## **Expectation and Variance**

## **Definition: Expectation**

The expectation of a *real* random variable X is defined as

$$\mathbb{E}[X] := \sum_{x \in \mathcal{X}} P_X(x) \cdot x.$$

Note that if X is not real, then we can still consider the expectation of some function  $f:\mathcal{X} \to \mathbb{R}$ , where

$$\mathbb{E}[f(X)] = \sum_{x \in \mathcal{X}} P_X(x) \cdot f(X).$$

## **Definition: Variance**

The variance of a *real* random variable X is defined as

$$\operatorname{Var}[X] := \mathbb{E}[(X - \mathbb{E}[X])^2].$$

The variation is a measure for the deviation of the mean. Hoeffding's inequality (here stated for binary random variables) states that for a list of i.i.d. random variables, the average of the random variables is close to the expectation, except with very small probability. We state it here without proof.

## Theorem: Hoeffding's inequality

Let  $X_1,\ldots,X_n$  be independent and identically distributed binary random variables with  $P_{X_i}(0)=1-\mu$  and  $P_{X_i}(1)=\mu$ , and thus  $\mathbb{E}[X_i]=\mu$ . Then, for any  $\delta>0$ 

$$P\left[\sum_i X_i > (\mu + \delta) \cdot n
ight] \leq \exp(-2\delta^2 n).$$

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