through the original optimal bundle $(15, 22.5)$ and have the same slope as the blue B^1 budget line.



Step 3

Now, we find the value of the Slutsky income M^s. We know the budget equation and we know that $(15, 22.5)$ is on it, so we plug in this point into the equation:

$$\begin{aligned} 3(15) + 2(22.5) &= M^s \\ 45 + 45 &= M^s \\ \therefore M^s &= 90 \end{aligned}$$

Step 4

To find the optimal Slutsky bundle, we just plug in the parameters (new prices and Slutsky income) into the demand function:

$$\begin{aligned} x_1^s(3, 2, 90) &= \frac{5 \times 90}{8 \times 3} = 18.75 \\ x_2^s(3, 2, 90) &= \frac{3 \times 90}{8 \times 2} = 16.875 \end{aligned}$$

Step 5

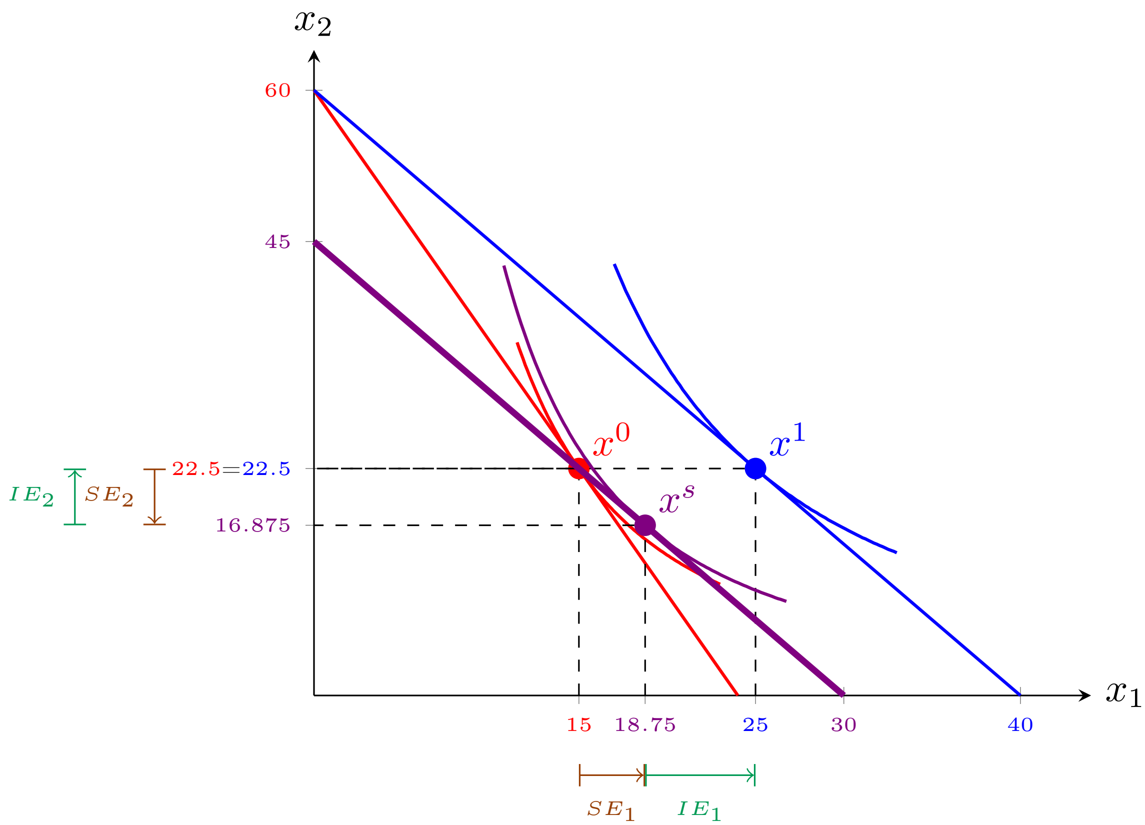
Finally, we decompose the price effect:

$$\begin{aligned} SE_1 &= 18.75 - 15 = 3.75 & SE_2 &= 16.875 - 22.5 = -5.625 \\ IE_1 &= 25 - 18.75 = 6.25 & IE_2 &= 22.5 - 16.875 = 5.625 \end{aligned}$$

As a final check, we should make sure that $TE_i = SE_i + IE_i$:

$$TE_1 = 3.75 + 6.25 = 10 \checkmark$$
$$TE_2 = (-5.625) + 5.625 = 0 \checkmark$$

Overall, for Cobb-Douglas, we see that a price change in good i does not affect the demand for good j. However, that does not necessarily mean there is no effect! Notice that with Cobb-Douglas, the substitution and income effect exactly cancel out for x_2.



3.2 Quasi-Linear

Consider the following setup:

- $u(x_1, x_2) = 6x_1^{1/3} + x_2$
- $p_1 = 2, p_2 = 1, M = 20$
- Then p_2 increases to $p_2 = 4$

Step 1

First, derive the demand function:

$$
\begin{aligned}
|MRS| = \frac{6 \times \frac{1}{3} x_1^{-2/3}}{1} &= \frac{p_1}{p_2} \\
x_1^{-2/3} &= \frac{p_1}{2p_2} \\
x_1 &= \left(\frac{2p_2}{p_1} \right)^{3/2}
\end{aligned}
$$

For an interior solution, the demands are: (let's not worry about corners for this question)

$$
\begin{aligned}
x_1(p, M) &= \left(\frac{2p_2}{p_1} \right)^{3/2} \\
\implies x_2(p, M) &= \frac{1}{p_2} \left(M - x_1(p, M) \right) \\
&= \frac{1}{p_2} \left(M - p_1 \left(\frac{2p_2}{p_1} \right)^{3/2} \right)
\end{aligned}
$$

Then, we calculate the optimal bundles before and after the price change:

Before:

$$
\begin{aligned}
x_1(2, 1, 20) &= \left(\frac{2 \times 1}{2} \right)^{3/2} = 1^{3/2} = 1 \\
x_2(2, 1, 20) &= \frac{1}{1} \left(20 - 2(1) \right) = 18
\end{aligned}
$$

After :

$$
\begin{aligned}
x_1(2, 4, 20) &= \left(\frac{2 \times 4}{2} \right)^{3/2} = 4^{3/2} = 8 \\
x_2(2, 4, 20) &= \frac{1}{4} \left(20 - 2(8) \right) = 1
\end{aligned}
$$

From this, we can see the total effect: $TE_1 = 8 - 1 = 7$ and $TE_2 = 1 - 18 = -17$.

Step 2

Next, we determine the Slutsky budget line. This is the new prices with an income level M^s, which we will need to solve for:

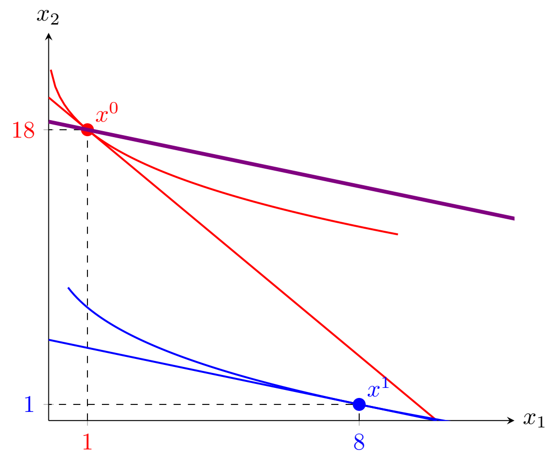
$$
2x_1 + 4x_2 = M^s
$$

Drawing this in the diagram, we know that this budget line has to pass through the original optimal bundle $(1, 18)$ and have the same slope as the blue B^1 budget line.

Step 3

Now, we find the value of the Slutsky income M^s. We know the budget equation and we know that $(1, 18)$ is on it, so we plug in this point into the equation:

$$
\begin{aligned}
2(1) + 4(18) &= M^s \\
2 + 72 &= M^s \\
\therefore M^s &= 74
\end{aligned}
$$



Step 4

To find the optimal Slutsky bundle, we just plug in the parameters (new prices and Slutsky income) into the demand function:

$$x_1^s(2,4,74) = \left(\frac{2\times 4}{2}\right)^{3/2} = 4^{3/2} = 8$$
$$x_2^s(2,4,74) = \frac{1}{4}\left(74 - 2(8)\right) = 14.5$$

As we expect, since this is quasi-linear, the change in income between the alternate and new world has no effect on x_1 (the non-numeraire good).

Step 5

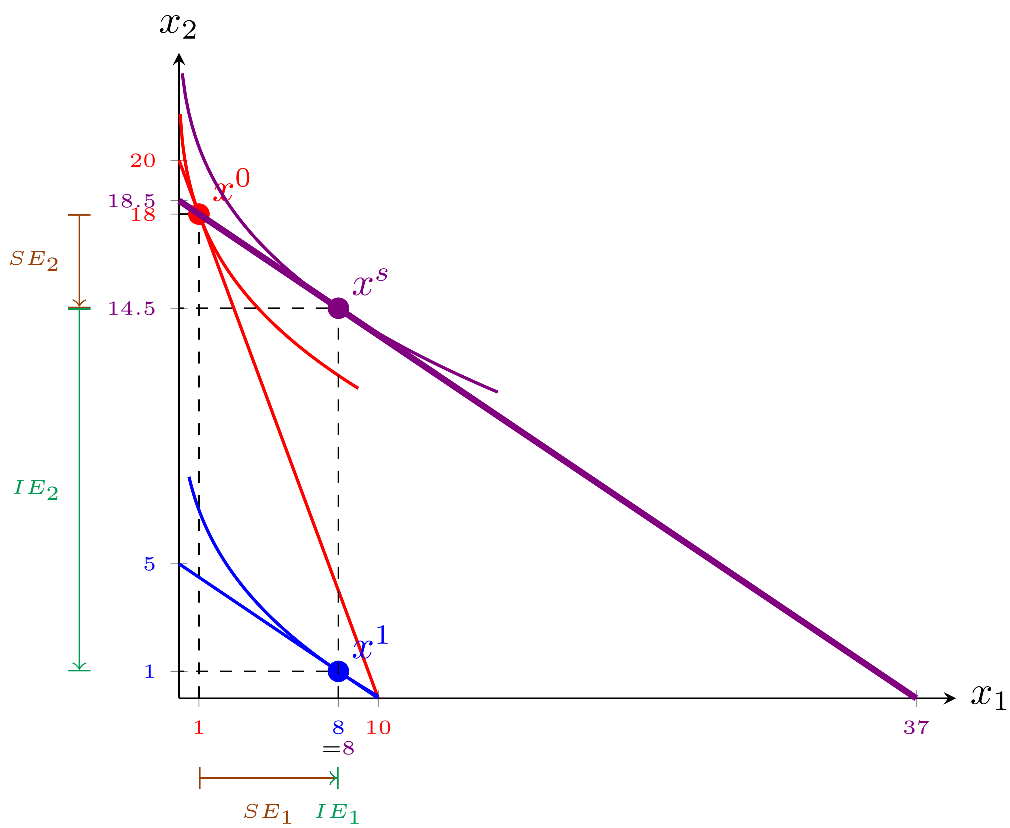
Finally, we decompose the price effect:

$$SE_1 = 8 - 1 = 7 \qquad SE_2 = 14.5 - 18 = -3.5$$
$$IE_1 = 8 - 8 = 0 \qquad IE_2 = 1 - 14.5 = -13.5$$

As a final check, we should make sure that $TE_i = SE_i + IE_i$:

$$TE_1 = 7 + 0 = 7 \checkmark$$
$$TE_2 = (-3.5) + (-13.5) = -17 \checkmark$$

So for quasi-linear, we see that the change in x_1 is entirely due to the substitution effect. This is a property of quasi-linear that we are already familiar with: the optimal level of the non-numeraire good is independent of income (at an internal solution). Similarly, we can say that there is "no income effect" for this good. For the numeraire x_2, we see that there is both a substitution and income effect. It also works in the way we would expect for a normal good: prices have increased so the income effect is negative. However, it is interesting to see here that most of the change came from the income effect than the substitution effect.



3.3 Perfect Complements

Consider the following setup:

- $u(x_1, x_2) = \min\{3x_1, 4x_2\}$
- $p_1 = 1, p_2 = 2, M = 30$
- Then p_1 increases to $p_1 = 3$

Step 1

First, derive the demand function. Since this is perfect complements, we don't use the tangency condition but instead find where the "kinks line" and budget line intersect:

$$\begin{aligned} 3x_1 =& 4x_2 \\ x_2 =& \frac{3}{4}x_1 \\ \therefore p_1x_1 + p_2\left(\frac{3}{4}x_1\right) =& M \\ x_1\left(4p_1 + 3p_2\right) =& 4M \end{aligned}$$

$$
\begin{aligned}
x_1(p,M) =& \frac{4M}{4p_1+3p_2} \\
\implies x_2(p,M) =& \frac{3M}{4p_1+3p_2}
\end{aligned}
$$

Then, we calculate the optimal bundles before and after the price change:

Before:

$$
\begin{aligned}
x_1(1,2,30) =& \frac{4\times 30}{4\times 1+3\times 2} = 12 \\
x_2(1,2,30) =& \frac{3\times 30}{4\times 1+3\times 2} = 9
\end{aligned}
$$

After :

$$
\begin{aligned}
x_1(3,2,30) =& \frac{4\times 30}{4\times 3+3\times 2} = \frac{20}{3} \approx 6.67 \\
x_2(3,2,30) =& \frac{3\times 30}{4\times 3+3\times 2} = 5
\end{aligned}
$$

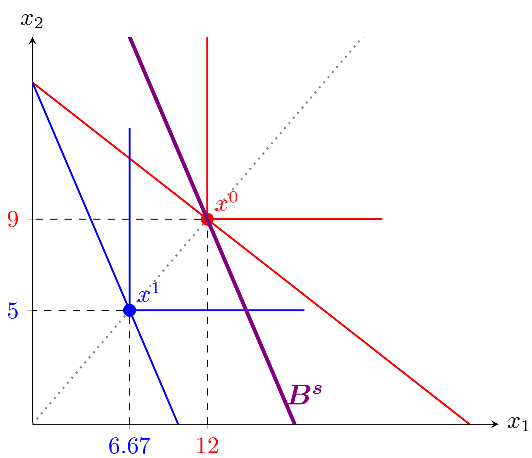
From this, we can see the total effect: $TE_1 = \frac{20}{3} - 12 = -\frac{16}{3} \approx -5.3$ and $TE_2 = 5 - 9 = -4$.

Step 2

Next, we determine the Slutsky budget line. This is the new prices with an income level M^s, which we will need to solve for:

$$3x_1 + 2x_2 = M^s$$

Drawing this in the diagram, we know that this budget line has to pass through the original optimal bundle $(12, 9)$ and have the same slope as the blue B^1 budget line.



Step 3

Now, we find the value of the Slutsky income M^s. We know the budget equation and we know that $(12, 9)$ is on it, so we plug in this point into the equation:

$$
\begin{aligned}
3(12) + 2(9) &= M^s \\
36 + 18 &= M^s \\
\therefore M^s &= 54
\end{aligned}
$$

Step 4

To find the optimal Slutsky bundle, we just plug in the parameters (new prices and Slutsky income) into the demand function:

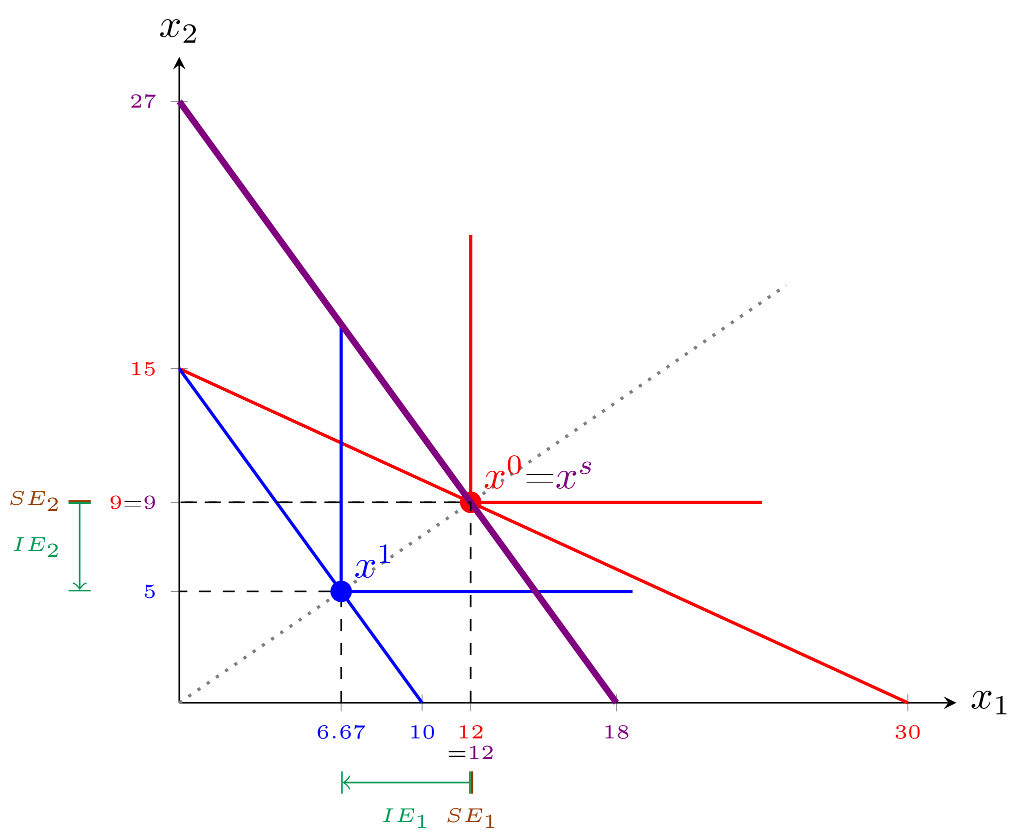
$$x_1^s(3,2,54) = \frac{4\times 54}{4\times 3 + 3\times 2} = 12$$
$$x_2^s(3,2,54) = \frac{3\times 54}{4\times 3 + 3\times 2} = 9$$

It's exactly the same as the old bundle! If you look at the diagram it makes complete sense. We know that the optimal bundle occurs where the budget line intersects the kinks line. And we've constructed the Slutsky budget line to necessarily intersect the kinks line at the old bundle x^0 , so of course this is where x^s will be as well!

Step 5

Finally, we decompose the price effect:

$$SE_1 = 12 - 12 = 0 \qquad SE_2 = 9 - 9 = 0$$
$$IE_1 = 6.67 - 12 = -5.33 \qquad IE_2 = 5 - 9 = -4$$



As a final check, we should make sure that $TE_i = SE_i + IE_i$:

$$TE_1 = 0 + (-5.33) = -5.33 \checkmark$$

$$TE_2 = 0 + (-4) = -4 \checkmark$$

So the entire change was driven by the income effect. Does this make sense? Of course! We have perfect *complements*, which means that the consumer will not substitute away from x_1 just because p_1 increased. Since they are *perfect* complements, then the substitution effect must be exactly zero.

3.4 Perfect Substitutes

Consider the following setup:

- $u(x_1, x_2) = 2x_1 + x_2$
- $p_1 = 3, p_2 = 5, M = 15$
- First, consider a change of p_2 decreasing to $p_2 = 3$
- Second, consider instead a change of p_2 decreasing to $p_2 = 1$

Step 1

First, derive the demand function. Since this is perfect substitutes, we just need to compare the MRS and price ratio. If we have $|MRS| < \frac{p_1}{p_2}$, then the optimal choice is to only purchase x_2. If we have $|MRS| > \frac{p_1}{p_2}$, then the optimal choice is to only purchase x_1. The MRS is constant and equal to:

$$|MRS| = \frac{2}{1} = 2$$

Therefore, the demand function is:

$$x_1(p, M) = \begin{cases} 0 & \text{if } 2 < \frac{p_1}{p_2} \\ \frac{M}{p_1} & \text{if } 2 > \frac{p_1}{p_2} \\ \in \left[0, \frac{M}{p_1}\right] & \text{if } 2 = \frac{p_1}{p_2} \end{cases} \qquad x_2(p, M) = \begin{cases} \frac{M}{p_2} & \text{if } 2 < \frac{p_1}{p_2} \\ 0 & \text{if } 2 > \frac{p_1}{p_2} \\ \in \left[0, \frac{M}{p_2}\right] & \text{if } 2 = \frac{p_1}{p_2} \end{cases}$$

Then, we calculate the optimal bundles before and after the two price changes.

$$\begin{array}{ccc} \textbf{Before:} & \textbf{After 1}: & \textbf{After 2}: \\ \frac{p_1}{p_2} = \frac{3}{5} = 0.6 & \frac{p_1}{p_2} = \frac{3}{3} = 1 & \frac{p_1}{p_2} = \frac{3}{1} = 3 \\ x_1(3,5,15) = \frac{15}{3} = 5 & x_1(3,3,15) = \frac{15}{3} = 5 & x_1(3,1,15) = 0 \\ x_2(3,5,15) = 0 & x_2(3,3,15) = 0 & x_2(3,1,15) = \frac{15}{1} = 15 \end{array}$$

From this, we can see the total effect of the first price change is $TE_1 = 5 - 5 = 0$ and $TE_2 = 0 - 0 = 0$. For the second price change, the total effect is $TE_1 = 0 - 5 = -5$ and $TE_2 = 15 - 0 = 15$.

Step 2

Next, we determine the Slutsky budget line. This is the new prices with an income level M^s, which we will need to solve for. For the first price change this will be:

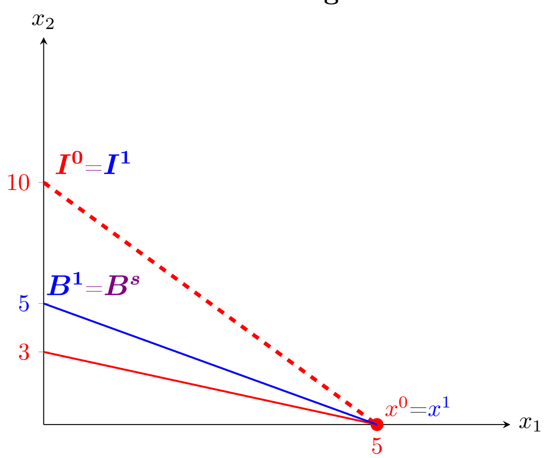
$$3x_1 + 3x_2 = M_1^s$$

For the second price change the Slutsky budget line will be:

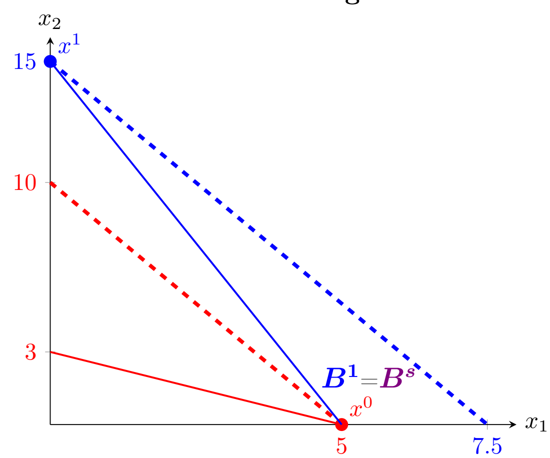
$$3x_1 + x_2 = M_2^s$$

Drawing this in the diagram, we know that both these budget lines have to pass through the original optimal bundle $(5, 0)$ and have the same slope as the blue B^1 budget line. I will draw these in separate diagrams, where the dashed line indicates the indifference curve (to distinguish it from the budget line)

First Price Change



Second Price Change



In the first price change, note that since the optimal bundle never changed, it must mean we are on the same indifference curve. This is why we have $I^0 = I^1$ and can only see one indifference curve.

Step 3

Now, we find the value of the Slutsky income M^s. We know the budget equation and we know that $(5, 0)$ is on it, so we plug in this point into the equation. For the first price change:

$$\begin{aligned} 3(5) + 3(0) &= M_1^s \\ 15 + 0 &= M_1^s \\ \therefore M_1^s &= 15 \end{aligned}$$

But notice that this gives us exactly the same budget line as after the price change (B^1). This is why in the diagram you can't see the purple B^s line, because it is exactly the same as the blue budget line.

And for the second price change, we know that $(5, 0)$ is also on it:

$$3(5) + (0) = M_1^s$$

$$15 + 0 = M_1^s$$
$$\therefore M_2^s = 15$$

Again, this is the same budget line as after the price change (B^1). This isn't just a coincidence. We are choosing the income so that the old optimal bundle is still affordable. But since the price changes are occurring on p_2 and the old optimal bundle only is only purchasing x_1 (i.e. $x_2 = 0$), that means that we have to keep the same income as before (i.e. the optimal bundle will always cost $p_1 x_1 = 3 \times 5 = 15$ no matter what p_2 is equal to).

Step 4

To find the optimal Slutsky bundle, we just plug in the parameters (new prices and Slutsky income) into the demand function. For the first price change:

$$x_1^s(3, 3, 15) = 5$$
$$x_2^s(3, 3, 15) = 0$$

As we can see from the diagram, it's the same as both the old and new optimal bundle. For the second price change:

$$x_1^s(3, 1, 15) = 0$$
$$x_2^s(3, 1, 15) = 15$$

And this is, of course, the same as the new optimal bundle (since it's the same budget line).

Step 5

Finally, we decompose the price effect. For the first price change:

$$SE_1 = 5 - 5 = 0 \qquad SE_2 = 0 - 0 = 0$$
$$IE_1 = 5 - 5 = 0 \qquad IE_2 = 0 - 0 = 0$$

Unsurprisingly, the price change does nothing. On all three budget lines (B^0, B^1, and B^s), we are always at the same bundle of $(5, 0)$. The price change affects a good the consumer wasn't purchasing and still will not purchase after the price decrease. So this means that there is no real effect of the price change.

However, now let's consider the second price change:

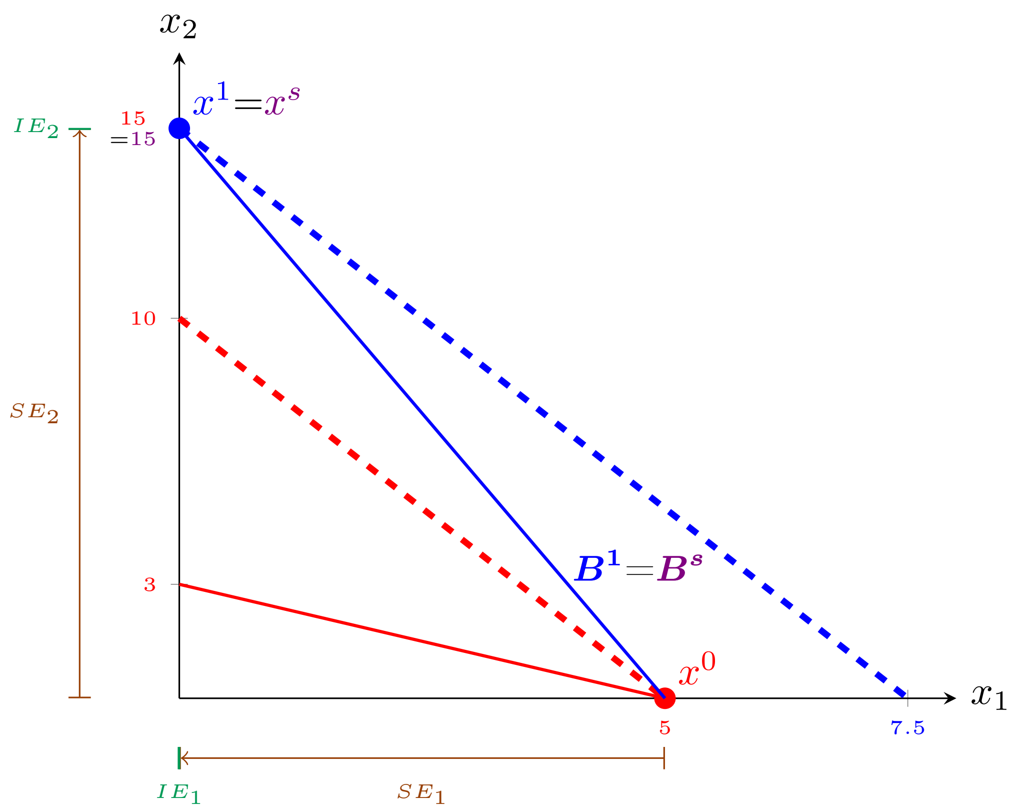
$$SE_1 = 0 - 5 = -5 \qquad SE_2 = 15 - 0 = 15$$
$$IE_1 = 0 - 0 = 0 \qquad IE_2 = 15 - 15 = 0$$

As a final check, we should make sure that $TE_i = SE_i + IE_i$:

$$TE_1 = (-5) + 0 = -5 \checkmark$$
$$TE_2 = 15 + 0 = 15 \checkmark$$

In this price change, everything happens through the substitution effect. This makes sense for two reasons. First, the goods are perfect substitutes, which means the substitution effect will likely play an important role. Second, the old optimal bundle was still affordable after the price change. Recall the thought experiment motivating this decomposition, where we create an alternate world where the consumer is "as rich as they were before the price change". By the Slutsky definition, since the optimal bundle is still affordable, the consumer is still as rich as they were before, and hence there is no income effect that we need to control for.

Since there isn't much to see for the first price change, here is the diagram for the second price change.



However, don't think just because we have perfect substitutes that there can never be an income effect. Consider the following price change:

- p_1 decreases to 1 and p_2 stays the same (i.e. $p_1 = 1, p_2 = 5, M = 15$)

Given that the new price ratio is $\frac{1}{5}$, the new bundle after the price change would be:

$$x_1(1,5,15) = \frac{15}{1} = 15$$
$$x_2(1,5,15) = 0$$

This means that the total effect is $TE_1 = 15 - 5 = 10$ and $TE_2 = 0 - 0 = 0$.

In this case, the Slutsky budget line would be:

$$x_1 + 5x_2 = M^s$$

Solving for the Slutsky income (using the fact that it passes through the original bundle $(5, 0)$):

$$\begin{aligned}(5) + 5(0) &= M^s \\ 5 + 0 &= M^s \\ \therefore M^s &= 5\end{aligned}$$

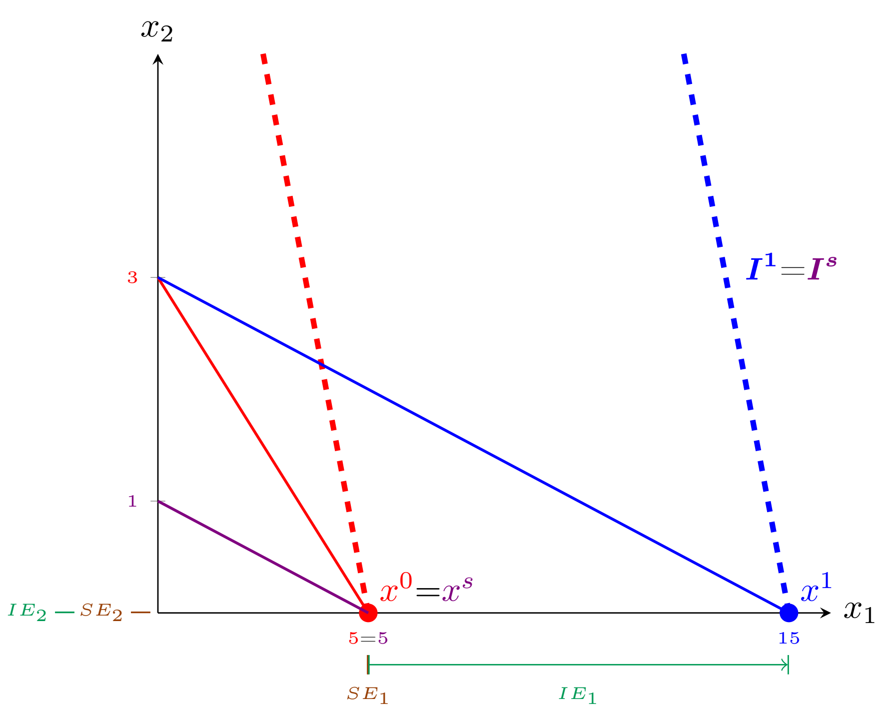
Now we can solve for the optimal Slutsky bundle:

$$\begin{aligned}x_1^s(1, 5, 5) =& \frac{5}{1} = 5 \\ x_2^s(1, 5, 5) =& 0\end{aligned}$$

Finally, we decompose the price effect.

$$SE_1 = 5 - 5 = 0 \qquad SE_2 = 0 - 0 = 0$$
$$IE_1 = 15 - 5 = 10 \qquad IE_2 = 0 - 0 = 0$$

Now the entire change is being caused by the income effect. The price change did affect the price ratio, but since we still have $|MRS| > \frac{p_1}{p_2}$, the consumer will still only purchase x_1. This means there cannot be a substitution effect. However, since the price change was in p_1, even though the mix of goods hasn't changed, the consumer is able to purchase *more* x_1. Therefore, the change is entirely an income effect.



As this example shows, at a corner, we may have:

1. No SE or IE (change 1)
2. Only SE and no IE (change 2)
3. Only IE and no SE (change 3)

You can see all of this summarized in one table:

		Change 1		**Change 2**		**Change 3**	
	Old	**New 1**	**Slutsky 1**	**New 2**	**Slutsky 2**	**New 3**	**Slutsky 3**
$\boldsymbol{p_1}$	3	3	3	3	3	1	1
$\boldsymbol{p_2}$	5	3	3	1	1	5	5
$\boldsymbol{M}$	15	15	3×5=15	15	3×5=15	15	1×5=5
$\boldsymbol{\frac{p_1}{p_2}}$	$\frac{3}{5} = 0.6$	$\frac{3}{3} = 1$	$\frac{3}{3} = 1$	$\frac{3}{1} = 3$	$\frac{3}{1} = 3$	$\frac{1}{5} = 0.2$	$\frac{1}{5} = 0.2$
Only purchase	x_1 $(2 > 0.6)$	x_1 $(2 > 1)$	x_1 $(2 > 1)$	x_2 $(2 < 3)$	x_2 $(2 < 3)$	x_1 $(2 > 0.2)$	x_1 $(2 > 0.2)$
Optimal Bundle $\boldsymbol{(x_1^*, x_2^*)}$	$(\frac{15}{3}, 0) = (5, 0)$	$(\frac{15}{3}, 0) = (5, 0)$	$(\frac{15}{3}, 0) = (5, 0)$	$(0, \frac{15}{1}) = (0, 15)$	$(0, \frac{15}{1}) = (0, 15)$	$(\frac{15}{1}, 0) = (15, 0)$	$(\frac{5}{1}, 0) = (5, 0)$
SE	–	None		All of TE		None	
IE	–	None		None		All of TE	