Definition: Strong Graph Product

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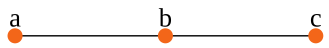
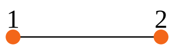
Let G, H be two graphs. We define the strong graph product $G \boxtimes H$ as follows. The set of vertices is $$ V(G \boxtimes H) := V(G) \times V(H). $$ The set of edges is $$ \begin{split} E(G \boxtimes H) := \big\{\{(x,y),(x',y')\} \mid (x,y) \neq (x',y') &\text{ and } \left(x = x' \text{ or } \{x,x'\} \in E(G) \right) &\text{ and } \left(y=y' \text{ or } \{y,y'\} \in E(H) \right) \big\}, \end{split} $$ i.e., there is an edge between (x,y) and $((x',y')$ if and only if the vertices of G are confusable (or equal) *and* the vertices of H are confusable (or equal).

Example

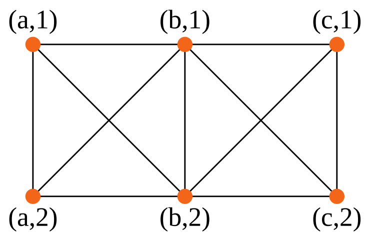
Consider the graph G:



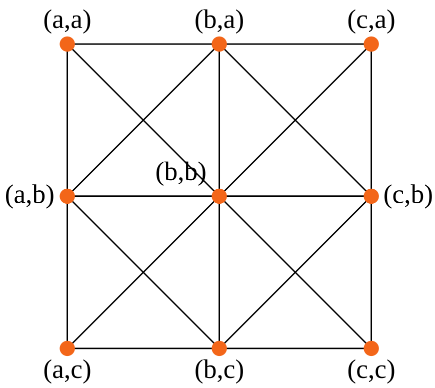
and the graph H:



The strong graph product of $G \boxtimes H $ is



The independence number of this graph is 2. The strong product $ G \boxtimes G $ of G with itself is



The independence number of this graph is 4. As the graphs get bigger, the independence number is increasingly hard to compute.

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For our application, we will often be interested in the strong graph product of a graph G with itself, possibly many times. Therefore it is useful to work out the definition of $G^{\boxtimes n}$, based on the above definition:

$$\begin{align*} V(G^{\boxtimes n}) &= V(G) \times \cdots \times V(G)\\ E(G^{\boxtimes n})&= \big\{ \{(x_1, \ldots, x_n),(v_1, \ldots, v_n)\} \mid (x_1, \ldots, x_n) \neq (v_1, \ldots, v_n) \text{ and } \forall i : x_i = v_i \vee \{x_i,v_i\} \in E(G)\} \end{align*}$$

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