

Some Important Distributions

We end the theoretic preliminaries with a (non-exhaustive) list of common probability distributions that you will come across throughout the course.

- The distribution of a biased coin with probability $P_X(1) = p$ to land heads, and a probability of $P_X(0) = 1 - p$ to land tails is called **Bernoulli(p) distribution**. The expected value is $\mathbb{E}[X] = p$ and the variance is $\text{Var}[X] = p(1 - p)$.
- When n coins X_1, X_2, \dots, X_n are flipped independently and every X_i is Bernoulli(p) distributed, let $S = \sum_{i=1}^n X_i$ be their sum, i.e., the number of heads in n throws of a biased coin. Then, S has the **binomial(n, p) distribution**:

$$P_S(k) = \binom{n}{k} p^k (1 - p)^{n-k} \quad \text{where } k = 0, 1, 2, \dots, n.$$

From simple properties of the expected value and variance, one can show that $\mathbb{E}[S] = np$ and $\text{Var}[S] = np(1 - p)$.

- The **geometric(p) distribution** of a random variable Y is defined as the number of times one has to flip a Bernoulli(p) coin before it lands heads:

$$P_Y(k) = (1 - p)^{k-1} p \quad \text{where } k = 1, 2, 3, \dots$$

There is another variant of the geometric distribution used in the literature, where one excludes the final success event of landing heads in the counting:

$$P_Z(k) = (1 - p)^k p \quad \text{where } k = 0, 1, 2, 3, \dots$$

While the expected values are slightly different, namely $\mathbb{E}[Y] = \frac{1}{p}$ and $\mathbb{E}[Z] = \frac{1-p}{p}$, their variances are the same: $\text{Var}[Y] = \text{Var}[Z] = \frac{1-p}{p^2}$.