Definition: Achievable Rate

As stated, the channel capacity reflects the maximum amount of information that could *in principle* be sent over a noisy channel per use of that channel. The question remains whether this capacity is **achievable** by an actual code in the following sense:

Definition: Achievable rate

For a given channel, a rate R is achievable if there exists a sequence of $(2^{n \cdot R}, n)$ codes (for $n = 1, 2, 3, \ldots$) such that $\lambda^{(n)} \xrightarrow{n \to \infty} 0$. Here, $\lambda^{(n)}$ is the **maximum error probability** as defined in the previous module.

Note that any $(2^{n \cdot R}, n)$ code has rate R, since $\frac{1}{n} \cdot \log 2^{n \cdot R} = R$. Thus, a rate R is achievable if there exists a sequence of rate R codes such that increasing the number of channel uses n (often called the block size) reduces the maximum error asymptotically to zero.

This final module will be concerned with proving **Shannon's noisy-channel coding theorem**, which states that any rate R that is strictly below the capacity C is achievable, and conversely, that any rate strictly above is not achievable.