Expectation and Variance

Definition: Expectation

The expectation of a *real* random variable X is defined as

$$\mathbb{E}[X] := \sum_{x \in \mathcal{X}} P_X(x) \cdot x.$$

Note that if X is not real, then we can still consider the expectation of some function $f : \mathcal{X} \to \mathbb{R}$, where

$$\mathbb{E}[f(X)] = \sum_{x \in \mathcal{X}} P_X(x) \cdot f(X).$$

Definition: Variance

The variance of a *real* random variable X is defined as

$$\mathrm{Var}[X] := \mathbb{E}[(X - \mathbb{E}[X])^2].$$

The variation is a measure for the deviation of the mean. Hoeffding's inequality (here stated for binary random variables) states that for a list of i.i.d. random variables, the average of the random variables is close to the expectation, except with very small probability. We state it here without proof.

Theorem: Hoeffding's inequality

Let $X_1, \ldots, X_n$ be independent and identically distributed binary random variables with $P_{X_i}(0) = 1 - \mu$ and $P_{X_i}(1) = \mu$, and thus $\mathbb{E}[X_i] = \mu$. Then, for any $\delta > 0$

$$P\left[\sum_i X_i > (\mu + \delta) \cdot n\right] \leq \exp(-2\delta^2 n).$$