

# Types of Discrete Channels

## Definition: Deterministic channel

A channel is deterministic if  $H(Y|X) = 0$ . In other words,

$$\forall x \in \mathcal{X} \exists y \in \mathcal{Y} : P_{Y|X}(y|x) = 1.$$

## Definition: Lossless channel

A channel is lossless (or **ideal**) if  $H(X|Y) = 0$  for all input distributions  $P_X$ . In other words,

$$\forall y \in \mathcal{Y} \exists! x \in \mathcal{X} : P_{Y|X}(y|x) > 0.$$

(the notation  $\exists! x$  means that there exists *exactly* one such  $x$ .)

In a deterministic channel, the output is completely determined by the input, whereas in a lossless channel, the input is completely determined by the output.

A **noiseless** channel is a channel that is both deterministic and lossless.