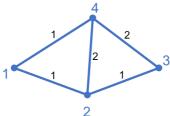
Random Walks on Graphs

An important and widely applicable example of a time-invariant Markov process is a random walk on a connected graph G with strictly positive symmetric edge weights $W_{ij}=W_{ji}$. The random walk is defined as follows: at node i, walk to node j with probability $\frac{W_{ij}}{W_i}$ where $W_i:=\sum_j W_{ij}$ is the sum of the weights of all edges involving node i, and $W:=\frac{1}{2}\sum_i W_i$ is the total of all edge weights.

Example



For the following graph , we have that $W_1=2, W_2=4, W_3=3, W_4=5$ and $2\cdot W=\sum_i W_i=2\cdot 7$.

The stationary distribution of this random walk is given by $\mu_i:=\frac{W_i}{2W}$, because indeed, at every node i, we have that the sum of all incoming weight is

$$\sum_{j} \mu_{j} \frac{W_{ij}}{W_{j}} = \sum_{j} \frac{W_{j}}{2W} \frac{W_{ij}}{W_{j}} = \frac{W_{i}}{2W} = \mu_{i} .$$

We continue to compute the entropy rate of this random walk. Assuming we start in the stationary distribution, we can compute the entropy rate as follows.

$$H(\lbrace X_{i}\rbrace) = \sum_{i} \mu_{i} H(\dots \frac{W_{ij}}{W_{i}} \dots) = -\sum_{i} \mu_{i} \sum_{j} \frac{W_{ij}}{W_{i}} \log \frac{W_{ij}}{W_{i}}$$

$$= -\sum_{i,j} \frac{W_{i}}{2W} \cdot \frac{W_{ij}}{W_{i}} \log \left(\frac{W_{ij}}{2W} \cdot \frac{2W}{W_{i}}\right)$$

$$= -\sum_{i,j} \frac{W_{ij}}{2W} \log \left(\frac{W_{ij}}{2W}\right) + \sum_{i,j} \frac{W_{ij}}{2W} \log \left(\frac{W_{i}}{2W}\right)$$

$$= -\sum_{i,j} \frac{W_{ij}}{2W} \log \left(\frac{W_{ij}}{2W}\right) + \sum_{i} \frac{W_{i}}{2W} \log \left(\frac{W_{i}}{2W}\right)$$

$$= H(\dots \frac{W_{ij}}{2W} \dots) - H(\dots \frac{W_{i}}{2W} \dots)$$

which is the difference of the entropy of the edge distribution and the entropy of the stationary distribution.

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Information Theory | Random Walks on Graphs **Example, continued**

In the example above, the edge distribution is $\frac{1}{14}(1,1,1,2,2,1,1,1,2,2)$ and the stationary distribution is $\frac{1}{14}(2,4,3,5)$, resulting in

$$H(\{X_i\}) = H(\frac{1}{14}(1,1,1,2,2,1,1,1,2,2)) - H(\frac{1}{14}(2,4,3,5)) \approx 1.312$$

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