Definition: Conditional Entropy

Let X be a random variable and $\mathcal{A}$ an event. Applying **the definition of entropy** to the conditional probability distribution $P_{X|\mathcal{A}}$ allows us to naturally define the entropy of X conditioned on the event $\mathcal{A}$:

$$H(X|\mathcal{A}) := \sum_{x \in \mathcal{X}} P_{X|\mathcal{A}}(x) \cdot \log \frac{1}{P_{X|\mathcal{A}}(x)}.$$

This leads to the following notion:

Definition: Conditional entropy

Let X and Y be random variables, with respective images $\mathcal{X}$ and $\mathcal{Y}$. The conditional entropy $H(X|Y)$ of X given Y is defined as

$$H(X|Y) := \sum_{y \in \mathcal{Y}} P_Y(y) \cdot H(X|Y{=}y)\,,$$

with the convention that the corresponding argument in the summation is 0 for $y \in \mathcal{Y}$ with $P_Y(y) = 0$.

Note that the conditional entropy $H(X|Y)$ is not the entropy of a probability distribution but an expectation: the average uncertainty about X when given Y.