Intermediate Micro: Final Exam Review

Motaz Al-Chanati

May 12, 2020

1 Overview

1.1 Monopoly

Solving for the Equilibrium

For a question on a monopolist, you are given the following information: the demand curve $Q(P)$ and the cost function $C(Q)$. You will be asked to find the firm's optimal quantity Q^* and price P^*

1. Invert the demand function (i.e. solve for P) to get the inverse demand function: $Q = Q(P) \rightarrow P = P(Q)$
2. Calculate the marginal revenue. You have a few options here:
   - If the inverse demand function is linear, i.e. in the form $P(Q) = A - BQ$, then you can simply double the coefficient on Q, i.e. $MR(Q) = A - 2BQ$
   - In general, you can use this formula: $MR(Q) = P(Q) + P'(Q) \times Q$, where $P'(Q) = \frac{dP(Q)}{dQ}$ (the derivative of the inverse demand function)
   - Alternatively, calculate revenue as $Rev(Q) = P(Q) \times Q$ (multiply by the inverse demand function by Q - there should be no P's here!). Then, take the derivative of revenue: $MR(Q) = \frac{dRev(Q)}{dQ}$
3. Calculate the marginal cost: $MC(Q) = \frac{dC(Q)}{dQ}$
4. The firm's optimal quantity occurs where marginal revenue equals marginal cost
   - Set: $MR(Q) = MC(Q)$
   - Solve for Q^*
5. Plug in the Q^* you found into the inverse demand function to get the optimal price: $P^* = P(Q^*)$
   - Do *not* plug it into MR or MC!

Deadweight Loss

The monopolist will charge higher price and sell a lower quantity than the perfectly competitive market. The perfectly competitive market is efficient (because it maximizes surplus), while the monopolist outcome is *inefficient*. We capture this idea by deadweight loss. To calculate the DWL of the monopolist:

1. Solve for the monopolist's equilibrium (see above). Call this Q_M, P_M.
2. Solve for the (short run) perfect competition equilibrium. Call this Q_E, P_E.
   - In the short run, you can think of the firm's MC curve as its supply curve.
   - Therefore the equilibrium will be where the firm's MC intersects the *inverse* demand curve, i.e. $MC(Q) = P(Q)$. Find this intersection to give you the equilibrium under perfect competition.
3. The DWL is the difference between the total surplus under perfect competition versus the total surplus under monopoly. Total surplus is equal to consumer surplus (CS) plus producer surplus (PS)
   - You can calculate CS and PS under each setting to get total surplus and then take the difference:

$$DWL = \underbrace{(CS_E + PS_E)}_{\substack{\text{Perf Comp} \\ \text{Total Surplus}}} - \underbrace{(CS_M + PS_M)}_{\substack{\text{Monopoly} \\ \text{Total Surplus}}}$$

   - Alternatively, use the following formula: (graphically, note that this represents the DWL triangle)

$$DWL = \frac{1}{2} \times (P_M - MC(Q_M)) \times (Q_E - Q_M)$$

     where $MC(Q_M)$ is the value on the MC curve at the monopolist quantity Q_M

Note: If you are asked to compare the monopolist to the long run perfectly competitive market, then the approach is different. For the monopolist, nothing changes. For perfect competition, the firm will operate at q_{MES} (where $MC = AC$). Use this condition to get the market price ($P^* = MC(q_{MES}) = AC(q_{MES})$), and then the market price to get the market quantity ($Q^* = P(Q^*)$). The number of firms will likely be more than 1 ($N = \frac{Q^*}{q_{MES}}$) indicating that firms enter in the LR and the market is no longer under a monopoly.

1.2 Price Discrimination

Two-Part Tariffs

For these questions, you are given the demand curve $Q(P)$ and the cost curve $C(Q)$. You will be asked for the optimal price P^* and the access fee F.

1. Find the inverse demand function by solving for P, i.e. $Q = Q(P) \rightarrow P = P(Q)$
2. Find the marginal cost function $MC(Q) = \frac{dC(Q)}{dQ}$

3. Set the optimal price using the rule $P = MC$
   - Set the inverse demand function equal to the MC and solve for Q: $P(Q) = MC(Q) \rightarrow Q^*$.
   - Plug the quantity into the inverse demand curve to get the price: $P^* = P(Q^*)$
4. Set the access fee equal to the consumer surplus at the point Q^*, P^*:
   - Solve for the y-intercept of the *inverse* demand curve. For example, if the inverse demand curve is of the form $P(Q) = A - BQ$, then the y-intercept is A.
   - Calculate the consumer surplus: $CS = \frac{1}{2} \times (A - P^*) \times Q^*$
   - Set the access fee equal to the consumer surplus $F = CS$

Market Segmentation

For these questions, the market will be made up of two groups. The first group has the demand function $Q_1(P)$ and the second group has the demand function $Q_2(P)$. You will also be given the firm's cost function $C(Q)$.

You could be asked to find the monopolist's optimal quantity and price under no price discrimination.

1. For each group, find the *inverse* demand function: $P_1(Q)$ and $P_2(Q)$
2. Find the total market demand $Q(P)$
   (a) Find the y-intercept of each inverse demand function (i.e. just plug in $Q = 0$). This is the "choke price". Call this A_1 and A_2 for each group.
   (b) For simplicity, call group 1 that one with the higher choke price (i.e. $A_1 > A_2$). Calculate the market demand as follows:

$$Q(P) = \begin{cases} Q_1(P) + Q_2(P) & \text{if } P \leq A_2 \\ Q_1(P) & \text{if } A_2 < P \leq A_1 \\ 0 & \text{if } P > A_1 \end{cases}$$

   (c) If you need to draw this, note that this curve has a y-intercept at A_1 and a kink at price A_2
3. Find the marginal revenue of the market demand curve
   (a) Find the quantity where the kink in the demand occurs. Call this Q_K and you can find it by plugging in A_2 into group 1's demand curve, i.e. $Q_K = Q_1(A_2) = Q(A_2)$
   (b) Find the *inverse* market demand curve by solving for P in each part of the demand curve

$$P(Q) = \begin{cases} P_{12}(Q) & \text{if } P \leq A_2 \implies Q \geq Q_K \\ P_1(Q) & \text{if } A_2 < P \leq A_1 \implies Q < Q_K \end{cases}$$

   where $P_{12}(Q)$ is calculated by solving for P from $Q_1(P) + Q_2(P)$ and $P_1(Q)$ is just the inverse demand curve you calculated in step 1.

(c) Calculate marginal revenue as you usually do (see the steps under Monopoly) for each segment of the inverse demand curve

$$MR(Q) = \begin{cases} MR_{12}(Q) & \text{if } Q \geq Q_K \\ MR_1(Q) & \text{if } Q < Q_K \end{cases}$$

(d) If you need to draw this, note that the there is a jump at Q_K

4. Find the marginal cost function $MC(Q) = \frac{dC(Q)}{dQ}$

5. Calculate the optimal quantity Q^* by setting $MR(Q) = MC(Q)$

- Try setting $MR_{12}(Q) = MC(Q)$ and then try setting $MR_1(Q) = MC(Q)$
- Pick the one that does not give you a contradiction. For example, if you set $MR_1(Q) = MC(Q)$ and find a $Q > Q_K$, this would be a contradiction (see how the MR curve is defined).

6. Find the optimal price by plugging in Q^* into the inverse market demand curve from 3(b): $P^* = P(Q^*)$

Next, you might be asked what the firm's optimal strategy is with <u>price discrimination</u>

1. For each group, take the following steps:

   (a) Find the inverse demand curves: $P_1(Q), P_2(Q)$

   (b) Find the marginal revenue curves: $MR_1(Q), MR_2(Q)$

2. Find the marginal cost function $MC(Q) = \frac{dC(Q)}{dQ}$

3. Use the optimality condition: $MR_1(Q_1) = MR_2(Q_2) = MC(Q)$

   (a) Set $MR_1(Q_1) = MR_2(Q_2)$ and solve for Q_1. This will give you the optimal Q_1 as a function of Q_2

   - You can instead solve for Q_2 as a function of Q_1. The next steps would be analogous, just using MC_1 instead of MC_2 and vice versa

   (b) Set $MR_2(Q_2) = MC(Q)$. Note that $Q = Q_1+Q_2$, so within MC you can replace Q with (Q_1+Q_2).

   (c) Next, within MC, you can replace the Q_1 with the function you found in 3(a). Now everything will be in terms of Q_2 (there should be no Q_1 or Q)

4. Solve for the optimal Q_2^*. Plug this into the function from 3(a) to get the optimal Q_1^*

5. To get the prices, plug in each quantity into their respective inverse demand curves: $P_1^* = P_1(Q_1^*)$ and $P_2^* = P_2(Q_2^*)$

6. To check you have the correct answer, plug in the quantities into the respective marginal revenue curves ($MR_1^* = MR_1(Q_1^*)$ and $MR_2^* = MR_2(Q_2^*)$). These should be equal. Moreover, if you plug in the *total* quantity ($Q^* = Q_1^* + Q_2^*$) into the marginal cost curve ($MC(Q^*)$), this should also equal those values.

1.3 Game Theory

Normal Form Game

You will be given a game in normal form (a table). Tips on reading the table:

- Each row represents the actions that player 1 can choose
- Each column represents the actions that player 2 can choose
- Each cell will contain two numbers (A, B). This represents the outcome or payoffs as a result of the corresponding row and column being chosen. The first number corresponds to player 1's payoff. The second number corresponds to player 2's payoff.

The first thing you should do with these tables is to find the best response. The best response is the best action that a player can take, *given* the other player's action. This is not necessarily the best action overall, it is simply the best thing a player can do *in response* to the other player's choice.[1]

- To find player 1's best response, start at the first column. Check each *row* within that column and only look at the *first* number in each row. Underline the highest number among them (only underline the first number within a cell).
  - Repeat this action for the second, third etc columns.
  - For a given action by player 2 (the column), the row which contains the underlined number tells you what player 1's best response would be
- To find player 2's best response, start at the first row. Check each *column* within that row and only look at the *second* number in each column. Underline the highest number among them (only underline the second number within a cell).
  - Repeat this action for the second, third etc row.
  - For a given action by player 1 (the row), the column which contains the underlined number tells you what player 2's best response would be

With the underlines, you can now quickly see a lot of useful information

- The Nash Equilibrium occurs in any cell where both numbers are underlined (there could be multiple!). The Nash Equilibrium is where player 1 chooses the action in the corresponding row and player 2 chooses the action in the corresponding column
- A dominant strategy is an action that is always a best response no matter what the other player does
  - To check if player 1 has a dominant strategy, see if there is a row whose number is underlined in every column
  - To check if player 2 has a dominant strategy, see if there is a column whose number is underlined in every row

[1] For example, consider "Rock, Paper, Scissors". If I play "Rock", your best response would be "Paper". If I play "Scissors", your best response would be "Rock".

Oligopoly - Cournot

For Cournot competition, two firms will compete on quantity. You will get an inverse demand curve that is a function of both firms' quantity $P(Q_1, Q_2)$.[2] For each firm will you also get a cost function: $C_1(Q_1)$ and $C_2(Q_2)$. To get the Nash Equilibrium:

1. Write out each firm's payoff. This is simply their profit: (note that the inverse demand curve is being plugged in for P)

$$\pi_1 = P(Q_1, Q_2) \times Q_1 - C_1(Q_1)$$
$$\pi_2 = P(Q_1, Q_2) \times Q_2 - C_2(Q_2)$$

2. As firms maximize profits, next you take FOCs. This means taking the partial derivative with respect to the firm's own quantity, holding fixed the other firm's quantity.

$$\frac{\partial \pi_1}{\partial Q_1} = \frac{\partial P(Q_1, Q_2)}{\partial Q_1} Q_1 + P(Q_1, Q_2) Q_1 - MC_1 = 0$$
$$\frac{\partial \pi_2}{\partial Q_2} = \frac{\partial P(Q_1, Q_2)}{\partial Q_2} Q_2 + P(Q_1, Q_2) Q_2 - MC_2 = 0$$

3. For each equation above, solve for respective Q. This gives you the best response function for each firm. Note that the best response function for a firm is a function of the other firm's quantity

$$\frac{\partial \pi_1}{\partial Q_1} = 0 \rightarrow Q_1 = R_1(Q_2)$$
$$\frac{\partial \pi_2}{\partial Q_2} = 0 \rightarrow Q_2 = R_2(Q_1)$$

4. Plug in firm 2's best response function into firm 1's best response function. Now you will have one equation where everything is in terms of Q_1.

$$Q_1 = R_1\left(R_2(Q_1)\right)$$

5. Solve for Q_1^* to get the equilibrium quantity for firm 1. Plug this into firm 2's best response to get the equilibrium quantity for firm 2: $Q_2^* = R_2(Q_1^*)$

Labor Supply

See Recitation 6 notes

[2] If the demand curve is only a function of Q, you can plug in $Q_1 + Q_2$ in place of Q

2 Exercises

2.1 Monopoly

Question: The demand for a monopolist's product is $Q(P) = 60 - \frac{1}{2}P$. The firm's cost function is $C(Q) = Q^2$. Find the market equilibrium.

Solution: The inverse demand function is:

$$P(Q) = 120 - 2Q$$

The marginal revenue is therefore:

$$MR(Q) = 120 - 4Q$$

The marginal cost is:

$$MC(Q) = 2Q$$

The firm's optimal rule is:

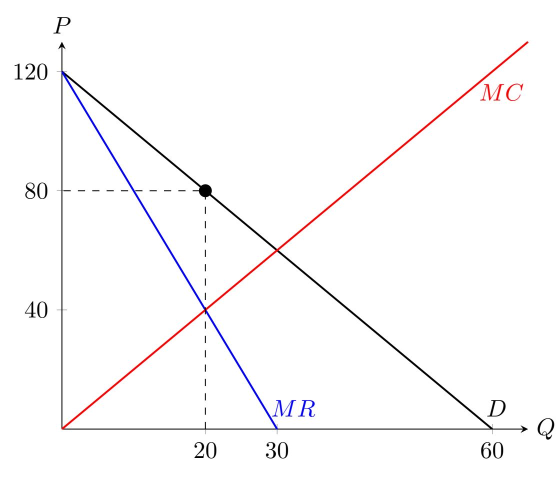
$$\begin{aligned} MR(Q) &= MC(Q) \\ 120 - 4Q &= 2Q \\ Q^* &= 20 \end{aligned}$$

The firm's optimal price is:

$$P^* = 120 - 2Q^* = 80$$

Question: What is the consumer and producer surplus in this market? What is the deadweight loss?

Solution: It's easier to draw the diagram before starting this question:



The consumer surplus is the top triangle above the market price and below the demand curve:

$$CS = \frac{1}{2} \times (120 - 80) \times 20 = 400$$

The producer surplus is the lower trapezoid below the market price and above the MC curve:

$$PS = \frac{(80 - 0) + (80 - 40)}{2} \times 20 = 1200$$

Therefore, total surplus is 1600.

To get DWL, we have to compare this to perfect competition, where the equilibrium would have occurred where MC crosses the demand curve (which happens at $Q = 30, P = 60$). This would have given us a total surplus equal to the triangle below the demand curve and above the MC curve.

$$TS_E = \frac{1}{2} \times 120 \times 30 = 1800$$

Therefore, DWL is the difference in total surpluses:

$$DWL = TS_E - TS_M = 1800 - 1600 = 200$$

Alternatively, DWL is equal to the triangle to the right of the market price:

$$\begin{aligned} DWL &= \frac{1}{2} \times (80 - MC(20)) \times (30 - 20) \\ &= \frac{1}{2} \times (80 - 40) \times 10 = 200 \end{aligned}$$

Question: Consider a monopolist with an inverse demand curve of $P(Q) = 200 - Q$. It has a constant marginal cost equal to 10. The government introduces a tax of \$28 per unit. How does CS and PS change as a result of the tax? What is the tax revenue collected?

Solution: The firm's marginal revenue is:

$$MR(Q) = 200 - 2Q$$

The firm's marginal cost before is $MC(Q) = 10$. After the tax it becomes $MC(Q) = 38$

Before the tax, the firm's optimal point is:

$$\begin{aligned} MR(Q) &= MC(Q) \\ 200 - 2Q &= 10 \\ Q^* &= 95 \end{aligned}$$

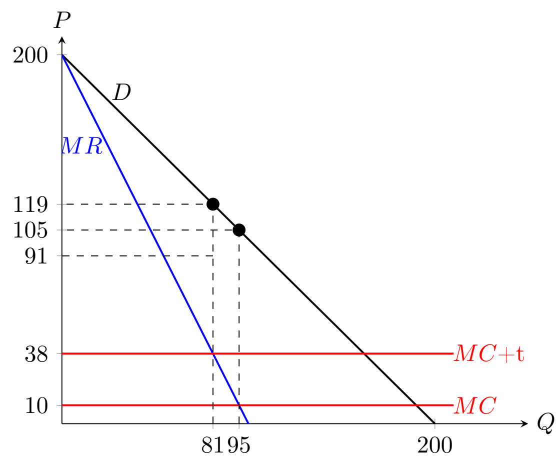
$$\begin{aligned} \implies P^* &= P(95) \\ &= 105 \end{aligned}$$

After the tax, the firm's optimal point is:

$$\begin{aligned} MR(Q) &= MC(Q) \\ 200 - 2Q &= 38 \\ Q^* &= 81 \end{aligned}$$

$$\begin{aligned} \implies P^* &= P(81) \\ &= 119 \end{aligned}$$

We can represent this change in the diagram



The old CS is the triangle above the old market price (105) and below the demand curve:

$$CS = \frac{1}{2} \times (200 - 105) \times 95 = 4512.5$$

The old PS is the rectangle below the old market price (105) and above the MC curve, up to the equilibrium quantity (95):

$$PS = (105 - 10) \times 95 = 9025$$

The new CS is the triangle above the new market price (119) and below the demand curve:

$$CS = \frac{1}{2} \times (200 - 119) \times 81 = 3280.5$$

The new PS is the rectangle below the price that sellers receive (91) and above the original MC curve, up to the equilibrium quantity (81):[3]

$$PS = (91 - 10) \times 81 = 6561$$

The government tax revenue is equal to the tax (28) multiplied by the number of units sold (81). This is

[3] The price sellers receive is equal to the new market price (119) minus the tax (28), which gives $119 - 28 = 91$

equivalent to the rectangle between the new CS and new PS:

$$G = 28 \times 81 = 2268$$

2.2 Price Discrimination

Market Segmentation

Question: A monopolist has two groups of consumers. The first group has the demand function: $Q_1 = 80 - P_1$. The second group has the demand function $Q_2 = 100 - 2P_2$. The firm's cost function is $MC(Q) = 10 + 2Q$. What is the equilibrium if the firm does not price discriminate?

Solution: First, we find the inverse demand curves:

$$P_1(Q_1) = 80 - Q_1$$
$$P_2(Q_2) = 50 - \frac{1}{2}Q_2$$

We can see that group 1's choke price is 80 while group 2's is 50.

The total market demand is then:

$$Q(P) = \begin{cases} (80 - P) + (100 - 2P) = 180 - 3P & \text{if } P \leq 50 \\ 80 - P & \text{if } 50 < P \leq 80 \\ 0 & \text{if } P > 80 \end{cases}$$

Note that this is a kinked demand curve, where a curve occurs at price $P = 50$ and quantity $Q = 30$. This means we can express the inverse demand curve as:

$$P(Q) = \begin{cases} 60 - \frac{1}{3}Q & \text{if } Q \geq 30 \\ 80 - Q & \text{if } Q < 30 \end{cases}$$

The marginal revenue curve is then:

$$MR(Q) = \begin{cases} 60 - \frac{2}{3}Q & \text{if } Q \geq 30 \\ 80 - 2Q & \text{if } Q < 30 \end{cases}$$

Let's check the optimal condition for each part of the MR curve:

$$\begin{aligned} MR(Q) &= MC(Q) \\ 60 - \frac{2}{3}Q &= 10 + 2Q \\ 50 &= \frac{8}{3}Q \\ Q^* &= 18.75 \end{aligned} \qquad\qquad \begin{aligned} MR(Q) &= MC(Q) \\ 80 - 2Q &= 10 + 2Q \\ 70 &= 4Q \\ Q^* &= 17.5 \end{aligned}$$

However, note that there is a contradiction. The first part of the curve gives an optimal point where $Q^* = 18.75$, but this formula is only valid for when $Q \geq 30$. Therefore, this cannot be the optimal point. Instead, the firm's optimal quantity is $Q^* = 17.5$. This means its optimal price is:

$$P^* = P(17.5) = 80 - 17.5 = 62.5$$

Note that at this price only group 1 is purchasing the good.

Question: What is the equilibrium if the firm does price discriminate?

Solution: Above, we found the inverse demand curves:

$$\begin{aligned} P_1(Q_1) &= 80 - Q_1 \\ P_2(Q_2) &= 50 - \frac{1}{2}Q_2 \end{aligned}$$

This means the marginal revenue curves are:

$$\begin{aligned} MR_1(Q_1) &= 80 - 2Q_1 \\ MR_2(Q_2) &= 50 - Q_2 \end{aligned}$$

The optimality condition is:

$$MR_1(Q_1) = MR_2(Q_2) = MC(Q)$$

Starting with the first equality, let's solve for Q_2

$$\begin{aligned} MR_1(Q_1) &= MR_2(Q_2) \\ 80 - 2Q_1 &= 50 - Q_2 \\ Q_2 &= 2Q_1 - 30 \end{aligned}$$

Next, we take $MC(Q)$ and plug in $Q = Q_1 + Q_2$ and then the equation for Q_2 we found above:

$$\begin{aligned} MC(Q) &= 10 + 2Q \\ &= 10 + 2(Q_1 + Q_2) \\ &= 10 + 2Q_1 + 2(2Q_1 - 30) \\ &= 6Q_1 - 50 \end{aligned}$$

Next, we take $MR_1(Q_1) = MC(Q)$ (notice that everything is in terms of Q_1 now)

$$\begin{aligned} MR_1(Q_1) &= MC(Q) \\ 80 - 2Q_1 &= 6Q_1 - 50 \\ 130 &= 8Q_1 \\ \implies Q_1^* &= \frac{130}{8} = 16.25 \end{aligned}$$

This means that the optimal Q_2 is

$$\begin{aligned} Q_2^* &= 2Q_1^* - 30 \\ &= 2.5 \end{aligned}$$

Plug these back into their respective inverse demand curves to get the price:

$$\begin{aligned} P_1^* &= P_1(Q_1^*) = 63.75 \\ P_2^* &= P_2(Q_2^*) = 48.75 \end{aligned}$$

We can check to make sure these numbers are correct:

$$\begin{aligned} MR_1(Q_1^*) &= 80 - 2(16.25) = 47.5 \\ MR_2(Q_2^*) &= 50 - (2.5) = 47.5 \\ MC(Q^*) &= 10 + 2(16.25 + 2.5) = 47.5 \end{aligned}$$

And as expected, the condition $MR_1(Q_1) = MR_2(Q_2) = MC(Q)$ holds.

2.3 Game Theory

Question: Two firms simultaneously choose their quantities Q_1, Q_2 to produce. The market's inverse demand curve is $P = 13 - Q_1 - Q_2$. Assume each firm has marginal cost equal to 1 (and has no fixed cost).

First, suppose each firm can produce only two possible quantities: 3 units or 4 units. Create a table to represent this game in normal form. Then find the Nash Equilibrium. If the firms could collude, what actions would they take in order to both increase their profits relative to the Nash Equilibrium? Using terminology from game theory, explain why you think this collusion is unstable.

Solution: Firm 1's profit is: $\pi_1 = PQ_1 - Q_1 = (P-1)Q_1$. Firm 2's profit is $\pi_1 = PQ_2 - Q_2 = (P-1)Q_2$. Since each firm has two possible actions, this leave four possibilities, as shown in the table below:

	3 units	**4 units**
3 units		
4 units		

This is the normal form of the game, but we now need to fill out each cell. Let's run through each scenario:

- When $Q_1 = Q_2 = 3$, then $P = 7$. This means $\pi_1 = \pi_2 = 6 \times 3 = 18$
- When $Q_1 = 3, Q_2 = 4$, then $P = 6$. This means $\pi_1 = 5 \times 3 = 15$ and $\pi_2 = 5 \times 4 = 20$
- When $Q_1 = 4, Q_2 = 3$, then $P = 6$. This means $\pi_1 = 5 \times 4 = 20$ and $\pi_2 = 5 \times 3 = 15$
- When $Q_1 = Q_2 = 4$, then $P = 5$. This means $\pi_1 = \pi_2 = 4 \times 4 = 16$

So, the normal form of this game is as follows:

	3 units	**4 units**
3 units	18, 18	15, 20
4 units	20, 15	16, 16

To find the Nash Equilibrium, we underline each of the best response's. You should then get the following table:

	3 units	**4 units**
3 units	18, 18	15, $\underline{20}$
4 units	$\underline{20}$, 15	$\underline{16}$, $\underline{16}$

This is just like the Prisoner's Dilemma.

- For firm 1: If firm 2 chooses 3, they should choose 4. If firm 2 chooses 4, they should choose 4.

- For firm 2: If firm 1 chooses 3, they should choose 4. If firm 1 chooses 4, they should choose 4.

We can see that the Nash Equilibrium is where each firm plays 4 units each. If the firms colluded and agreed to keep quantities at 3 each, they would both have higher payoff. However, this couldn't be an equilibrium because both firms have an incentive to deviate. This is because the best response to a firm playing 3 units is for the other firm to play 4 units.

Question: Suppose that demand falls to $P = 12 - Q_1 - Q_2$. What is the Nash Equilibrium now?

Solution: Repeating as above, let's run through each scenario:

- When $Q_1 = Q_2 = 3$, then $P = 6$. This means $\pi_1 = \pi_2 = 5 \times 3 = 15$
- When $Q_1 = 3, Q_2 = 4$, then $P = 5$. This means $\pi_1 = 4 \times 3 = 12$ and $\pi_2 = 4 \times 4 = 16$
- When $Q_1 = 4, Q_2 = 3$, then $P = 5$. This means $\pi_1 = 4 \times 4 = 16$ and $\pi_2 = 4 \times 3 = 12$
- When $Q_1 = Q_2 = 4$, then $P = 4$. This means $\pi_1 = \pi_2 = 3 \times 4 = 12$

So, the normal form of this game (with the best response underlined) is as follows:

	3 units	**4 units**
3 units	15, 15	$\underline{12}$, $\underline{16}$
4 units	$\underline{16}$, $\underline{12}$	$\underline{12}$, $\underline{12}$

Now we see there are three cells where both numbers are underlined. This means there are three Nash Equilibria: (3,4); (4,3); and (4,4).

Question: Suppose demand goes back to its original level: $P = 13 - Q_1 - Q_2$. Now let's give the firm more options. They can now play: 3, 4, 5, or 6 units. Show the game in normal form. Do you still find the same Nash Equilibrium as before? Is there any action that we would *never* expect the firms to choose?

Solution: The full table looks as follows:

	3	**4**	**5**	**6**
3	(18,18)	$(15,\underline{20})$	$(\underline{12},\underline{20})$	$(\underline{9},18)$
4	$(\underline{20},15)$	$(\underline{16},\underline{16})$	$(\underline{12},15)$	(8,12)
5	$(\underline{20},\underline{12})$	$(15,\underline{12})$	(10,10)	(5,6)
6	$(18,\underline{9})$	(12,8)	(6,5)	(0,0)

We can see that (4,4) is still a Nash Equilibrium. Moreover, we can see that 6 is never a best response. So we should never expect to see the (rational) firms play 6 under any circumstance.

Question: Now let's allow firms to choose any quantities they want. Find the Nash Equilibrium to the game.

Solution: Recall, we have that firm 1's profit is: $\pi_1 = PQ_1 - Q_1$ and that firm 2's profit is $\pi_1 = PQ_2 - Q_2$.

The first thing we do is plug in the inverse demand function so that the profits are only in terms of Qs:

$$\pi_1 = (13 - Q_1 - Q_2)Q_1 - Q_1 = 12Q_1 - Q_1^2 - Q_1Q_2$$
$$\pi_2 = (13 - Q_1 - Q_2)Q_2 - Q_2 = 12Q_2 - Q_2^2 - Q_1Q_2$$

Taking FOCs:

$$\frac{\partial \pi_1}{\partial Q_1} = 12 - 2Q_1 - Q_2 = 0$$
$$\frac{\partial \pi_2}{\partial Q_2} = 12 - 2Q_2 - Q_1 = 0$$

Solving for the best response functions:

$$\frac{\partial \pi_1}{\partial Q_1} = 0 \implies Q_1 = \frac{12 - Q_2}{2}$$
$$\frac{\partial \pi_2}{\partial Q_2} = 0 \implies Q_2 = \frac{12 - Q_1}{2}$$

Plug in firm 2's best response function into firm 1's:

$$
\begin{aligned}
Q_1 &= \frac{12 - \left(\frac{12-Q_1}{2}\right)}{2} \\
&= 6 - \left(3 - \frac{1}{4}Q_1\right) \\
&= 3 + \frac{1}{4}Q_1 \\
\frac{3}{4}Q_1 &= 3 \\
\therefore Q_1^* &= 4 \\
Q_2^* &= 6 - \frac{1}{2}Q_1^* \\
&= 4
\end{aligned}
$$

Again, we have found that (4,4) is the Nash Equilibrium. This says that firm 1 is best responding to firm 2, who in turn is also best responding to firm 1, and so on. Graphically, we can see this as the intersection of the best response functions: