Definition: Mutual Information

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Let X and Y be random variables. The mutual information $I(X;Y)$ of X and Y is defined as

$$I(X;Y) = H(X) - H(X|Y).$$

Thus, in a sense, mutual information reflects the reduction in uncertainty about X when we learn Y. Verify the following properties of the mutual information:

$$\begin{aligned}
&I(X;Y) = H(X) + H(Y) - H(XY) \\
&I(X;Y) = I(Y;X) && (\text{“symmetry ”}) \\
&I(X;Y) \geq 0 && (\text{“positivity ”}) \\
&I(X;Y) = 0 \text{ iff } X \text{ and } Y \text{ are independent} \\
&I(X;X) = H(X) && (\text{“self-information ”})
\end{aligned}$$