

# Definition: Conditional Entropy

Let  $X$  be a random variable and  $\mathcal{A}$  an event. Applying **the definition of entropy** to the conditional probability distribution  $P_{X|\mathcal{A}}$  allows us to naturally define the entropy of  $X$  conditioned on the event  $\mathcal{A}$ :

$$H(X|\mathcal{A}) := \sum_{x \in \mathcal{X}} P_{X|\mathcal{A}}(x) \cdot \log \frac{1}{P_{X|\mathcal{A}}(x)}.$$

This leads to the following notion:

## Definition: Conditional entropy

Let  $X$  and  $Y$  be random variables, with respective images  $\mathcal{X}$  and  $\mathcal{Y}$ . The conditional entropy  $H(X|Y)$  of  $X$  given  $Y$  is defined as

$$H(X|Y) := \sum_{y \in \mathcal{Y}} P_Y(y) \cdot H(X|Y=y),$$

with the convention that the corresponding argument in the summation is 0 for  $y \in \mathcal{Y}$  with  $P_Y(y) = 0$ .

Note that the conditional entropy  $H(X|Y)$  is not the entropy of a probability distribution but an expectation: the average uncertainty about  $X$  when given  $Y$ .