Definition: Independence Number

Definition: Undirected Graph

A undirected simple graph G consists of a set V(G) of **vertices** and a set E(G) of **edges**. The edges are unordered pairs of vertices: each edge connects two different vertices of the graph.

Definition: Independence number

The independence number $\alpha(G)$ of a graph G is the size of the largest **independent set** of G, where an independent set of G is a set $S \subseteq V(G)$ such that

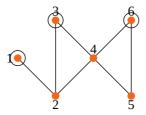
$$orall x, x' \in S: (x,x')
otin E(G).$$

That is, an independent set S in G is a set of vertices such that there is no edge between any of the vertices.

Finding the independence number of a graph is an NP-hard problem, meaning there is no known efficient method for finding the independence number of an arbitrary graph.

Example

Consider the following graph with $V(G) = \{1,2,3,4,5,6\}$ and $E(G) = \{\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{4,5\},\{4,6\},\{5,6\}\}$:



A maximal independent set $\{1,3,6\}$ is marked in the graph. As there is no independent set of size 4 (can you prove that?), the independence number of this graph is 3.

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