

# The Chain Rule for Mutual Information

Similarly to the chain rule for entropy, we can prove a chain rule for mutual information:

## Corollary: Chain rule for mutual information

Let  $W, X, Y$  and  $Z$  be random variables. Then

$$I(WX; Y|Z) = I(X; Y|Z) + I(W; Y|ZX).$$

Proof hint

Apply the generalized chain rule.

Show full proof

$$\begin{aligned} I(WX; Y|Z) &= H(WX|Z) - H(WX|YZ) \\ &= (H(X|Z) + H(W|XZ)) - (H(X|YZ) + H(W|XYZ)) \\ &= H(X|Z) - H(X|YZ) + H(W|XZ) - H(W|XYZ) \\ &= I(X; Y|Z) + I(W; Y|XZ). \end{aligned}$$