

# Definition: Achievable Rate

As stated, the channel capacity reflects the maximum amount of information that could *in principle* be sent over a noisy channel per use of that channel. The question remains whether this capacity is **achievable** by an actual code in the following sense:

## Definition: Achievable rate

For a given channel, a rate  $R$  is achievable if there exists a sequence of  $(2^{n \cdot R}, n)$  codes (for  $n = 1, 2, 3, \dots$ ) such that  $\lambda^{(n)} \xrightarrow{n \rightarrow \infty} 0$ . Here,  $\lambda^{(n)}$  is the maximum error probability as defined in the previous module.

Note that any  $(2^{n \cdot R}, n)$  code has rate  $R$ , since  $\frac{1}{n} \cdot \log 2^{n \cdot R} = R$ . Thus, a rate  $R$  is achievable if there exists a sequence of rate  $R$  codes such that increasing the number of channel uses  $n$  (often called the block size) reduces the maximum error asymptotically to zero.

This final module will be concerned with proving **Shannon's noisy-channel coding theorem**, which states that any rate  $R$  that is strictly below the capacity  $C$  is achievable, and conversely, that any rate strictly above is not achievable.