Definition: Mutual Information

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Let X and Y be random variables. The mutual information $I(X;Y)$ of X and Y is defined as

$$I(X;Y) = H(X) - H(X|Y).$$

Thus, in a sense, mutual information reflects the reduction in uncertainty about X when we learn Y. Verify the following properties of the mutual information:

$$I(X;Y) = H(X) + H(Y) - H(XY)$$
$$I(X;Y) = I(Y;X) \qquad\qquad (\text{``symmetry''})$$
$$I(X;Y) \geq 0 \qquad\qquad (\text{``positivity''})$$
$$I(X;Y) = 0 \text{ iff } X \text{ and } Y \text{ are independent}$$
$$I(X;X) = H(X) \qquad\qquad (\text{``self-information''})$$

created: 2019-10-21