

# Definition: Independence Number

## Definition: Undirected Graph

A undirected simple graph  $G$  consists of a set  $V(G)$  of **vertices** and a set  $E(G)$  of **edges**. The edges are unordered pairs of vertices: each edge connects two different vertices of the graph.

## Definition: Independence number

The independence number  $\alpha(G)$  of a graph  $G$  is the size of the largest **independent set** of  $G$ , where an independent set of  $G$  is a set  $S \subseteq V(G)$  such that

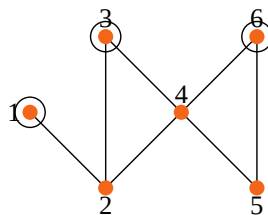
$$\forall x, x' \in S : (x, x') \notin E(G).$$

That is, an independent set  $S$  in  $G$  is a set of vertices such that there is no edge between any of the vertices.

Finding the independence number of a graph is an NP-hard problem, meaning there is no known efficient method for finding the independence number of an arbitrary graph.

## Example

Consider the following graph with  $V(G) = \{1, 2, 3, 4, 5, 6\}$  and  $E(G) = \{\{1, 2\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{4, 5\}, \{4, 6\}, \{5, 6\}\}$ :



A maximal independent set  $\{1, 3, 6\}$  is marked in the graph. As there is no independent set of size 4 (can you prove that?), the independence number of this graph is 3.