Some Important Distributions

We end the theoretic preliminaries with a (non-exhaustive) list of common probability distributions that you will come across throughout the course.

- The distribution of a biased coin with probability $P_X(1) = p$ to land heads, and a probability of $P_X(0) = 1 - p$ to land tails is called **Bernoulli(p)** distribution. The expected value is $\mathbb{E}[X] = p$ and the variance is $\mathrm{Var}[X] = p(1-p)$.
- When n coins $X_1, X_2, \ldots, X_n$ are flipped independently and every X_i is Bernoulli(p) distributed, let $S = \sum_{i=1}^{n} X_i$ be their sum, i.e., the number of heads in n throws of a biased coin. Then, S has the **binomial(n, p)** distribution:

  $$P_S(k) = \binom{n}{k} p^k (1-p)^{n-k} \qquad \text{where } k = 0, 1, 2, \ldots, n\,.$$

  From simple properties of the expected value and variance, one can show that $\mathbb{E}[S] = np$ and $\mathrm{Var}[S] = np(1-p)$.
- The **geometric(p)** distribution of a random variable Y is defined as the number of times one has to flip a Bernoulli(p) coin before it lands heads:

  $$P_Y(k) = (1-p)^{k-1} p \qquad \text{where } k = 1, 2, 3, \ldots\,.$$

  There is another variant of the geometric distribution used in the literature, where one excludes the final success event of landing heads in the counting:

  $$P_Z(k) = (1-p)^{k} p \qquad \text{where } k = 0, 1, 2, 3, \ldots\,.$$

  While the expected values are slightly different, namely $\mathbb{E}[Y] = \frac{1}{p}$ and $\mathbb{E}[Z] = \frac{1-p}{p}$, their variances are the same: $\mathrm{Var}[Y] = \mathrm{Var}[Z] = \frac{1-p}{p^2}$.