Logarithms

Throughout the course, we will be working heavily with the logarithm function. Familiarize yourself with its definition, and make sure you are able to manipulate expressions and solve equations involving logarithms.

Convention

We will write $\log(x)$ for the base-2 logarithm of x. That is, $\log x = \frac{\ln(x)}{\ln(2)}$.

In particular, you should know the following facts:

$$
\begin{aligned}
\log(1) &= 0. \text{For } 0 < x < 1 \text{ it holds that } \log(x) < 0, \text{ and for } x > 1, \text{ we have that } \log(x) > 0. \\
\log(2) &= 1. \\
\log(2^x) &= x. \\
2^{\log(x)} &= x. \\
\log(x \cdot y) &= \log(x) + \log(y). \\
\log\left(\frac{x}{y}\right) &= \log(x) - \log(y). \\
\log(x^y) &= y \cdot \log(x). \\
x^{\log(y)} &= y^{\log(x)}.
\end{aligned}
$$

- The logarithm function is only defined on $\mathbb{R}_+$.
- The logarithm function is strictly increasing: $\log(x) > \log(y)$ whenever $x > y$.
- $\log(x+y)$ and $\log(x-y)$ do not generally have a simpler form.