ec normală: $\frac{Ax+By+Cz+D}{\pm\sqrt{A^2+B^2+C^2}}=0$

$M_0(x_0,y_0,z_0)$ - abaterea la planul π

$$\delta(M_0,\pi)=\frac{Ax_0+By_0+Cz_0+D}{\pm\sqrt{A^2+B^2+C^2}}$$

$d(M_0,\pi)=|\delta(M_0,\pi)|$; $(\sphericalangle \pi_1,\pi_2)=(\sphericalangle \vec{n_1},\vec{n_2})$; $\cos\pi_1=\cos\pi_2 \Rightarrow$ plane bis.

$\pi_1: A_1x+B_1y+C_1z+D_1=0 \Rightarrow \vec{n_1}(A_1,B_1,C_1)$

$\pi_2: A_2x+B_2y+C_2z+D_2=0 \Rightarrow \vec{n_2}(A_2,B_2,C_2)$; $\cos\alpha=\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\cdot\sqrt{A_2^2+B_2^2+C_2^2}}$

$\pi_1\perp\pi_2 \Leftrightarrow A_1A_2+B_1B_2+C_1C_2=0$; $\pi_1 \parallel \pi_2 \Leftrightarrow \frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$

$$\begin{cases}x=x_0+l\cdot t\\ y=y_0+m\cdot t\\ z=z_0+n\cdot t\end{cases}$$
$\rightarrow$ ec param a dr Δ ce trece prin M_0, $\parallel \vec{a}$

$\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}$: ec canonică

convenție: $\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{0}$

$z-z_0=0$

Pr punct pe dreaptă
$$\begin{cases}\frac{x-x_0}{l}=\frac{y-y_0}{m}=\frac{z-z_0}{n}\\ (x-x_1)\cdot l+(y-y_1)\cdot m+(z-z_1)\cdot n=0\end{cases}$$

$\vec{a}=\overrightarrow{M_0M_1}(x_1-x_0,y_1-y_0,z_1-z_0) \Rightarrow \frac{x-x_0}{x_1-x_0}=\frac{y-y_0}{y_1-y_0}=\frac{z-z_0}{z_1-z_0}$; $\pi_1\cap\pi_2:\begin{pmatrix}A_1&B_1&C_1\\A_2&B_2&C_2\end{pmatrix}$

$\pi_1\cap\pi_2=d: \vec{c}=\vec{n_1}\times\vec{n_2}$; $d(M_1,\Delta)=\frac{\|(r_1-r_0)\times\vec{a}\|}{\|\vec{a}\|}$

dist 2 dr stâmbe: $\frac{\|\overrightarrow{AB}\cdot(\vec{v_1}\times\vec{v_2})\|}{\|\vec{v_1}\times\vec{v_2}\|}$; $\frac{\|\overrightarrow{AP}\times\vec{v}\|}{\|\vec{v}\|}$

$d_1\perp d_2 \Leftrightarrow l_1l_2+m_1m_2+n_1n_2=0$

$d_1\parallel d_2 \Leftrightarrow \frac{l_1}{l_2}=\frac{m_1}{m_2}=\frac{n_1}{n_2}$

π_1,π_2 $\rightarrow$ tăiate după o dr $\Leftrightarrow$ rang$\begin{pmatrix}A_1&B_1&C_1\\A_2&B_2&C_2\end{pmatrix}=2$

$\rightarrow$ paralele $\Leftrightarrow \frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}\neq\frac{D_1}{D_2}$

$\rightarrow$ coincid $\Leftrightarrow \frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\frac{D_1}{D_2}$

Δ_1,Δ_2 conc $M_0(x_0,y_0,z_0) \Rightarrow \begin{vmatrix}x-x_0&y-y_0&z-z_0\\l_1&m_1&n_1\\l_2&m_2&n_2\end{vmatrix}$

$\Delta,\pi \rightarrow Al+Bm+Cn\neq0 \Rightarrow \Delta\cap\pi=P$

$\rightarrow Al+Bm+Cn=0$, $Ax_0+By_0+Cz_0+D\neq0 \Rightarrow$
$\Rightarrow \Delta\parallel\pi$

$\rightarrow Al+Bm+Cn=0$, $Ax_0+By_0+Cz_0+D=0 \Rightarrow$
$\Rightarrow \Delta\in\pi$

$$\begin{cases}Ax+By+Cz+D=0\\ \frac{x-x_0}{A}=\frac{y-y_0}{B}=\frac{z-z_0}{C}\end{cases}$$

Proiecția unui pct pe plan

$\pi_1,\pi_2,\pi_3,\Delta,m,M$(extinsă)

a) $\Delta\neq0$, planele $\cap$ într-un punct

b) $\Delta=0$, rang $m=2$, rang $M=3 \Rightarrow$ vect. norm: 2 câte 2 necol, π între 2-2 $\rightarrow$ 3 dr $\parallel$

c) rang $m=2$, rang $M=3 \Rightarrow$ 2 din 3 vect. norm. sunt col.,
2 π paral între ele, al 3-lea între după câte o dreaptă

d) rang $m=2$, rang $M=2 \Rightarrow$ v norm 2-2 necol $\Rightarrow \pi$ 2-2 dist. trec prin aceeași dreaptă

e) rang $m=2$, rang $M=2$, 2/3 v norm col. $\Rightarrow$ 2 π coincid, al 3-lea le inters.

f) rang $m=1$, rang $M=2 \Rightarrow$ 2/3 plane coincid, $\pi_3\parallel\pi_1$ și π_2

g) rang $m=1$, rang $M=3 \Rightarrow$ plane distincte și parал.

h) rang $m=1$, rang $M=1 \Rightarrow \pi$ coincid.

$\vec{AB}=(x_B-x_A)\vec{i}+(y_B-y_A)\vec{j}$

$\vec{AB}\sim\vec{CD} \Leftrightarrow$ au ac. modul, sens, direcție

$\vec{p}\perp\vec{a} \Leftrightarrow p\cdot a=0$

$a\cdot b=\|a\|\cdot\|b\|\cdot\cos\alpha$

$\cos\alpha=\frac{a\cdot b}{\|a\|\cdot\|b\|}$

$d(\pi_1,\pi_2)=\frac{|D_2-D_1|}{\sqrt{A^2+B^2+C^2}}$

$a\cdot b=a_1\cdot b_1+a_2\cdot b_2+a_3\cdot b_3$

$\|a\|=\sqrt{a_1^2+a_2^2+a_3^2}$

$\cos\alpha=\frac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}}$

$a\perp b \Leftrightarrow a\cdot b=0$

$\vec a\times\vec b=\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3\end{vmatrix}=\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3\end{vmatrix} i-\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3\end{vmatrix} j+\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2\end{vmatrix}\vec k$; a, b coliniare $\Leftrightarrow \vec a\times\vec b=\vec 0$

$\|a\times b\|=\|a\|\cdot\|b\|\cdot\sin\alpha$; $\vec a\times\vec b\perp\vec a$, $\vec a\times\vec b\perp\vec b$; $\vec a\times\vec b\neq\vec b\times\vec a$; $\vec a\times\vec b=$

$a\times(b\times c)\neq(a\times b)\times c$; $(\lambda a+\mu b)\times c=\lambda(a\times c)+\mu(b\times c)$; $A_\Delta=\frac{1}{2}\|\vec{AB}\times\vec{AC}\|$ sau $\frac{1}{2}\|$

$A_\square=\|\vec{AB}\times\vec{AC}\|$; $h_{B,AC}=\frac{\|\vec{AB}\times\vec{AC}\|}{\|\vec{AC}\|}$; $(a\times b)\cdot c=(a,b,c)$; a,b,c coplanare

$\Leftrightarrow (a,b,c)=0$; a,b,c reper $\Leftrightarrow (a,b,c)>0$; $V_T=\frac{1}{6}|(a,b,c)|$, $V_{paral.}=|(a,b,c$

$(a,b,c)=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3\end{vmatrix}$

vda $\pm=\pm\frac{a}{\|a\|}$

$\vec r=\vec r_0+t\vec a$ - vect de poz.

$\begin{cases} x=x_0+l\cdot t \\ y=y_0+m\cdot t\end{cases}$ $M(x,y)$

ec param.

ec canonică: $\frac{x-x_0}{l}=\frac{y-y_0}{m}$

$Ax+By+C=0$ - ec gen a dreptei

$\vec n(A,B)$, vect normal

$\vec a(-B,A)$, vect dir $\perp\vec n$

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ec prin tăieturi: $\frac{x}{a}+\frac{y}{b}-1=0$

a, b punctele de tăiere

$A_1x+B_1y+C_1+\lambda(A_2x+B_2y+C_2)=0$ ec fasc de drepte

$\left.\begin{matrix} d_1: A_1x+B_1y+C_1=0 \\ d_2: A_2x+B_2y+C_2=0\end{matrix}\right\}$ concurente $\Rightarrow \lambda d_1+\mu d_2=0$ (dreaptă care trece prin $d_1\cap d_2$)

dist 2 drepte paral: $d(d_1,d_2)=\frac{|c_2-c_1|}{\sqrt{a^2+b^2}}$

$d_1: A_1x+B_1y+C_1=0$

$d_2: A_2x+B_2y+C_2=0$

$\cos\theta=\pm\frac{A_1A_2+B_1B_2}{\sqrt{A_1^2+B_1^2}\cdot\sqrt{A_2^2+B_2^2}}$

$m_{AB}=-\frac{A}{B}$

$tg\,\theta=\pm\frac{k_1-k_2}{1+k_1k_2}$

$d_1\perp d_2 \Leftrightarrow k_1k_2=-1$

$d_1\parallel d_2 \Leftrightarrow k_1=k_2$

$d(M_0,\Delta)=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$

$A_\Delta=\frac{1}{2}\begin{vmatrix} x_A & y_A \\ x_B & y_B \\ x_C & y_C\end{vmatrix}$

π plan, $M(x_0,y_0,z_0)$ punct, $\vec a_1(l_1,m_1,n_1)$, $\vec a_2(l_2,m_2,n_2)$ vect necol și $\parallel$ cu π

$\vec r=\vec r_0+u\vec a_1+v\vec a_2$ - vect de poziție al M_0

$\begin{cases} x=x_0+l_1\cdot u+l_2\cdot v \\ y=y_0+m_1\cdot u+m_2\cdot v \\ z=z_0+n_1u+n_2v\end{cases}$ - ec param

$\vec n=\vec v_1\times\vec v_2$

$\left.\begin{matrix}\vec n(A,B,C) \\ A(a_1,a_2,a_3)\end{matrix}\right\}\Rightarrow A(x-a_1)+B(y-a_2)+C(z-a_3)=0$

x	0	30°	45°	60°	90°	120°	135°	150°	180°	360°
sin x	0	$\frac{1}{2}$	$\frac{\sqrt2}{2}$	$\frac{\sqrt3}{2}$	1	$\frac{\sqrt3}{2}$	$\frac{\sqrt2}{2}$	$\frac{1}{2}$	0	0
cos x	1	$\frac{\sqrt3}{2}$	$\frac{\sqrt2}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt2}{2}$	$-\frac{\sqrt3}{2}$	-1	1

$tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}$ $\sin\alpha=\frac{c.\,op}{ip}$

$ctg\,\alpha=\frac{\cos\alpha}{\sin\alpha}$ $\cos\alpha=\frac{c.\,al}{ip}$

$\vec u,\vec v$ col $\Leftrightarrow \vec u=\alpha\cdot\vec v$

$\vec u\parallel\vec v \Leftrightarrow \frac{x_1}{x_2}=\frac{y_1}{y_2}$

$Ax+By+Cz+D=0$

$\vec n(A,B,C)$ vect. norm

$M_1,M_2,M_3\in\pi$, $\vec{M_1M_2}\parallel\pi$, $\vec{M_1M_3}\parallel\pi$

$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_2 & z_3-z_2\end{vmatrix}=0$

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}-1=0$

a,b,c tăieturi