Definition: Strong Graph Product

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Let G, H be two graphs. We define the strong graph product $G \boxtimes H$ as follows. The set of vertices is
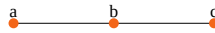
$$V(G \boxtimes H) := V(G) \times V(H).$$

The set of edges is

$$E(G \boxtimes H) := \Big\{ \{(x,y),(x',y')\} \mid (x,y) \neq (x',y') \text{ and } (x = x' \text{ or } \{x,x'\} \in E(G)) \text{ and } (y = y' \text{ or } \{y,y'\}$$
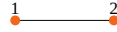
i.e., there is an edge between (x,y) and (x',y') if and only if the vertices of G are confusable (or equal) *and* the vertices of H are confusable (or equal).
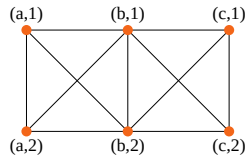
Example

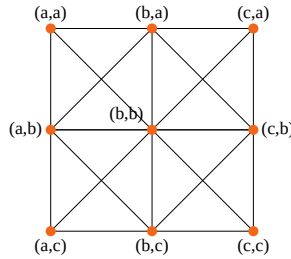Consider the graph G:

and the graph H:

The strong graph product of $G \boxtimes H$ is

The independence number of this graph is 2. The strong product $G \boxtimes G$ of G with itself is

The independence number of this graph is 4. As the graphs get bigger, the independence number is increasingly hard to compute.

For our application, we will often be interested in the strong graph product of a graph G with itself, possibly many times. Therefore it is useful to work out the definition of $G^{\boxtimes n}$, based on the above definition:

$$V(G^{\boxtimes n}) = V(G) \times \cdots \times V(G)$$
$$E(G^{\boxtimes n}) = \big\{ \{(x_1,\ldots,x_n),(v_1,\ldots,v_n)\} \mid (x_1,\ldots,x_n) \neq (v_1,\ldots,v_n) \text{ and } \forall i : x_i = v_i \vee \{x_i,v_i\} \in E(G)]$$