## The Chain Rule for Mutual Information

Similarly to the **chain rule for entropy**, we can prove a chain rule for mutual information:

## **Corollary: Chain rule for mutual information**

Let W, X, Y and Z be random variables. Then

$$I(WX;Y|Z) = I(X;Y|Z) + I(W;Y|ZX).$$

**Proof hint** 

Apply the generalized chain rule.

Show full proof

$$\begin{split} I(WX;Y|Z) &= H(WX|Z) - H(WX|YZ) \\ &= (H(X|Z) + H(W|XZ)) - (H(X|YZ) + H(W|XYZ)) \\ &= H(X|Z) - H(X|YZ) + H(W|XZ) - H(W|XYZ) \\ &= I(X;Y|Z) + I(W;Y|XZ). \end{split}$$

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