

Definition: Mutual Information

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Let X and Y be random variables. The mutual information $I(X; Y)$ of X and Y is defined as

$$I(X; Y) = H(X) - H(X|Y).$$

Thus, in a sense, mutual information reflects the reduction in uncertainty about X when we learn Y . Verify the following properties of the mutual information:

$$I(X; Y) = H(X) + H(Y) - H(XY)$$

$$I(X; Y) = I(Y; X) \quad (\text{“symmetry ”})$$

$$I(X; Y) \geq 0 \quad (\text{“positivity ”})$$

$$I(X; Y) = 0 \text{ iff } X \text{ and } Y \text{ are independent}$$

$$I(X; X) = H(X) \quad (\text{“self-information ”})$$