Intermediate Micro: Midterm 2 Review

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<u>Note:</u> **There are many worked examples in my notes for Recitations 7-10. Make sure you also work through all of those too.**

1 Exercises

Q1: Fuel Tax

Jaime has Cobb-Douglas utility and always spends 60% of income on food (f) and the remainder on gas (g). Denote p_f and p_g as the prices of food and gas, respectively, and M as income.

1. Suppose $p_f = 3$, $p_g = 2$, and $M = 100$. How much food and gas does Jaime consume?
2. Does Jamie consider food a normal good or an inferior good? If you answer normal, is it a luxury or a necessity?
3. True or False: Jamie has homothetic preferences.
4. True or False: Jamie's price elasticity of demand for gas is unit elastic.
5. To combat climate change, the government has decided to impose a unit tax of $2 on gas. Is the change in Jaime's demand for gas driven mostly by the substitution effect?
6. Draw Jamie's p_f price offer curve before and after the fuel tax.
7. How much tax revenue does the government raise from taxing Jamie? If they simply took this amount money from Jamie instead of imposing a tax, would Jamie be better or worse off?
8. How much would the government have to pay Jaime to fully compensate for the fuel tax?
9. Instead of the fuel tax, the government is considering levying an income tax of $10. How much is Jamie willing to pay to have the income tax instead of the fuel tax?

Q2: Peanut Butter and Jelly

Billie loves PB&J sandwiches, but they must have an exact ratio of 2:1 peanut butter (b) to jelly (j). Anything beyond that ratio brings Billie no extra utility (but also no disutility, as Billie will just scrape off any extra spread).

1. Derive Billie's demand functions for peanut butter and jelly
2. Find the equation of the line that represents Billie's p_b price offer curve, p_j price offer curve, and income expansion path.
3. True or False: Billie's own-price elasticity for b is as elastic as the cross-price elasticity for b
4. True or False: Billie's own-price elasticity for b is as elastic as the cross-price elasticity for j
5. Let's consider Billie's Engel curves
   (a) True or False: Billie's Engel curves for b and for j have the same slope.
   (b) Plot the Engel curves when $p_b = 6$, $p_j = 4$. Label the point where $M = 40$.
6. Suppose we start off with $p_b = 6$, $p_j = 4$, and $M = 40$. And then the price of peanut butter drops by half to $p_b = 3$. Decompose the price change described into the income and substitution effect. Draw this in a diagram and clearly label the changes due to each effect.
7. What is the compensating variation for the price change described in (5)? Draw a diagram showing the CV
8. Now imagine that we start at $p_b = 2$, $p_b = 4$, and $M = 40$. Then p_j changes such that the EV is \$20. Can you work out how much the price of jelly changed?

Q3: Labor-Leisure

Lee has 24 hours each day to spend on working (denoted by L hours of labor) and relaxing (denoted by ℓ hours of leisure). Lee earns a wage of w dollars per hour and spends it on a variety of goods (this total consumption expenditure is denoted by c). Lee's utility function is $u(c, \ell) = c\ell$. We can express Lee's budget constraint as $c = wL$.

1. Express Lee's labor (L) as a function of leisure (ℓ). Using this, express Lee's budget constraint as a function of c and ℓ. What is the price of leisure? How do you interpret this?
2. Using the utility function and the budget constraint from (1), find Lee's demand for leisure ℓ and consumption c.
3. Suppose that wage is set at $w = 5$.
   (a) Determine Lee's, choice of leisure and consumption. Draw this in a diagram with a budget constraint and indifference curve.

(b) Suppose that Lee's wage increases to $w = 10$. How does Lee's choice of leisure and consumption change? Does Lee work more or less after the wage increase?

4. Decompose the change described in (3) into the substitution and income effect. Draw a diagram representing this decomposition

5. Consider the following change instead (i.e. keep $w = 5$). Suppose that all prices in the economy, except wages, fall by half. Write the new budget constraint and solve for the new optimal bundle.

6. Is there anything odd about the income effect you found in (4)? How can you explain it? (hint: look at your answer from (5)).

7. Lee's friend Dee works the same job. However, when the wage increases from \$5 to \$10, Dee increases amount of hours worked. Does Dee consider leisure a Giffen good?

8. Lee's boss wants to get Lee working 16 hours per day. Raising the wage to \$10 did not work. So, the boss has decided to give Lee a fixed amount b on top of the regular wage (of $w = 10$). Write Lee's now budget constraint and demand functions. How much should b be set at to get Lee working 16 hours per day?

9. Is Lee worse off than before the wage changes? (i.e. compare Lee's situation in (8) to the situation in (3)). What w and b should the boss set to get Lee working 16 hours per week while being as well off as before the wage changes (i.e. as well off as in (3))?

Q4: Hicksian Demand

In this exercise, we will see the usefulness for the Hicksian demand. Note that this question involves taking integrals. However, don't get too bogged down in the math - the key lesson here is understanding the intuition behind the Hicksian demand and how it relates to EV/CV.

Consider a consumer with the utility function $u(x_1, x_2) = x_1 x_2$. The (Marshallian) demand functions are:

$$x_1(p, M) = \frac{M}{2p_1} \qquad\qquad x_2(p, M) = \frac{M}{2p_2}$$

The Hicksian demand functions are: (you are not expected to know how to derive these)

$$h_1(p, u) = \sqrt{\frac{p_2 u}{p_1}} \qquad\qquad h_2(p, u) = \sqrt{\frac{p_1 u}{p_2}}$$

Where u represents a utility level and h_i is quantity of good i.

1. Suppose we start at the parameter values $p_1 = 1, p_2 = 1, M = 8$. Then p_1 increases to 4. Calculate the old and new bundles, as well as the old and new utility. Let's call the old utility u_0 and the new utility u_1. (Note: "old" means before the price increase and "new" means after the price increase)

2. Calculate the EV and CV of the price change.

3. Plot the Marshallian demand curve for good 1 $x_1(p, M)$, with p_1 on the y-axis and x_1 on the x-axis. Shade the change in net consumer surplus due to the price increase.

4. Plot the Hicksian demand function for good 1 $h_1(p, u)$, with p_1 on the y-axis and x_1 on the x-axis. Do this twice (on the same axis): once for demand curve at the old utility u_0 and once for the demand curve at the new utility u_1. Indicate where the old and bundles are on the demand curves.

5. Compare your pictures in (3) and (4). When p_1 increases to 4, what happens on the Marshallian demand curve? What about for the Hicksian demand curve? Can you explain the difference?

6. Find the point on the Hicksian demand curve for old utility $h_1(p, u_0)$ where $p_1 = 2$.

   (a) What is the quantity of x_1 at this point? This point represents the quantity of x_1 needed at a price of $p_1 = 2$ to get a utility of u_0.

   (b) Shade the change in net "consumer surplus" on the Hicksian demand curve $h_1(p, u_0)$. How would you interpret this area? (hint: is it EV or CV?)

7. Use integration to calculate the area that you shaded in (6b).

8. Repeat (6) and (7) for the Hicksian demand curve for new utility $h_1(p, u_1)$.

Q5: Eating Healthy

Doris allocates a part of her monthly budget to purchase healthy foods from a supermarket near her college. She chooses between avocados (a) and broccoli (b). She'll only buy broccoli if it's less than half the price of avocados, otherwise she'll only purchase avocados (at exactly half, she's indifferent between all combinations).

1. Give a utility function that would explain Doris' behavior. Derive the demand functions.

2. Suppose Doris allocates \$30 for healthy food, and the prices are $p_a = 5$, $p_b = 3$. At these parameter values, plot the avocados demand curve with p_a on the y-axis. In a different axis, plot it with p_b on the y-axis. Indicate a point on each of the demand curves representing the current price and quantity demanded.

3. True or False: Doris' income expansion path will always be a vertical or horizontal line

4. Doris' parents would like her to eat a mix of vegetables. They give her the option of two monthly care packages. Option 1 is a box with 8 broccolis. Option 2 is a box with 1 avocado and 3 broccolis.

   (a) Draw how each of these care packages would affect her budget line

   (b) Which option would Doris choose?

5. Doris decides to forgo both care packages. Instead, she's heard of a weekend farmers market which sells broccoli for a dollar each (avocados are still \$5 there too). However, her parents would like her to stay on campus and not travel too far away.

   (a) What is minimum amount of income her parents could give her so that she prefers to shop at her local supermarket?

   (b) What is the maximum Doris is willing to pay to travel to the farmers market?

Q6: Market Demand

In the town of Townsville, there are two types of consumers who purchase two goods x and y. Type A consumers have utility $u(x, y) = x - \frac{1}{40}x^2 + \frac{1}{5}y$. Type B consumers have utility $u(x, y) = 6x - \frac{1}{10}x^2 - y$. There are 100 type A consumers and 50 type B consumers.

1. Determine the demand for good x for each type of consumer
2. Fix $p_y = 1$. For simplicity, let's re-write the demands into p-q notation, where $p = p_x$ and q_A = quantity demanded of x by a Type A consumers and q_B = quantity demanded of x by a Type B consumers. Write the total market demand Q.
3. Plot the market demand function that you derived in (2)
4. Suppose $p = 4$. Calculate the gross and net consumer surplus.
5. Suppose the number of Type A consumers drops to 50. Derive and draw the new market demand curve if that were to happen.
6. In another town, Samesburg, the market demand for x is: $Q = 4000 - 500p$. All 100 residents of Samesburg have the exact same preferences, that are similar in form to the Townsville residents. If the price of x decreases from $p = 4$ to $p = 2$, how much does the average Samesburg resident change their quantity demanded? Decompose this into the change due to the substitution and income effects.
7. True or False: Townsville is always more price sensitive than Samesburg.

2 Solutions

Q1: Fuel Tax

1. Since this is Cobb-Douglas preferences with a 60/40 split (i.e. $\frac{3}{5}/\frac{2}{5}$ split), we know that the demand functions are:

$$f(p, M) = \frac{3M}{5p_f}$$
$$g(p, M) = \frac{2M}{5p_g}$$

Using the given parameter values, we can calculate Jamie's chosen bundle:

$$f(3, 2, 100) = \frac{3 \times 100}{5 \times 3} = \frac{100}{5} = 20$$
$$g(3, 2, 100) = \frac{2 \times 100}{5 \times 2} = \frac{100}{5} = 20$$

2. For Cobb-Douglas, both goods are always considered normal goods, but they are neither luxury nor necessity. This is because while a higher income leads to a higher quantity demanded, the share spent on each good stays the same at all income levels.

3. True. Cobb-Douglas preferences are an example of homothetic preferences.

4. True. This is a property of Cobb-Douglas preferences. To see this:

$$\begin{aligned}\varepsilon &= \frac{dQ}{dP} \times \frac{P}{Q} \\ &= \frac{\partial g(p, M)}{\partial p_f} \times \frac{p_g}{g} \\ &= \left(-\frac{3M}{5p_g^2}\right) \times \frac{p_g}{g} \\ &= -\frac{3M}{5p_g g}\end{aligned}$$

We need to get rid of the g and have this only as a function of p_g. But note that $p_g g = 0.6M$ (i.e. Jamie always spends 60% of income on gas). Plugging this in:

$$\varepsilon = -\frac{0.6M}{0.6M} = -1$$

So it is indeed always unit elastic

5. With the tax, the gas price becomes $p_g = 4$. This obviously has no impact on the demand of food, but the new quantity of gas demanded is:

$$g(3, 4, 100) = \frac{2 \times 100}{5 \times 4} = \frac{200}{20} = 10$$

So we see that the total change is -10. To decompose this, we first need to find the income level that makes the old bundle affordable at the new prices:

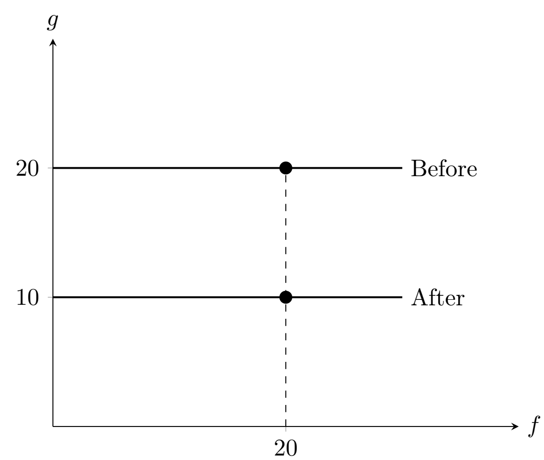
$$\begin{aligned} M^s &= 3 \times 20 + 4 \times 20 \\ &= 60 + 80 \\ &= 140 \end{aligned}$$

At the new prices and this income level, the demand for gas would be:

$$g(3, 4, 140) = \frac{2 \times 140}{5 \times 4} = \frac{280}{20} = 14$$

Therefore, the substitution effect causes a change in demand of $14 - 20 = -6$, while the income effect is $10 - 14 = -4$. Therefore, most of the effect is coming from the substitution effect.

6. For the p_f price offer curve, note that g doesn't depend on p_f. Before the fuel tax, we always have $g = 20$. After the fuel tax, we always have $g = 10$. Therefore, the price offer curve should just be horizontal lines at the optimal g value:



7. Since Jamie purchases 10 units of gas and the tax is \$2 per unit, the government gets $10 \times 2 = 20$ dollars of tax revenue from Jamie. If they just took away \$20, Jamie's income be 80. At this income level and without a tax (i.e. at the old prices), Jamie's demand would be:

$$f(3, 2, 80) = \frac{3 \times 80}{5 \times 3} = \frac{80}{5} = 16$$
$$g(3, 2, 80) = \frac{2 \times 80}{5 \times 2} = \frac{80}{5} = 16$$

Now, we need to compare utilities. Any utility function that gives the demand functions derived in (1) would work. For example, you could use $u(f, g) = f^{0.6}g^{0.4}$. Or you could use $u(f, g) = f^3g^2$. Or any monotonic transformation of these - as long as you stick to only using the same one! The point is to see the sign of the change (i.e. positive or negative) not the magnitude. Let's take the first utility function and compare Jamie's utility under the fuel tax and the income tax. We could assume that utility is higher under the fuel tax and see if we get a contradiction

$$\begin{aligned}
\text{Utility under Fuel Tax} &> \text{Utility under Income Tax} \\
u(20, 10) &> u(16, 16) \\
20^{0.6}10^{0.4} &> 16^{0.6}16^{0.4} \\
2^{0.6}10^{0.6}10^{0.4} &> 16^{0.6+0.4} \\
2^{0.6}10 &> 16 \\
2^{0.6} &> 1.6 \\
\approx 1.516 &\not> 1.6
\end{aligned}$$

With some simplification, we can see that the left hand side is <u>not</u> larger than the right hand side, i.e. utility under fuel tax is actually smaller than utility under the income tax. Therefore, the government should just take the 20 away from Jamie; this gets the same revenue for the government while hurting Jamie less.

8. This is the CV of the price change. We need to find the income that gets Jamie back to the old utility level. The utility before the change is:

$$u(20, 20) = 20^{0.6}20^{0.4} = 20$$

The demand for the goods at the new post-tax prices with an income M^c (that we will want to solve for) is:

$$\begin{aligned}
f &= \frac{3M^c}{5 \times 3} = \frac{M^c}{5} \\
g &= \frac{2M^c}{5 \times 4} = \frac{M^c}{10}
\end{aligned}$$

To make the utilities equal we would need:

$$\begin{aligned}
u\left(\frac{M^c}{5}, \frac{M^c}{10}\right) &= u(20, 20) \\
\left(\frac{M^c}{5}\right)^{0.6}\left(\frac{M^c}{10}\right)^{0.4} &= 20 \\
\frac{M^c}{5}\left(\frac{1}{2}\right)^{0.4} &= 20 \\
M^c &= 100(2^{0.4}) \\
M^c &\approx 132
\end{aligned}$$

So to fully compensate Jaime, the government would have to give $132 - 100 = 32$.

9. Under an income tax of \$10, Jamie's demand would be:

$$\begin{aligned}
f(3, 2, 90) &= \frac{3 \times 90}{5 \times 3} = \frac{90}{5} = 18 \\
g(3, 2, 90) &= \frac{2 \times 90}{5 \times 2} = \frac{90}{5} = 18
\end{aligned}$$

Now, suppose that on top of this income tax, Jamie is willing to pay some amount W. Then Jamie's demand would be:

$$f(3,2,90-W) = \frac{3\times(90-W)}{5\times 3} = \frac{90-W}{5} = 18 - \frac{W}{5}$$
$$g(3,2,90-W) = \frac{2\times(90-W)}{5\times 2} = \frac{90-W}{5} = 18 - \frac{W}{5}$$

So, the question is how much would W have to be so that Jamie's utility under the income tax (with an extra payment of W) is the same as the utility under the fuel tax (without the payment W). But this is exactly the equivalent variation! Let $M^e = 90 - W$ be the income we want to solve for. The EV just means that we want the following:

$$\begin{aligned}
u\left(\frac{M^e}{5}, \frac{M^e}{5}\right) &= u\,(20, 10) \\
\left(\frac{M^e}{5}\right)^{0.6}\left(\frac{M^e}{5}\right)^{0.4} &= 20^{0.6}10^{0.4} \\
\frac{M^e}{5} &= 2^{0.6}10 \\
M^e &= 2^{0.6}50 \\
&\approx 75.8
\end{aligned}$$

Therefore, the EV is $90 - 75.8 = -14.2$. In other words, the maximum that Jamie is willing to pay is $W = 14.2$.

Q2: Peanut Butter and Jelly

1. First, let's think about what the utility function would look like. The story suggests perfect complements and since the ratio is 2:1, we have $\alpha = 2$ and $\beta = 1$. In this case, two possible utility representations would be:

$$\begin{aligned}
u(b,j) &= \min\left\{\frac{1}{\alpha}b, \frac{1}{\beta}j\right\} = \min\left\{\frac{b}{2}, j\right\} \\
\underline{\text{or}}\ u(b,j) &= \min\{\beta b, \alpha j\} = \min\{b, 2j\}
\end{aligned}$$

Let's work with the second utility function. To derive the demand functions, we want to find the line where all the kinks occur:

$$b = 2j$$

We plug this into the budget constraint:

$$\begin{aligned}
p_b b + p_j j &= M \\
p_b(2j) + p_j j &= M \\
(2p_b + p_j)\, j &= M \\
j &= \frac{M}{2p_b + p_j} \\
\implies b &= \frac{2M}{2p_b + p_j}
\end{aligned}$$

2. The offer curves all represent the locations of the optimal bundle as we change the stated parameter. However, for perfect substitutes, all the optimal bundles must occur at the kink of an indifference curve. So all the offer curves can just simply be represent by the "line of kinks", which in this case is: $b = 2j$

3. False. First, let's calculate the own-price elasticity:

$$\begin{aligned}
\varepsilon^b_{p_b} &= \frac{dQ_b}{dP_b} \times \frac{P_b}{Q_b} \\
&= \frac{\partial b(p, M)}{\partial p_b} \times \frac{p_b}{b(p, M)} \\
&= \left(-\frac{2M}{(2p_b + p_j)^2} \times 2 \right) \times \frac{p_b}{\left(\frac{2M}{2p_b + p_j}\right)} \\
&= -\frac{4M}{(2p_b + p_j)^2} \times \frac{p_b\,(2p_b + p_j)}{2M} \\
&= -\frac{2p_b}{2p_b + p_j}
\end{aligned}$$

Note that in the third line, I am using the chain rule. Next, let's do the cross-price elasticity:

$$\begin{aligned}
\varepsilon^b_{p_j} &= \frac{dQ_b}{dP_j} \times \frac{P_j}{Q_b} \\
&= \frac{\partial b(p, M)}{\partial p_j} \times \frac{p_j}{b(p, M)} \\
&= \left(-\frac{2M}{(2p_b + p_j)^2} \times 1 \right) \times \frac{p_j}{\left(\frac{2M}{2p_b + p_j}\right)} \\
&= -\frac{2M}{(2p_b + p_j)^2} \times \frac{p_j\,(2p_b + p_j)}{2M} \\
&= -\frac{p_j}{2p_b + p_j}
\end{aligned}$$

So, for the elasticities to be equal we would need:

$$\begin{aligned}
\varepsilon^b_{p_b} &= \varepsilon^b_{p_j} \\
-\frac{2p_b}{2p_b + p_j} &= -\frac{p_j}{2p_b + p_j} \\
2p_b &= p_j
\end{aligned}$$

So it depends on the prices. If this is not true, then the elasticities will be different.

4. True. Following the same steps as above, let's do the cross-price elasticity of j:

$$\begin{aligned}
\varepsilon^j_{p_b} &= \frac{dQ_j}{dP_b} \times \frac{P_b}{Q_j} \\
&= \frac{\partial j(p, M)}{\partial p_b} \times \frac{p_b}{j(p, M)} \\
&= \left(-\frac{M}{(2p_b + p_j)^2} \times 2 \right) \times \frac{p_b}{\left(\frac{M}{2p_b+p_j}\right)} \\
&= -\frac{2M}{(2p_b + p_j)^2} \times \frac{p_b\,(2p_b + p_j)}{M} \\
&= -\frac{2p_b}{2p_b + p_j}
\end{aligned}$$

Indeed, we find that $\varepsilon^b_{p_b} = \varepsilon^j_{p_b}$.

5.

(a) False. Starting with the demand functions, let's solve for M to get the Engel curves

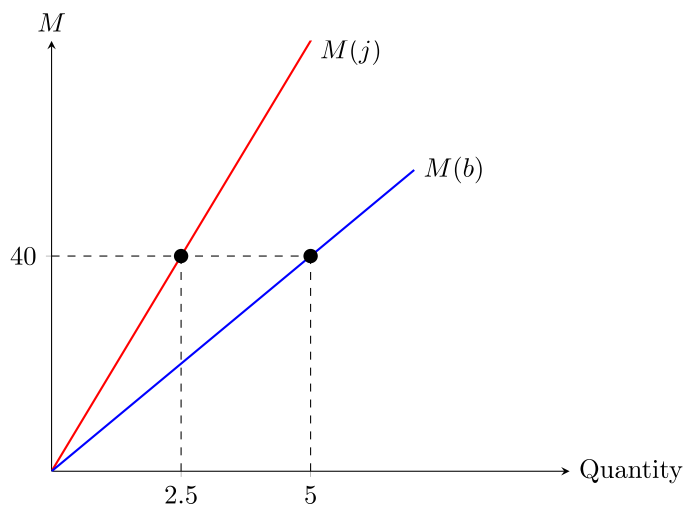
$$\begin{aligned}
j(M; p_b, p_j) &= \frac{M}{2p_b + p_j} \qquad &\Longrightarrow\; M(j; p_b, p_j) = (2p_b + p_j)\,j \\
b(M; p_b, p_j) &= \frac{2M}{2p_b + p_j} \qquad &\Longrightarrow\; M(b; p_b, p_j) = \frac{1}{2}\,(2p_b + p_j)\,j
\end{aligned}$$

As we can see, the slopes are not the same (the Engel curve for b is flatter).

(b) At $p_b = 6$ and $p_j = 4$, the Engel curves become:

$$\begin{aligned}
M(j) &= M(j; 6, 4) = 16j \\
M(b) &= M(b; 6, 4) = 8j
\end{aligned}$$

Note that at $M = 40$, we have $j = \frac{40}{16} = 2.5$ and $b = \frac{40}{8} = 5$. This gives us the following graph:



6. Let's use the demand functions to calculate the change in optimal bundle:

Before:

$$b(6,4,40) = \frac{2 \times 40}{2 \times 6 + 4} = \frac{80}{16} = 5$$
$$j(6,4,40) = \frac{40}{2 \times 6 + 4} = \frac{40}{16} = 2.5$$

After :

$$b(3,4,40) = \frac{2 \times 40}{2 \times 3 + 4} = \frac{80}{10} = 8$$
$$j(3,4,40) = \frac{40}{2 \times 3 + 4} = \frac{40}{10} = 4$$

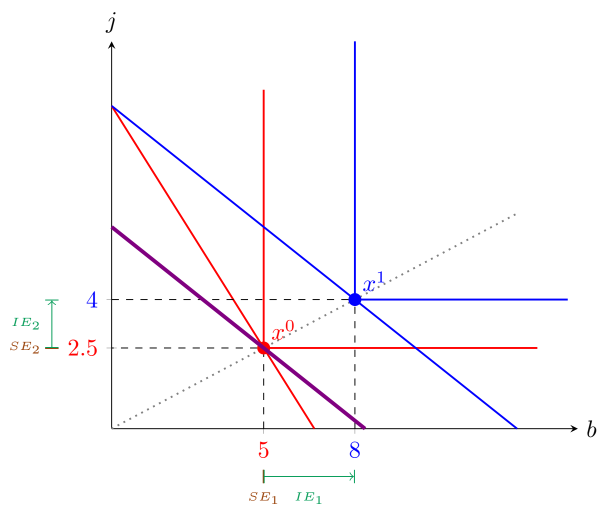
From this, we can see the total effect: $TE_1 = 8 - 5 = 3$ and $TE_2 = 4 - 2.5 = 1.5$. Since this perfect complements, we should already anticipate that all of this is driven by the income effect. But to be completely thorough, let's go through all the steps.
First, we want to find the income that makes the old bundle affordable at the new prices:

$$\begin{aligned} M^s &= 3 \times 5 + 4 \times 2.5 \\ &= 15 + 10 \\ &= 25 \end{aligned}$$

At this income and the new prices, the demand would be:

$$b(3,4,25) = \frac{2 \times 25}{2 \times 3 + 4} = \frac{50}{10} = 5$$
$$j(3,4,25) = \frac{25}{2 \times 3 + 4} = \frac{25}{10} = 2.5$$

Unsurprisingly, this shows us that $SE_1 = 5 - 5 = 0$ and $SE_2 = 2.5 - 2.5 = 0$. So, exactly as we expected, everything happens through the income effect. The diagram looks as follows:



7. For the CV, we want to find the income where we can achieve the old utility at the new prices. Let's call this income M^c, and the demands here will be:

$$b(3,4,M^c) = \frac{2 \times M^c}{2 \times 3 + 4} = \frac{2M^c}{10}$$
$$j(3,4,M^c) = \frac{M^c}{2 \times 3 + 4} = \frac{M^c}{10}$$

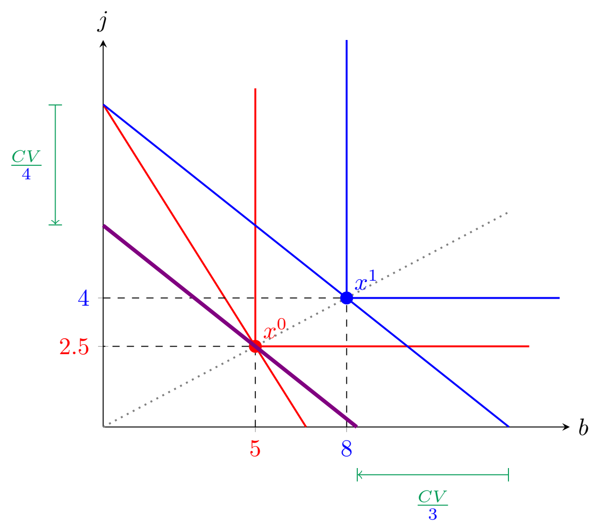
A trick with perfect complements is to use the fact that an optimal bundle only occurs at the kink. This means that to achieve the old utility, we have to be at the old bundle. This is different to other utility functions where we just have to be tangent to the old indifference curve. With perfect complements, the “tangency” can only occur at one place, regardless of the price ratio. Since the old bundle is $(b, j) = (5, 2.5)$, we want to find the M^c such that:

$$\left(\frac{2M^c}{10}, \frac{M^c}{10}\right) = (5, 2.5)$$

You can take either good and will get the same answer. For example, taking j, we get the equation:

$$\frac{M^c}{10} = 2.5$$
$$\therefore M^c = 25$$

So the CV is $25 - 40 = -15$. Taking away \$15 would fully undo the effects of the price decrease. The diagram looks as follows:



Notice that this is exactly the same diagram as before! This is not a coincidence. When we do the Slutsky decomposition, we ensure that the Slutsky budget line goes through the kink in the old IC (since we want that point to be affordable) but with a slope of the new price ratio. When we do the CV calculation, we also make sure that the CV budget line goes through kink in the old IC (since we want to be tangent to that IC) but with a slope of the new price ratio. So we get exactly the same line!

8. At these starting parameter values, we have the following demands:

$$b(2, 4, 40) = \frac{2 \times 40}{2 \times 2 + 4} = \frac{80}{8} = 10$$
$$j(2, 4, 40) = \frac{40}{2 \times 2 + 4} = \frac{40}{8} = 10$$

Recall that for equivalent variation, we have an income change while staying at the old prices. So the demand in this "EV world" would be:

$$
\begin{aligned}
b(2,4,40+EV) = b(2,4,60) =& \frac{2\times 60}{2\times 2+4} = \frac{120}{8} = 15\\
j(2,4,40+EV) = b(2,4,60) =& \frac{60}{2\times 2+4} = \frac{60}{8} = 7.5
\end{aligned}
$$

We know that this bundle $(b, j) = (15, 7.5)$ has to be the same bundle chosen as after the price change of p_j. In other words, the demand after the price change is:

$$
\begin{aligned}
b(2,p_j,40) =& \frac{2\times 40}{2\times 2+p_j} = \frac{80}{4+p_j}\\
j(2,p_j,40) =& \frac{40}{2\times 2+p_j} = \frac{40}{4+p_j}
\end{aligned}
$$

And it must be the case that:

$$\left(\frac{80}{4+p_j}, \frac{40}{4+p_j}\right) = (15, 7.5)$$

Again, we can take either one. Let's take b:

$$
\begin{aligned}
\frac{80}{4+p_j} &= 15\\
4+p_j &= \frac{80}{15}\\
p_j &= \frac{5}{3} - 4\\
\therefore p_j &= \frac{1}{3}
\end{aligned}
$$

So the price of jelly dropping from 4 to $\frac{1}{3}$ is what gives an EV of 20.

Q3: Labor-Leisure

1. In 24 hours, Lee can either work or relax. Therefore his 'time budget constraint' is: $\ell + L = 24$. Re-arranging this gives us $L = 24 - \ell$. Plugging this into the budget constraint gives us:

$$
\begin{aligned}
c &= w(24-\ell)\\
w\ell + c &= 24w
\end{aligned}
$$

The coefficient on ℓ is w - this represents the price of leisure. This represents the opportunity cost of leisure: one hour of relaxing means one hour of not working. This means an hour's worth of wages (i.e. w dollars) is foregone when Lee chooses to take an hour of leisure.

2. The utility maximization problem is:

$$
\begin{aligned}
&\max_{\ell,c} c\ell\\
&\text{s.t. } w\ell + c = 24w
\end{aligned}
$$

This is just a Cobb-Douglas utility function, so we can use the usual formulas to get the demand functions

$$x_1 = \frac{\alpha}{\alpha+\beta} \cdot \frac{M}{p_1} \qquad\qquad x_2 = \frac{\beta}{\alpha+\beta} \cdot \frac{M}{p_2}$$

Let $x_1 = \ell$, $x_2 = c$, $p_1 = w$, $p_2 = 1$, $M = 24w$. From the utility function, we can see $\alpha = \beta = 1$. This means the demands are:

$$\ell = \frac{1}{2} \cdot \frac{24w}{w} = 12$$
$$c = \frac{1}{2} \cdot \frac{24w}{1} = 12w$$

3.

(a) With $w = 5$, we get $\ell = 12$ and $c = 60$.

(b) With $w = 10$, we get $\ell = 12$ and $c = 120$. Lee does not change the amount of leisure (and hence labor).

4. To decompose this, we need to find the level of income where the old bundle is affordable at the new prices. Let's call this income M^s. This is given by:

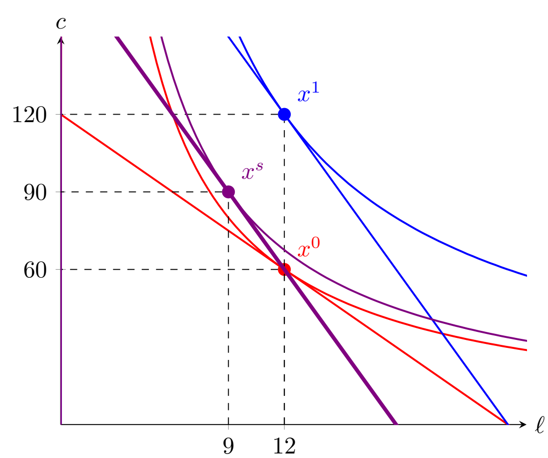
$$10 \cdot 12 + 1 \cdot 60 = M^s$$
$$120 + 60 = M^s$$
$$\therefore M^s = 180$$

The bundle at this level of income and new prices is given by:

$$\ell = \frac{1}{2} \cdot \frac{180}{10} = 9 \qquad\qquad c = \frac{1}{2} \cdot \frac{180}{1} = 90$$

This means that the substitution and income effect is given by:

$$SE_1 = 9 - 12 = -3 \qquad\qquad SE_2 = 90 - 60 = 30$$
$$IE_1 = 12 - 9 = 3 \qquad\qquad IE_2 = 120 - 90 = 30$$



5. Right now the price of consumption is 1. The question is essentially saying let $p_c = \frac{1}{2}$. The new budget constraint would be:

$$w\ell + \frac{1}{2}c = 24w$$

The demand for ℓ would still not change, since it is independent to the price of consumption. But the demand for consumption becomes:

$$c = \frac{1}{2} \cdot \frac{24w}{\frac{1}{2}} = 24w = 120$$

6. What's odd is the prices have increased (since w increased), which usually means we have a *negative* income effect (the person feels overall poorer since the purchasing power of their income has fallen). But we found that when w increases, there is a *positive* income effect. How can we explain this? Notice that w is both the price of leisure, but it also determines Lee's income. So the fact that prices have increased is offset by the fact that income has also increased. In fact, it's more than offset.

   When w increased from 5 to 10, the parameters change from $(p_\ell, p_c, M) = (5, 1, 120)$ to $(10, 1, 240)$. In (6), the parameter values are $(5, 0.5, 120)$ - and notice that under this, we get the same optimal bundle as when wage increased to 10. As we have learnt earlier, if we scale up all prices and income by the factor, that has no impact on the consumer's demand. So the wage increase is effectively the same as prices in the economy (except wages) falling. This fall in price is why we are getting a positive income effect.

7. For Lee, the income and substitution effect cancelled out, which is why the optimal level of ℓ did not change. For Dee, since L has increased, then ℓ must have decreased. For this to have happened, then the (positive) income effect must have been smaller in absolute value than the (negative) substitution effect.

   Dee does not consider leisure to be a Giffen good. You may be tempted to think so because wage (price of leisure) has increased but quantity of leisure demanded has decreased. But there a few issues with this thinking. Firstly, a Giffen good must be an inferior good, which leisure is not (it increases with income). Secondly, as explained before, when wage increases, there is also a change in income, so it's not a pure price change.

8. The new budget constraint would be:

$$w\ell + c = 24w + b$$

   This means that the demand functions would be:

$$\ell = \frac{1}{2} \cdot \frac{24w + b}{w} = 12 + \frac{b}{2w}$$
$$c = \frac{1}{2} \cdot \frac{24w + b}{1} = 12w + \frac{b}{2}$$

   At $w = 10$, this would be:

$$\ell = 12 + \frac{b}{20}$$
$$c = 120 + \frac{b}{2}$$

To get Lee working at 16 hours per week, we need leisure to be 8 hours per week. So to get $\ell = 8$, we can solve for b:

$$\begin{aligned} 8 &= 12 + \frac{b}{20} \\ -4 &= \frac{b}{20} \\ \therefore b &= -80 \end{aligned}$$

So if the boss takes away \$80 from Lee, then Lee will actually work more. This means Lee's consumption would be: $c = 120 - 40 = 80$.

9. Lee is worse off:

$$\begin{aligned} u(12, 60) &= 12 \times 60 = 720 \\ u(8, 80) &= 8 \times 80 = 640 \end{aligned}$$

Since we want Lee as well off as before, we need $u = 720$. And since we want $\ell = 8$, then that means:

$$\begin{aligned} c\ell &= 720 \\ c &= \frac{720}{8} = 90 \end{aligned}$$

Therefore, we need the optimal bundle to be $(8, 90)$. At this bundle, Lee has an MRS of:

$$\frac{c}{\ell} = \frac{90}{8} = 11.25$$

Therefore, the price ratio has to be in this proportion too:

$$\begin{aligned} |MRS| &= \frac{p_\ell}{p_c} \\ 11.25 &= \frac{w}{1} \\ \therefore w &= 11.25 \end{aligned}$$

At a wage of 11.25, the demand functions become:

$$\begin{aligned} \ell &= 12 + \frac{b}{22.5} \\ c &= 135 + \frac{b}{2} \end{aligned}$$

So to get $\ell = 8$, we can solve for b:

$$\begin{aligned} 8 &= 12 + \frac{b}{22.5} \\ -4 &= \frac{b}{22.5} \\ \therefore b &= -90 \end{aligned}$$

To check, we can plug this into the demands to check (derived in (8)):

$$\ell = 12 - \frac{90}{2 \times 11.25} \qquad c = 24 \times 11.25 - \frac{90}{2}$$
$$= 12 - \frac{90}{22.5} \qquad = 135 - 45$$
$$= 12 - 4 = 8 \qquad = 90$$

So a wage of \$11.25 and a bonus of -\$90, gets Lee at the bundle $(8, 90)$, in which Lee works for 16 hours per day but achieves the same original level of utility.

Q4: Hicksian Demand

1. Calculate the optimal bundles and utility before and after the price change:

<u>Before</u>

$$x_1^0 = x_1(1,1,8) = \frac{8}{2 \times 1} = 4$$
$$x_2^0 = x_2(1,1,8) = \frac{8}{2 \times 1} = 4$$
$$u^0 = u(x_1^0, x_2^0) = 4 \times 4 = 16$$

<u>After</u>

$$x_1^1 = x_1(4,1,8) = \frac{8}{2 \times 4} = 1$$
$$x_2^1 = x_2(4,1,8) = \frac{8}{2 \times 1} = 4$$
$$u^1 = u(x_1^1, x_2^1) = 1 \times 4 = 4$$

2. To calculate the EV, we want to have the old prices at the new utility. The demand at the old prices for an income M^e (that we want to solve for) is:

$$x_1^e = x_1(1,1,M^e) = \frac{M^e}{2}$$
$$x_2^e = x_2(1,1,M^e) = \frac{M^e}{2}$$

Plugging this into the utility function and setting it equal to the new utility gives us:

$$u(x_1^e, x_2^e) = u(x_1^1, x_2^1)$$
$$\left(\frac{M^e}{2}\right)\left(\frac{M^e}{2}\right) = 4$$
$$\frac{(M^e)^2}{4} = 4$$
$$M^e = \sqrt{4 \times 4}$$
$$M^e = 4$$

So the EV is $M^e - M^0 = 4 - 8 = -4$.

To calculate the CV, we want to have the new prices at the old utility. The demand at the new prices for an income M^c (that we want to solve for) is:

$$x_1^c = x_1(4, 1, M^c) = \frac{M^c}{8}$$
$$x_2^c = x_2(4, 1, M^c) = \frac{M^c}{2}$$

Plugging this into the utility function and setting it equal to the new utility gives us:

$$\begin{aligned} u(x_1^c, x_2^c) &= u(x_1^1, x_2^1) \\ \left(\frac{M^c}{8}\right)\left(\frac{M^c}{2}\right) &= 16 \\ \frac{(M^c)^2}{16} &= 16 \\ M^c &= \sqrt{16 \times 16} \\ M^c &= 16 \end{aligned}$$

So the CV is $M^c - M^1 = 16 - 8 = 8$.

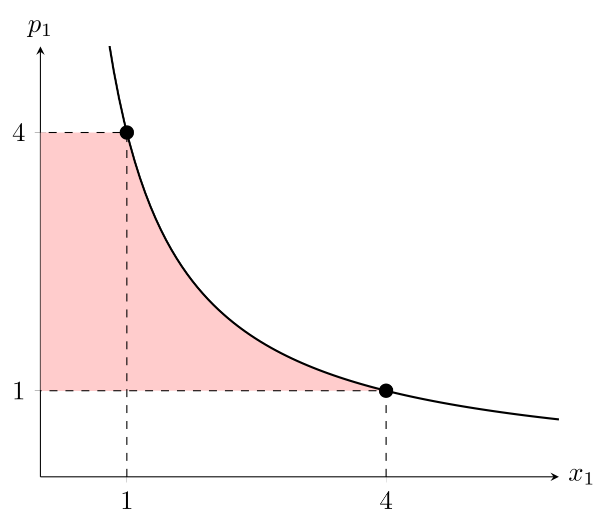
3. First, let's solve for the inverse demand function:

$$\begin{aligned} x_1 &= \frac{M}{2p_1} \\ \therefore p_1 &= \frac{M}{2x_1} \end{aligned}$$

At the current values (i.e. $M = 8$), this becomes:

$$p_1 = \frac{8}{2x_1} = \frac{4}{x_1}$$

Now we can plot the demand curve, where the red area is the decrease in net consumer surplus:



4. Just like the Marshallian demand, we start by solving for the inverse demand:

$$h_1 = \sqrt{\frac{p_2 u}{p_1}}$$
$$\sqrt{p_1} = \frac{\sqrt{p_2 u}}{h_1}$$
$$p_1 = \frac{p_2 u}{h_1^2}$$

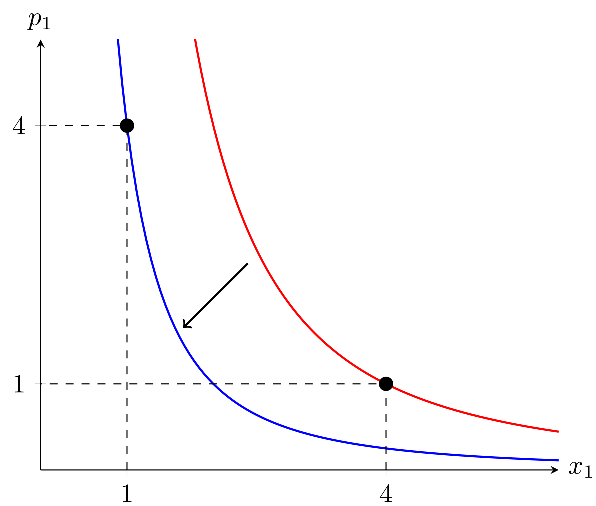
Before the price change, we have $p_2^0 = 1$ and $u^0 = 16$, which means the Hicksian demand under those parameter values is:

$$h_1 = \sqrt{\frac{16}{p_1}} = \frac{4}{\sqrt{p_1}} \qquad\qquad p_1 = \frac{16}{h_1^2}$$

After the price change, we have $p_2^1 = 1$ and $u^1 = 4$, which means the Hicksian demand under those parameter values is:

$$h_1 = \sqrt{\frac{4}{p_1}} = \frac{2}{\sqrt{p_1}} \qquad\qquad p_1 = \frac{4}{h_1^2}$$

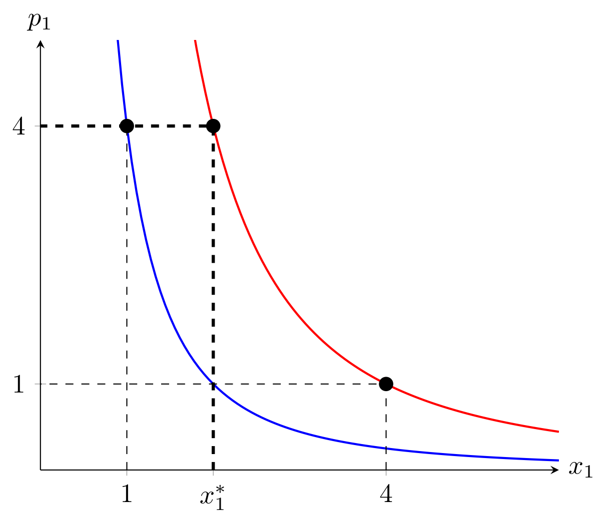
Notice that it is a completely different graph. In the Marshallian demand curve that we are used to, a change in income leads to a *shift* of the demand curve. Similarly, with the Hicksian demand, a change in utility leads to a *shift* of the demand curve. Let's plot this: (the old Hicksian demand curve $h_1(p, u_0)$ is in red and the new Hicksian demand curve $h_1(p, u_1)$ is in blue)



5. As we can see, when we have the change in p_1, we *move along* the Marshallian demand curve. However, for the Hicksian demand curve, not only do we move along the demand curve, but there is a *shift* in the demand curve too! Notice the start and end points are still the same regardless of which diagram we use. The difference is that the Hicksian demand is holding fixed the utility level (that's why u is a parameter in the Hicksian demand). So, when p_1 changes, the consumer re-optimizes and not only chooses a different x_1, they also end up with a different utility level. Therefore, we need to also shift the Hicksian demand curve to reflect this change too.

6. Now, we are going to be focusing on the red demand curve $h_1(p, u_0)$.

   (a) We want to find the point on the red demand curve where $p_1 = 4$. Let's draw the diagram to see what's going on:

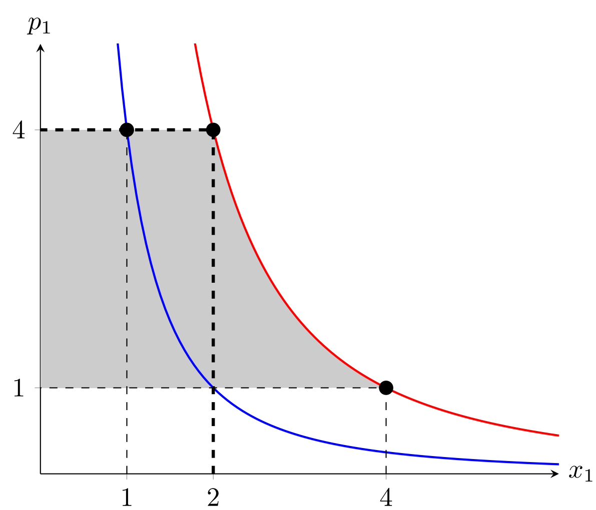


   To find this point x_1^*, we just need to plug in $p_1 = 4$ into the Hicksian demand function:

$$h_1 = \frac{4}{\sqrt{p_1}} = \frac{4}{\sqrt{4}} = 2$$

   Therefore $x_1^* = 2$

   (b) Let's shade the change in net "consumer surplus" in gray:

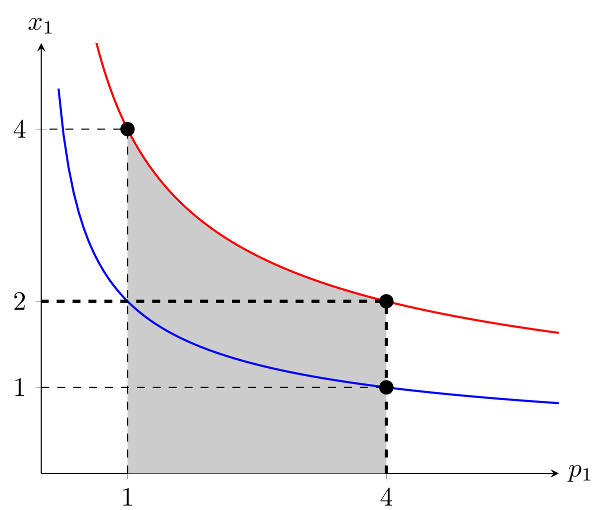


   I put consumer surplus in double-quotes because it isn't really consumer surplus, but it is quite analogous (let's call it CS2 for now). To help us understand it, let's compare this to actual consumer surplus:

- The CS is the area under the Marshallian demand curve.
  - The CS2 is area under the Hicksian demand curve
- In the Marshallian demand, we hold fixed prices and income (however, *utility* is allowed to vary along the curve)
  - In the Hicksian demand, we hold fixed prices and utility (however, *income* is allowed to vary along the curve)
- Therefore, CS represents a change in *utility* as we change price, and hold income fixed
  - Therefore, CS2 represents a change in *income* as we change price, and hold utility fixed

Since we are holding utility fixed at the old level (u_0), then this CS2 is exactly the compensating variation! It captures how much income we need to go from the old prices to the new prices while staying at the same utility (i.e. staying on the same Hicksian demand curve).

7. To use integration, it may be a little easier to flip the axis so that p_1 is on the x-axis and x_1 is on the y-axis: (but notice that this is still the same graph as before)



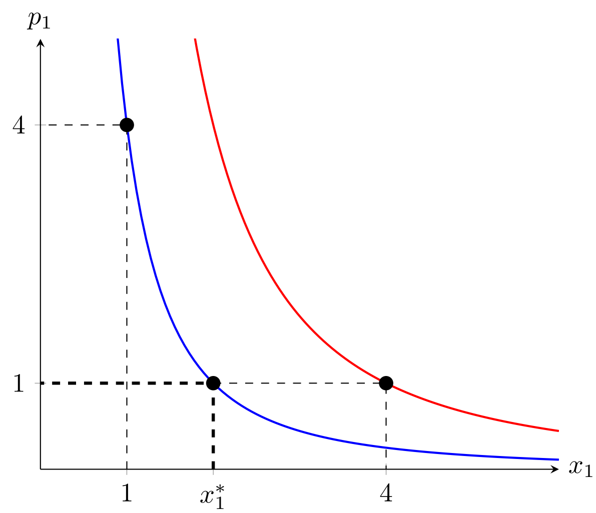
Now we can integrate as we usually do to find the area under graph. The function we are integrating is the Hicksian demand function $h_1(p, u_0)$ with respect to p_1. The limits are $p_1^0 = 1$ (the old price) to $p_1^1 = 4$ (the new price). This gives us:

$$
\begin{aligned}
CV &= \int_{p_1^0}^{p_1^1} \left(\sqrt{\frac{p_2^0 u^0}{p_1}} \right) dp \\
&= \int_1^4 \frac{4}{\sqrt{p}} dp \\
&= [2 \cdot 4\sqrt{p}]_1^4 \\
&= 8\sqrt{4} - 8\sqrt{1} \\
&= 16 - 8 \\
&= 8
\end{aligned}
$$

Notice that this is exactly what we found in (2)! So the CV is just the change in net "consumer surplus", using the old utility Hicksian demand.

8. Now let's repeat what we did using the other Hicksian demand curve $h_1(p, u_1)$, which I've plotted in blue.

   (a) We want to find the point on the blue demand curve where $p_1 = 1$. Let's draw the diagram to see what's going on:

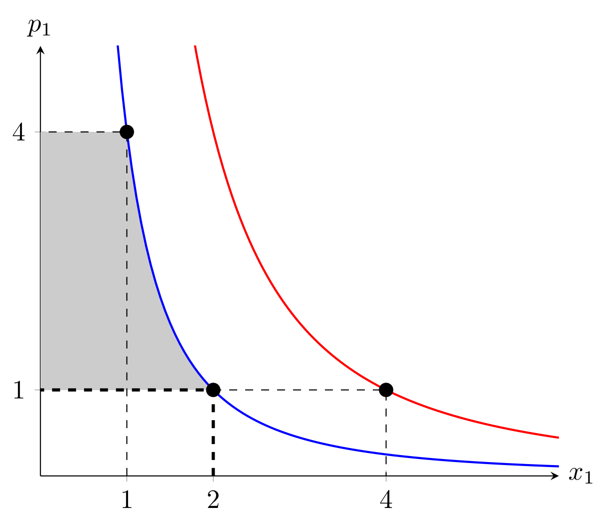


   To find this point x_1^*, we just need to plug in $p_1 = 1$ into the Hicksian demand function:

$$h_1 = \frac{2}{\sqrt{p_1}} = \frac{2}{\sqrt{1}} = 1$$

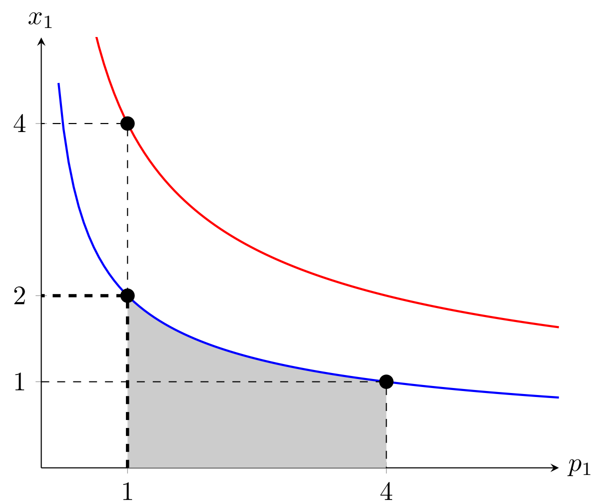
   Therefore $x_1^* = 2$

   (b) Let's shade the change in net "consumer surplus" in gray:



Since we are holding utility fixed at the new level (u_1), then this shaded area is exactly the equivalent variation! It captures how much income we need to go from the new price to the old prices while staying at the same utility (i.e. staying on the same Hicksian demand curve).

To use integration, let's flip the axis again:



Now we can integrate as we usually do to find the area under graph. The function we are integrating is the Hicksian demand function $h_1(p, u_1)$ with respect to p_1. The limits are $p_1^1 = 4$ (the new price) to $p_1^0 = 1$ (the old price). This gives us:

$$
\begin{aligned}
EV &= \int_{p_1^1}^{p_1^0} \left(\sqrt{\frac{p_2^1 u^1}{p_1}} \right) dp \\
&= \int_4^1 \frac{2}{\sqrt{p}} dp \\
&= -\left[2 \cdot 2\sqrt{p}\right]_1^4 \\
&= -\left(4\sqrt{4} - 4\sqrt{1}\right) \\
&= -(8-4) \\
&= -4
\end{aligned}
$$

And again, this is exactly what we got in (2). Note the negative sign comes in because we are moving from 4 to 1, so we make it a negative to capture the fact that we are moving backwards in the integration. But, this just shows us that EV is just the change in net "consumer surplus", using the new utility Hicksian demand.

Q5: Eating Healthy

1. This suggests perfect substitutes, i.e. $u(a, b) = \frac{1}{\alpha}a + \frac{1}{\beta}b$. So now we need to figure out the ratio $\alpha : \beta$ of how she substitutes between the two products. Note that the MRS here is $\frac{\beta}{\alpha}$. If $p_b < \frac{1}{2}p_a \implies 2p_b < p_a$,

then she only buys broccoli. Since she only buys broccoli when MRS is less than the price ratio, we can re-state it as:

$$\text{If } 2 < \frac{p_a}{p_b}, \text{ then: } \frac{\beta}{\alpha} < \frac{p_a}{p_b}$$

Similarly, if $p_b > \frac{1}{2}p_a \implies 2p_b > p_a$, then she only buys avocados (which must mean MRS is greater than the price ratio):

$$\text{If } 2 > \frac{p_a}{p_b}, \text{ then: } \frac{\beta}{\alpha} > \frac{p_a}{p_b}$$

Therefore, the only way for this to work is to have that $\frac{\beta}{\alpha} = 2$, which means $\alpha = 1$ and $\beta = 2$. In other words, 2 broccolis are perfect substitutes for 1 avocado. So possible utility functions you can write are:

$$u(a, b) = a + \frac{1}{2}b$$
$$u(a, b) = 2a + b$$

The demand functions are then easy to see:

$$a(p, M) = \begin{cases} 0 & \text{if } 2 < \frac{p_a}{p_b} \\ \frac{M}{p_a} & \text{if } 2 > \frac{p_a}{p_b} \\ \in \left[0, \frac{M}{p_a}\right] & \text{if } 2 = \frac{p_a}{p_b} \end{cases} \qquad b(p, M) = \begin{cases} \frac{M}{p_b} & \text{if } 2 < \frac{p_a}{p_b} \\ 0 & \text{if } 2 > \frac{p_a}{p_b} \\ \in \left[0, \frac{M}{p_b}\right] & \text{if } 2 = \frac{p_a}{p_b} \end{cases}$$

And we know that the edge-case where $2 = \frac{p_a}{p_b}$, the entire budget line is optimal.

2. At these parameter values, the avocados demand function becomes:

$$a(p_a; p_b, M) = a(p_a; 3, 30) = \begin{cases} 0 & \text{if } 2 < \frac{p_a}{3} \implies p_a > 6 \\ \frac{30}{p_a} & \text{if } 2 > \frac{p_a}{3} \implies p_a < 6 \\ \in [0, 5] & \text{if } 2 = \frac{p_a}{3} \implies p_a = 6 \end{cases}$$

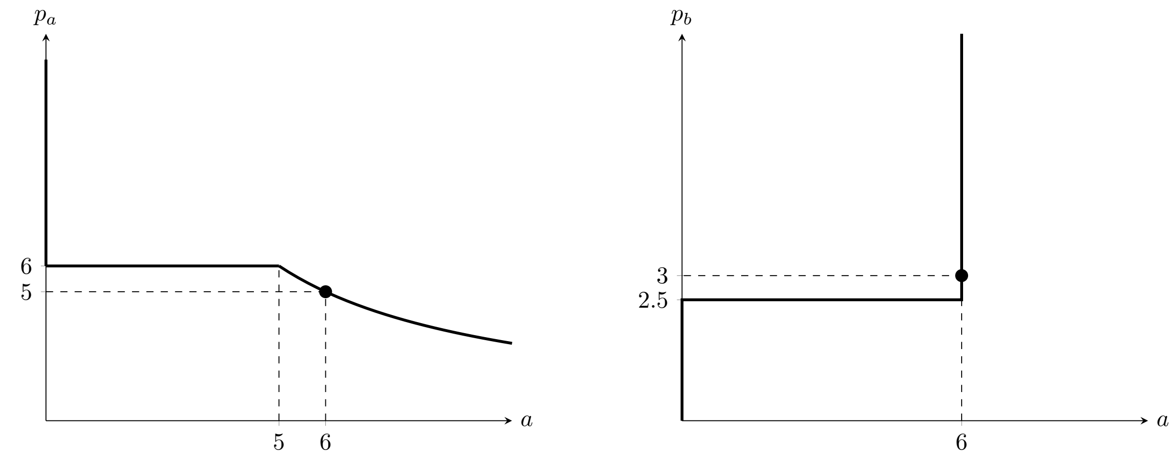
This gives us quantity of a as a function of p_a, which is what the first graph is asking for. But, as always, it's better to solve for the inverse demand function before plotting: (note, you can't really invert the first case since it's a vertical line)

$$p_a(a; p_b, M) = p_a(a; 3, 30) = \begin{cases} \frac{30}{x_a} & \text{if } a > 5 \\ 6 & \text{if } a \in [0, 5] \end{cases}$$

This is plotted below on the left. Now let's do the second graph. This requires $a$as a function of p_b:

$$a(p_b; p_a, M) = a(p_b; 5, 30) = \begin{cases} 0 & \text{if } 2 < \frac{5}{p_b} \implies p_b < 2.5 \\ 6 & \text{if } 2 > \frac{5}{p_b} \implies p_b > 2.5 \\ \in [0, 6] & \text{if } 2 = \frac{5}{p_b} \implies p_b = 2.5 \end{cases}$$

Let's plot this without inverting, since it is a little complicated and pretty unnecessary. This is in the figure below on the right.



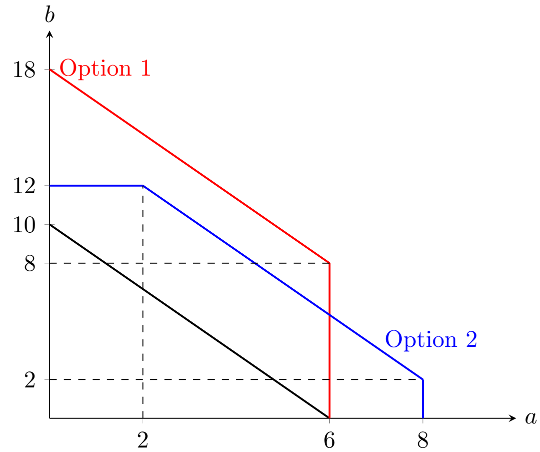
3. False. Changing income doesn't affect the optimal mix of goods. So if $2 > \frac{p_a}{p_b}$, then she will always choose avocados, and the income expansion path will be a horizontal line. Similarly, if $2 < \frac{p_a}{p_b}$, then she will only buy broccoli, and the income expansion path will be vertical. However, consider when $2 = \frac{p_a}{p_b}$, then the entire budget line is optimal. Which means that the income expansion path is in fact the entire space on the axis!

4. Note that without the care package, her budget line was $5a + 3b = 30 \implies b = 10 - \frac{5}{3}a$. Notice here that the maximum amount of avocado she could buy (the x-intercept) is 6 and the maximum amount of broccoli she could buy (the y-intercept) is 10.

   (a) For option 1, this just gives Doris an extra 8 broccolis at each and every level, so we just need to shift her budget constraint up by 8 at each and every level. However, the maximum number of avocados she can purchase is still 6, so we have to truncate the budget line at that point. In other words, if she spent all her money on avocados, she'd be at the bundle $(6, 8)$ since she gets 8 broccolis from her parents.

   For option 2, Doris gets an extra 2 a and 2 b at each and every level. This means that the maximum avocado she can buy is now 8 and the maximum broccoli is now 12. In particular, if she spends all her money on broccoli, she will be at the bundle $(2, 12)$, i.e. 2 avocados from her parents, 10 broccoli she purchases, plus 2 broccoli from her parents. Similarly, if she spends all her money on avocado, then she will be at the bundle $(8, 2)$. Anything in between those two points will involve her mixing her purchases (i.e. it will still be at a slope of $-5/3$).

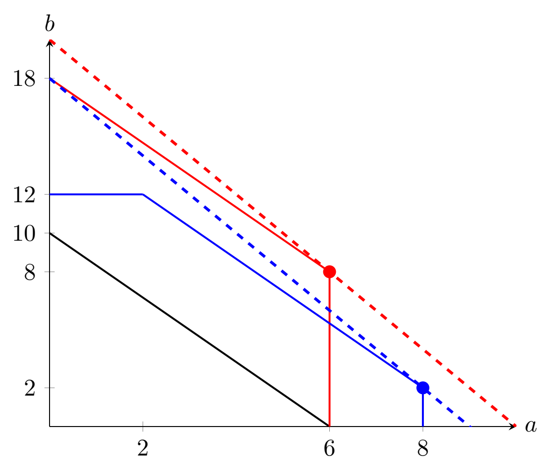
   Therefore, the diagram for this is shown below. In black is the original budget line. In red is the option 1 budget line and in blue is the option 2 budget line.



(b) Notice that the slope of the budget line really hasn't changed - it's just been shifted out and truncated. Which means that Doris will still want to mix in the same way she's always: she will still be choosing to spend all her money on avocados. So in option 1, her optimal bundle is $(6, 8)$. In option 2, her optimal bundle is $(8, 2)$. To see which one is better we just compare the utilities: (use whichever utility function you chose in question 1):

$$u(6, 8) = 2 \times 6 + 8 = 20$$
$$u(8, 2) = 2 \times 8 + 2 = 18$$

So, she would prefer option 1. You can also see this by drawing the diagram (the dashed lines are the indifference curves)



5. At the farmers market, the price ratio is $\frac{p_a}{p_b} = \frac{5}{1} = 5$, and since $2 < 5$, then she would only purchase broccoli there. In fact, she would be able to buy $\frac{30}{1} = 30$ units of broccoli, which would give her a utility of:

$$u(0, 30) = 2 \times 0 + 30 = 30$$

(a) This is just asking for the EV. At the farmers market, her optimal bundle would give her a utility of:

$$u(0, 30) = 2 \times 0 + 30 = 30$$

To find the EV, we need to find the income level M^e such that Doris achieves the same utility as in the farmers market but at the current prices she is facing. At the current prices, her demand is obviously $(a, b) = (\frac{M}{p_a}, 0)$, i.e. she only purchases avocados at this price ratio. Therefore, we can solve for M^e by doing the following:

$$\begin{aligned} u\left(\frac{M^e}{5}, 0\right) &= u(0, 30) \\ \frac{2M^e}{5} &= 30 \\ M^e &= \frac{150}{2} = 75 \end{aligned}$$

So, the EV is $75 - 30 = 45$. Therefore, Doris' parents would need to give her at least \$45 to get her to stay on campus.

(b) This is just asking for the CV. On campus, she chooses the bundle $(6, 0)$ and her utility is:

$$u(6, 0) = 2 \times 6 + 0 = 12$$

To find the CV, we need to find the income level M^c such that Doris achieves the same utility as on campus but at the farmers market prices. At those prices, her demand is $(a, b) = (0, \frac{M}{p_b})$, i.e. she only purchases broccolis at this price ratio. Therefore, we can solve for M^c by doing the following:

$$\begin{aligned} u\left(0, \frac{M^c}{1}\right) &= u(6, 0) \\ M^c &= 12 \end{aligned}$$

Therefore the CV is 12-30=-18. So Doris is willing to pay up to \$18 to travel to the farmers market.

Q6: Market Demand

1. Type A consumers:

$$\begin{aligned} MRS = \frac{1 - \frac{1}{20}x}{\frac{1}{5}} &= \frac{p_x}{p_y} \\ 1 - \frac{1}{20}x &= \frac{1}{5} \cdot \frac{p_x}{p_y} \\ \frac{1}{20}x &= 1 - \frac{1}{5} \cdot \frac{p_x}{p_y} \\ x &= 20 - 4\frac{p_x}{p_y} \end{aligned}$$

Type B consumers:

$$
\begin{aligned}
MRS = \frac{6 - \frac{1}{5}x}{1} &= \frac{p_x}{p_y} \\
6 - \frac{1}{5}x &= \frac{p_x}{p_y} \\
\frac{1}{5}x &= 6 - \frac{p_x}{p_y} \\
x &= 30 - 5\frac{p_x}{p_y}
\end{aligned}
$$

Note that these are both quasi-linear, so we don't need to plug into the budget constraint to get the demand functions. We do need to be a bit careful though, because the demand functions we wrote could have corner solutions, i.e. if $x < 0$. Let's check when that happens for Type A:

$$
\begin{aligned}
20 - 4\frac{p_x}{p_y} &< 0 \\
\frac{p_x}{p_y} &> \frac{20}{4} = 5
\end{aligned}
$$

And for Type B:

$$
\begin{aligned}
30 - 5\frac{p_x}{p_y} &< 0 \\
\frac{p_x}{p_y} &> \frac{30}{5} = 6
\end{aligned}
$$

Therefore, the demand functions are:

$$
\underline{\text{Type A}} \qquad\qquad\qquad\qquad \underline{\text{Type B}}
$$

$$
x = \begin{cases} 20 - 4\frac{p_x}{p_y} & \text{if } \frac{p_x}{p_y} < 5 \\ 0 & \text{if } \frac{p_x}{p_y} > 5 \end{cases} \qquad\qquad x = \begin{cases} 30 - 5\frac{p_x}{p_y} & \text{if } \frac{p_x}{p_y} < 6 \\ 0 & \text{if } \frac{p_x}{p_y} > 6 \end{cases}
$$

2. Let's re-write the demand functions using the given notation: (and fix $p_y = 1$)

$$
\underline{\text{Type A}} \qquad\qquad\qquad\qquad \underline{\text{Type B}}
$$

$$
q_A = \begin{cases} 20 - 4p & \text{if } p < 5 \\ 0 & \text{if } p > 5 \end{cases} \qquad\qquad q_B = \begin{cases} 30 - 5p & \text{if } p < 6 \\ 0 & \text{if } p > 6 \end{cases}
$$

The market demand is the total demanded by all consumers:

$$
Q = 100 \times q_A + 50 \times q_B
$$

Since there are 100 of Type A and 50 of Type B. We have to be careful because of these corners, i.e. if the price is too high then consumers will not demand anything. Notice that the consumers have different "choke prices". For Type A, they won't purchase above $p = 5$. But for Type B, their maximum price is $p = 6$. That means between 5 and 6, only Type B consumers are purchasing. This means the market demand function is:

$$Q(p) = \begin{cases} 100\,(20 - 4p) + 50\,(30 - 5p) & \text{if } p < 5 \\ 100\,(0) + 50\,(30 - 5p) & \text{if } p \in [5, 6] \\ 100(0) + 50(0) & \text{if } p > 6 \end{cases}$$

Simplifying this gives us:

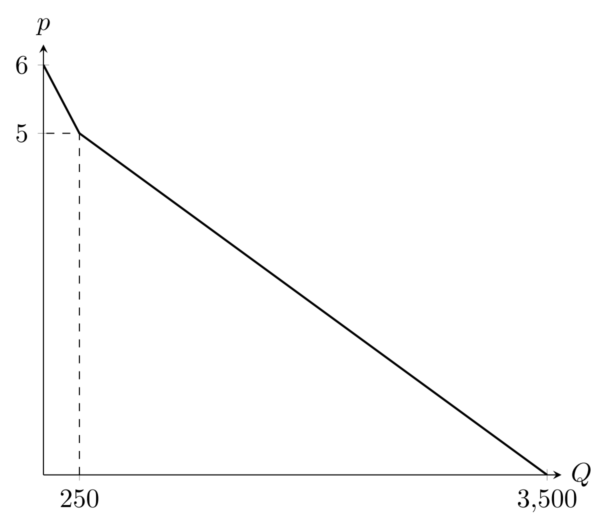
$$Q(p) = \begin{cases} 3500 - 650p & \text{if } p < 5 \\ 1500 - 250p & \text{if } p \in [5, 6] \\ 0 & \text{if } p > 6 \end{cases}$$

3. First, we want to find the inverse demand function. Note that at $p = 5$, $Q = 250$, which is where the kink occurs. So the inverse demand is:

$$p(Q) = \begin{cases} \frac{1}{650}(3500 - Q) & \text{if } Q > 250 \\ \frac{1}{250}(1500 - Q) & \text{if } Q < 250 \end{cases}$$

$$\therefore p(Q) = \begin{cases} \frac{70}{13} - \frac{Q}{650} & \text{if } Q > 250 \\ 6 - \frac{Q}{250} & \text{if } Q < 250 \end{cases}$$

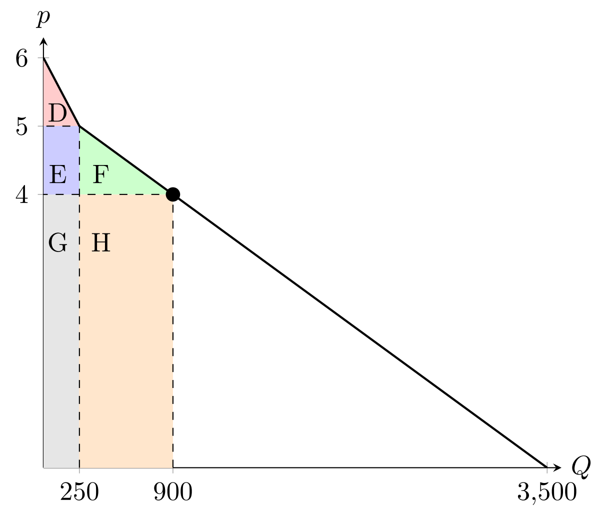
Let's plot this:



4. At $p = 4$, the quantity demanded in the market is:

$$Q(4) = 3500 - 650 \times 4 = 900$$

For the consumer surplus, we just need to be careful of the kink in the demand curve. Here is a graph with the areas of interest broken down into manageable chunks:



The net consumer surplus is $D + E + F$.

$$
\begin{aligned}
D &= \frac{1}{2} \times (6-5) \times (250-0) \\
&= \frac{1}{2} \times 1 \times 250 = 125 \\
E + F &= (5-4) \times \frac{1}{2}\left((250-0) + (900-0)\right) \\
&= 1 \times \frac{(250+900)}{2} = 575 \\
\therefore CS_{\text{Net}} &= 125 + 575 = 700
\end{aligned}
$$

The gross consumer surplus is $CS_{\text{Net}} + G + H$

$$
\begin{aligned}
G + H &= (4-0) \times (900-0) \\
&= 4 \times 900 = 3600 \\
\therefore CS_{\text{Gross}} &= 700 + 3600 = 4300
\end{aligned}
$$

5. Taking the same steps as before, the new demand curve is:

$$
Q(p) = \begin{cases} 50\,(20-4p) + 50\,(30-5p) & \text{if } p < 5 \\ 50\,(0) + 50\,(30-5p) & \text{if } p \in [5,6] \\ 50(0) + 50(0) & \text{if } p > 6 \end{cases}
$$

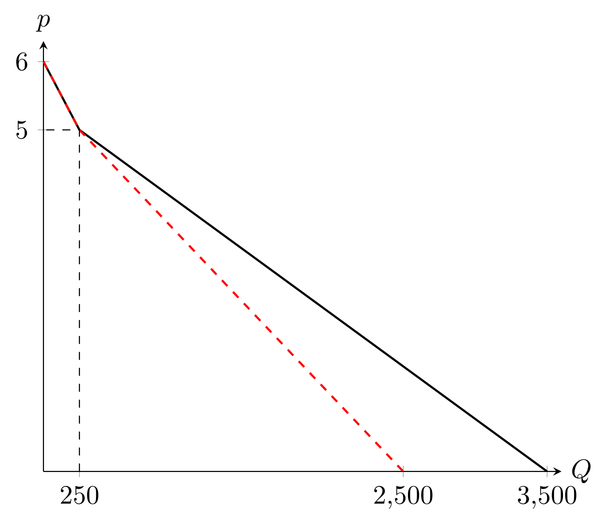
$$
\therefore Q(p) = \begin{cases} 2500 - 450p & \text{if } p < 5 \\ 1500 - 250p & \text{if } p \in [5,6] \\ 0 & \text{if } p > 6 \end{cases}
$$

Now we find the inverse demand curve:

$$p(Q) = \begin{cases} \frac{1}{450}(2500 - Q) & \text{if } Q > 250 \\ \frac{1}{250}(1500 - Q) & \text{if } Q < 250 \end{cases}$$

$$\therefore p(Q) = \begin{cases} \frac{50}{9} - \frac{Q}{450} & \text{if } Q > 250 \\ 6 - \frac{Q}{250} & \text{if } Q < 250 \end{cases}$$

If the number of Type A falls, this only affects the demand curve for $p < 5$. Notice it doesn't affect the location of the kink or anything above it since Type A people never purchased anything above $p = 5$ anyway. Plotting the new demand curve as a red dashed line gives us:



6. Since everybody is the same in Samesburg, we can infer that an individual resident's demand curve is:

$$q = \frac{Q}{100} = 40 - 5p$$

   If the price changes from 4 to 2, then quantity demanded increases from $q(4) = 40 - 20 = 20$ to $q(2) = 40 - 10 = 30$.

   To decompose this, we need to know what bundle they would choose at the new prices and with the compensated income. However, we know that they have quasi-linear utility, which means that income doesn't affect their demand functions for x_1 (just like the Townsville people). Hence, at the new prices, they will still demand $q(2) = 30$ regardless of the income. Therefore, all of the change is due to the substitution effect and not the income effect.

7. The question is asking about price elasticity. Being more sensitive to price just means having a larger price elasticity of demand (in absolute value). For Samesburg, the price elasticity demand of their

market demand curve is:

$$\begin{aligned}\varepsilon_S(P) &= \frac{dQ}{dP} \times \frac{P}{Q} \\ &= (-500) \times \frac{P}{(4000 - 500P)} \\ &= \frac{-500P}{4000 - 500P}\end{aligned}$$

For Townsville, the kink does complicate things a little bit, but it's still the same idea, so you should get the following:

$$\varepsilon_T(P) = \begin{cases} \frac{-650P}{3500-650P} & \text{if } p < 5 \\ \frac{-250P}{1500-250P} & \text{if } p \in (5, 6] \end{cases}$$

Now, let's compare the elasticities. Let's suppose Townsville is more elastic (i.e. larger in absolute value). First, let's check for $p < 5$:

$$\begin{aligned}|\varepsilon_T(P)| &> |\varepsilon_S(P)| \\ \frac{650P}{3500 - 650P} &> \frac{500P}{4000 - 500P} \\ \frac{1}{\frac{70}{13P} - 1} &> \frac{1}{\frac{8}{P} - 1} \\ \frac{70}{13P} - 1 &> \frac{8}{P} - 1 \\ \frac{70}{13} &> 8 \\ \approx 5.38 &\not> 8\end{aligned}$$

So clearly, we can already see that this is false. But just for completeness, let's also check when $p > 5$:

$$\begin{aligned}|\varepsilon_T(P)| &> |\varepsilon_S(P)| \\ \frac{250P}{1500 - 250P} &> \frac{500P}{4000 - 500P} \\ \frac{1}{\frac{6}{P} - 1} &> \frac{1}{\frac{8}{P} - 1} \\ \frac{6}{P} - 1 &> \frac{8}{P} - 1 \\ 6 &\not> 8\end{aligned}$$

But again, we get a contradiction. So at all points, Samesburg is more price sensitive.