Elipsa:

- $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $b=\sqrt{a^2-c^2}$ ← semiaxa mare, b - semiaxa mică
- ~~ex~~ excentricitate: $\varepsilon=\frac{c}{a}=\sqrt{1-\frac{b^2}{a^2}}$
- razele focale ale unui punct $M(x,y)$: $\begin{cases} r_1=a+\varepsilon x \\ r_2=a-\varepsilon x \end{cases}$ | ec tang. în M_0: $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}=1$
- ec tang paralele cu o dir dată: $y=kx+\sqrt{a^2k^2+b^2}$ | ec tang. vert: $x=\pm a$ | $a^2y_0x-b^2x_0y-(a^2-b^2)x_0y_0=0$ (ec. normale)
- ec tang ce trec printr-un punct ext planului:

a) $x_1\neq\pm a$ → tang au pantele: $k_1,k_2=\frac{-x_1y_1\pm\sqrt{b^2x_1^2+a^2y_1^2-a^2b^2}}{x_1^2-a^2}$

b) $x_1=\pm a$ tangenta vert cu panta $k=\pm\frac{y_1^2-b^2}{2ay_1}$, $tg=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

Hiperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, $b=\sqrt{c^2-a^2}$; a, b - semiaxele hiperbolei, 2c - dist focală

- Excentricitate: $\varepsilon=\frac{c}{a}=\sqrt{1+\frac{b^2}{a^2}}>1$; $\begin{cases} r_1=a+\varepsilon x \\ r_2=a-\varepsilon x \end{cases}$ | Asimptotele hiperbolei: $y=\pm\frac{b}{a}x$
- tangenta și normala într-un punct: $\frac{xx_0}{a^2}-\frac{yy_0}{b^2}=1$; $a^2y_0x+b^2x_0y-(a^2+b^2)x_0y_0=0$
- tangenta la hiperbolă ce trece printr-un punct ext:

a) $x_1\neq\pm a$ → $k_{1,2}=\frac{x_1y_1\pm\sqrt{a^2y_1^2-b^2x_1^2+a^2b^2}}{x_1^2-a^2}$ b) $x_1=\pm a$ → $k=\pm\frac{y_1^2+b^2}{2ay_1}$

Parabola: ec canonică:

$y^2=2px$, p = parametrul parabolei

- tangenta într-un punct: $yy_0=p(x+x_0)$ • tangenta dată de o pantă: $y=kx+\frac{p}{2k}$
- tangenta prin pct ext: a) $x_1=0$ → tang este Oy, cealaltă ec: $y-y_1=\frac{p}{2y_1}$

b) $x_1\neq0$ → $y-y_1=k(x-x_1)$, k soluție

Cuadrice pe ec reduse:

Elipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, a, b, c semiaxe, >0; plan tangent în $M(x_0,y_0,z_0)$: $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}+\frac{zz_0}{c^2}=1$

- planele coord sunt plane simetrice, vf elips sunt: $(\pm a,0,0)$, $(0,\pm b,0)$ $(0,0,\pm c)$

con de grad 2: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$ (con de rot $a=b$)

- intersecția cu planele de coord:

I xOy: $h=0$ → intersecția cu originea; $h\neq0$, $z=h$ → $\frac{x^2}{a^2h^2/c^2}+\frac{y^2}{b^2h^2/c^2}=1$

II xOz: $h=0$ → dreptele $\begin{cases}\frac{y}{b}\pm\frac{z}{c}=0\\ y_0\end{cases}$; $h\neq0$ → hiperbola $\frac{z^2}{c^2h^2/b^2}-\frac{x^2}{a^2h^2/b^2}=1$

III yOz: $h=0$ → dreptele $\begin{cases}\frac{y}{b}\pm\frac{z}{c}=0\\ x=0\end{cases}$; $h\neq0$ → hiperbola $\frac{y^2}{c^2h^2/b^2}-\frac{z^2}{c^2\cdot\frac{h^2}{a^2}}=1$

- plan tangent în $M(x_0,y_0,z_0)$: $\frac{xx_0}{a^2}+\frac{yy_0}{b^2}+\frac{zz_0}{c^2}=0$

Hiperboloid cu o pânză: $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$

Vf: $(\pm a,0,0)$, $(0,\pm b,0)$, planele de coord. sunt pl. de sim.
- originea este centrul de sim.; - intersecția cu planele:

I xOy: $h=0$ → elipsa de stricțiune, $z=h$: $h\neq 0 \Rightarrow$

$\Rightarrow \frac{x^2}{\left(a\sqrt{\frac{h^2}{c^2}+1}\right)^2}+\frac{y^2}{\left(b\sqrt{\frac{h^2}{c^2}+1}\right)^2}=1$

II yOz, $x=h$: $\frac{y^2}{b^2}-\frac{z^2}{c^2}=1-\frac{h^2}{a^2}$

plan tg:

$\frac{xx_0}{a^2}+\frac{yy_0}{b^2}-\frac{zz_0}{c^2}=1$

Hiperboloid cu 2 pânze:

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$

Vf: $x=y=0$

- planele, axele, coord de sim. identice cu hip. 1 pânză

I xOy: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{h^2}{c^2}-1$

II yOz: $\frac{z^2}{c^2}-\frac{y^2}{b^2}=\frac{h^2}{b^2}+1$ (hiperb.)

- același plan tg doar că $=-1$

$a\otimes b=\begin{pmatrix}b_1\\b_2\end{pmatrix}\cdot(a_1\ a_2)$ sau $\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}\cdot(a_1\ a_2\ a_3)$

Plan:

$\text{Transl}=\begin{pmatrix}1&0&w_1\\0&1&w_2\\0&0&1\end{pmatrix}$ $\text{Rot}(Q,\theta)=\begin{pmatrix}\cos\theta&-\sin\theta&q_1(1-\cos\theta)+q_2\sin\theta\\ \sin\theta&\cos\theta&-q_1\sin\theta+q_2(1-\cos\theta)\\0&0&1\end{pmatrix}$

$\text{Scale}(Q,s)=\begin{pmatrix}s&0&(1-s)q_1\\0&s&(1-s)q_2\\0&0&1\end{pmatrix}$ $\text{Scale}(Q,s_x,s_y)=\begin{pmatrix}s_x&0&(1-s_x)q_1\\0&s_y&(1-s_y)q_2\\0&0&1\end{pmatrix}$

A: $A(0,0)=\frac{1}{2}|x_B y_C - x_C y_B|$

$\text{Mirror}(Q,w)=\begin{pmatrix}I_2-2(w^\perp\otimes w^\perp)&2(w^\perp\otimes w^\perp)\cdot Q\\0&1\end{pmatrix}$

$\text{Shear}(Q,w,\theta)=\begin{pmatrix}I_2+\text{tg}\,\theta(w^\perp\otimes w)&-\text{tg}\,\theta(w^\perp\otimes w)\cdot Q\\0&1\end{pmatrix}$

$w^\perp=(-b,a)$
$w=(a,b)$

Spațiu:

$\text{Transl}(w)=\begin{pmatrix}1&0&0&w_1\\0&1&0&w_2\\0&0&1&w_3\\0&0&0&1\end{pmatrix}$, $\text{Rot}(Q,u,\theta)=\begin{pmatrix}\text{Rot}(u,\theta)&(I_3-\text{Rot}(u,\theta))\cdot Q\\0&1\end{pmatrix}$

$\text{Rot}(u,\theta)=\cos\theta\cdot I_3+(1-\cos\theta)(u\otimes u)+\sin\theta(u\times -)$, $u\times -=\begin{pmatrix}0&-u_3&u_2\\u_3&0&-u_1\\-u_2&u_1&0\end{pmatrix}$

$\text{Scale}(Q,s)=\begin{pmatrix}s&0&0&(1-s)q_x\\0&s&0&(1-s)q_y\\0&0&s&(1-s)q_z\\0&0&0&1\end{pmatrix}$, Scale $(Q,s_x,s_y,s_z)=\begin{pmatrix}s_x&0&0&(1-s_x)\cdot q_x\\0&s_y&0&(1-s_y)\cdot q_y\\0&0&s_z&(1-s_z)\cdot q_z\\0&0&0&1\end{pmatrix}$

$\text{Mirror}(Q,n)=\begin{pmatrix}I_3-2(n\otimes n)&2(n\otimes n)\cdot Q\\0&1\end{pmatrix}$

$\text{Shear}(Q,n,u,\theta)=\begin{pmatrix}I_3+\text{tg}\,\theta\cdot(n\otimes u)&-\text{tg}\,\theta\cdot(n\otimes u)\cdot Q\\0&1\end{pmatrix}$