

Definition: Binary Representation & Binary Intervals

Definition: Standard binary representation

The standard binary representation of a real number $r \in [0, 1)$ is a (possibly infinite) string of bits $c_1 c_2 \dots$ such that

$$r = \sum_i c_i \cdot 2^{-i},$$

where by convention, 0 is represented by the string 0.

Not all reals in $[0, 1)$ have a finite representation, but any interval $[a, b)$ with $0 \leq a < b \leq 1$ contains at least one number with a finite binary representation.

Example

The following table lists some numbers $r \in [0, 1)$ and their standard binary representation.

r	binary representation of r
$1/2$	1
$1/3$	01010101...
$1/4$	01
$3/4$	11
$13/16$	1101
$13/32$	01101

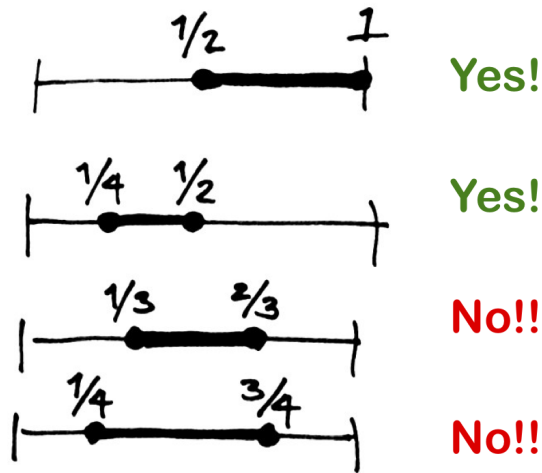
Note that 1101 is also the binary form of the natural number 13. Adding a 0 on the left divides the represented value by 2.

Definition: Binary interval

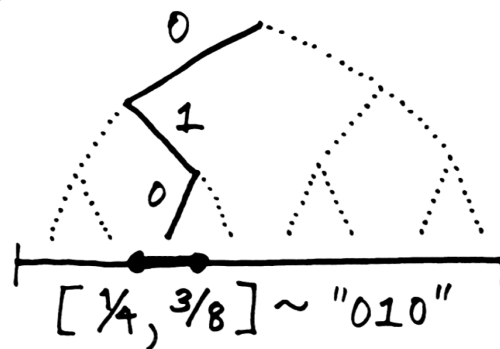
A binary interval is an interval of the form

$$\left[\frac{s}{2^\ell}, \frac{s+1}{2^\ell} \right)$$

with $s, \ell \in \mathbb{N}$ and $0 \leq s < 2^\ell$.



The **name** of the interval is the binary representation of s (as a natural number) padded with zeroes on the left to reach length ℓ . The name can also be interpreted as the path to follow from the root in order to reach the interval as follows:



[Images by Mathias Madsen, thanks a lot!]