Types of Discrete Channels

Definition: Deterministic channel

A channel is deterministic if $H(Y|X) = 0$. In other words,

$$\forall x \in \mathcal{X} \, \exists y \in \mathcal{Y} : P_{Y|X}(y|x) = 1.$$

Definition: Lossless channel

A channel is lossless (or **ideal**) if $H(X|Y) = 0$ for all input distributions P_X. In other words,

$$\forall y \in \mathcal{Y} \, \exists! x \in \mathcal{X} : P_{Y|X}(y|x) > 0.$$

(the notation $\exists! x$ means that there exists *exactly* one such x.)

In a deterministic channel, the output is completely determined by the input, whereas in a lossless channel, the input is completely determined by the output. A **noiseless** channel is a channel that is both deterministic and lossless.

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