The Weak Law of Large Numbers

The following general law states that if we sample several times from the same distribution, the average converges (in probability) to the expected value of the distribution.

Theorem: Weak Law of Large Numbers

Let $X_1, X_2, \ldots$ be real i.i.d. random variables with mean $\mu = \mathbb{E}[X_i]$ and variance $\sigma^2 = \mathbb{E}[(X_i - \mu)^2] < \infty$. Define the random variables

$$S_n := \frac{1}{n} \sum_{i=1}^{n} X_i.$$

Then $S_n \xrightarrow{p} \mu$, where we interpret μ as the constant random variable that is μ with probability 1.

created: 2018-12-12