# Definition: Independence Number

### Definition: Undirected Graph

A undirected simple graph $G$ consists of a set $V(G)$ of **vertices** and a set $E(G)$ of **edges**. The edges are unordered pairs of vertices: each edge connects two different vertices of the graph.

### Definition: Independence number

The independence number $\alpha(G)$ of a graph $G$ is the size of the largest **independent set** of $G$, where an independent set of $G$ is a set $S \subseteq V(G)$ such that

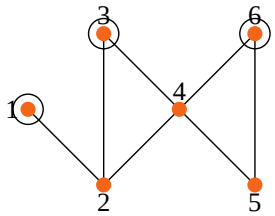
$$\forall x, x' \in S : (x, x') \notin E(G).$$

That is, an independent set $S$ in $G$ is a set of vertices such that there is no edge between any of the vertices.

Finding the independence number of a graph is an NP-hard problem, meaning there is no known efficient method for finding the independence number of an arbitrary graph.

**Example**

Consider the following graph with $V(G) = \{1, 2, 3, 4, 5, 6\}$ and $E(G) = \{\{1, 2\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{4, 5\}, \{4, 6\}, \{5, 6\}\}$:




A maximal independent set $\{1, 3, 6\}$ is marked in the graph. As there is no independent set of size 4 (can you prove that?), the independence number of this graph is 3.