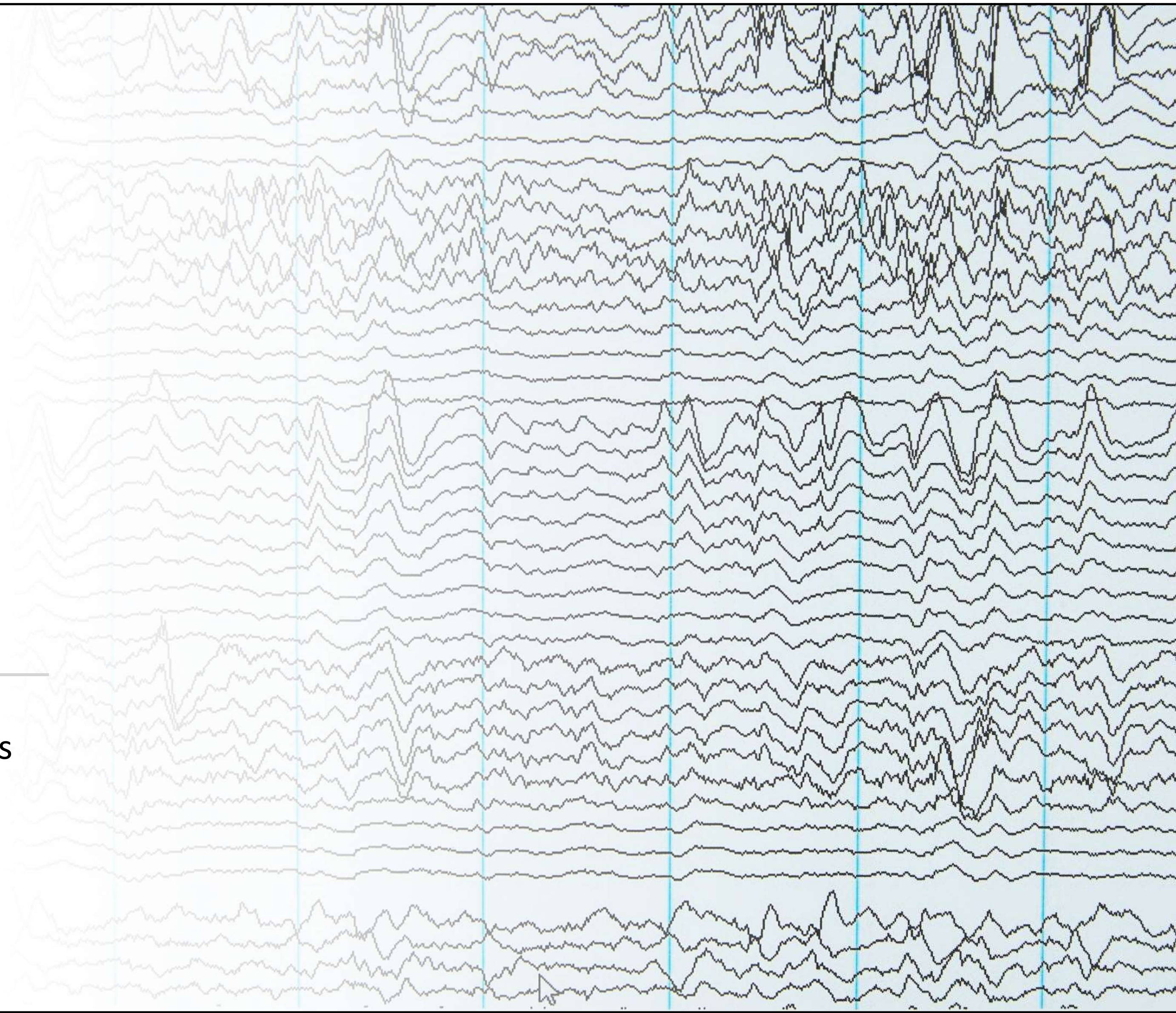
# Phase 3

Analysis of the effect of parameters on the system



# Our team members

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## Chaotic dynamics of the finance model

- At first, we plot the time response for different parameters for our Chaotic system, shown in equation (1), and examine the influence of its parameters. After that, we do this for our proposed system again.

$$\omega(t) = \begin{cases} \dot{\omega}_1(t) = [\omega_2(t) - \eta_1]\omega_1(t) + \omega_3(t) \\ \dot{\omega}_2(t) = 1 - \eta_2\omega_2(t) - \omega_1^2(t) \\ \dot{\omega}_3(t) = -\eta_3\omega_3(t) - \omega_1(t) \end{cases} \tag{1}$$

# Parameters Definitions

- $\omega_1$ = interest rate
- $\omega_2$ = demand for investment
- $\omega_3$ = index price
- $\eta_1$ = savings amount
- $\eta_2$ = cost of each investment
- $\eta_3$ = flexibility of commercial market demand = the ratio of changes in the demand of a product to changes in its price
- $\Lambda_i$ = time-varying exogenous disturbances acting on the CFM
- $u_i$ = control input vector
- $\alpha_i,\ \beta_i, \gamma_i$ = feedback controller gains
- $\sigma$ = controller parameter

# Fixed Points

$$\omega_{e1} = \left[0, \frac{1}{\eta_2}, 0\right], \omega_{e2,3}$$

$$= \left[\pm\sqrt{1 - \eta_1\eta_2 - \frac{\eta_2}{\eta_3}}, \left(\eta_1 + \frac{1}{\eta_3}\right), \mp\frac{1}{\eta_3}\sqrt{1 - \eta_1\eta_2 - \frac{\eta_2}{\eta_3}}\right]$$

# Impact of the parameter $n_1$

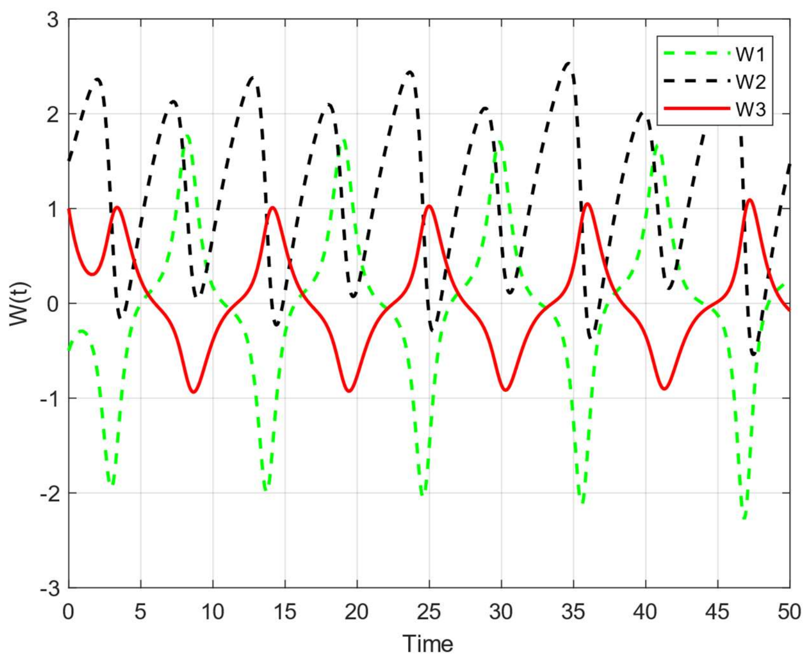
- Initial conditions:

  $w_1(0) = -0.5$, $W_2(0) = 1.5$, $W_3(0) = 1$;

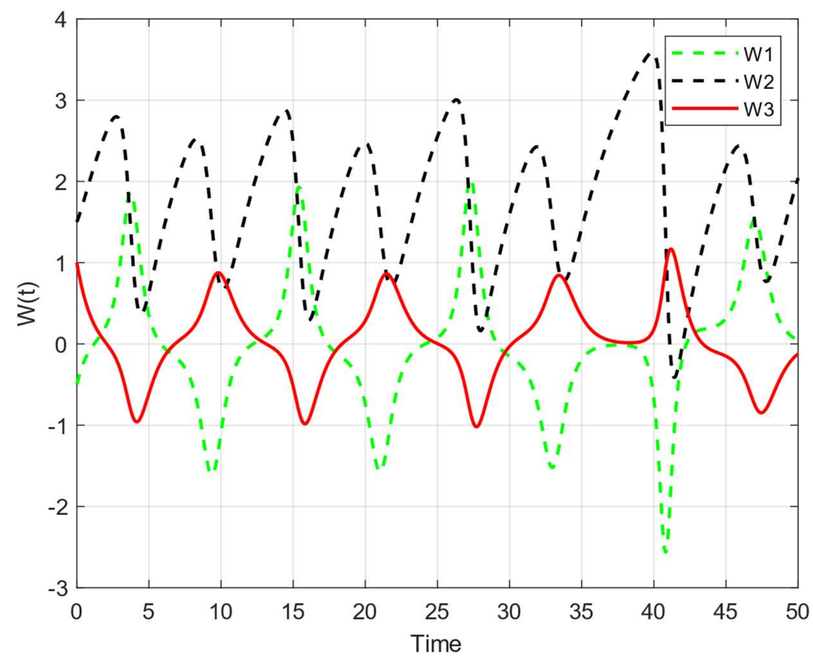
- $n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 1.5$

- Fixed points =
- [0, 5, 0] ,
- [sqrt(0.87 - 0.2$n_1$), $n_1$+ 0.6667, -0.6667sqrt(0.87 - 0.2$n_1$)],
- [-sqrt(0.87 - 0.2$n_1$), $n_1$+ 0.6667, 0.6667sqrt(0.87 - 0.2$n_1$)]

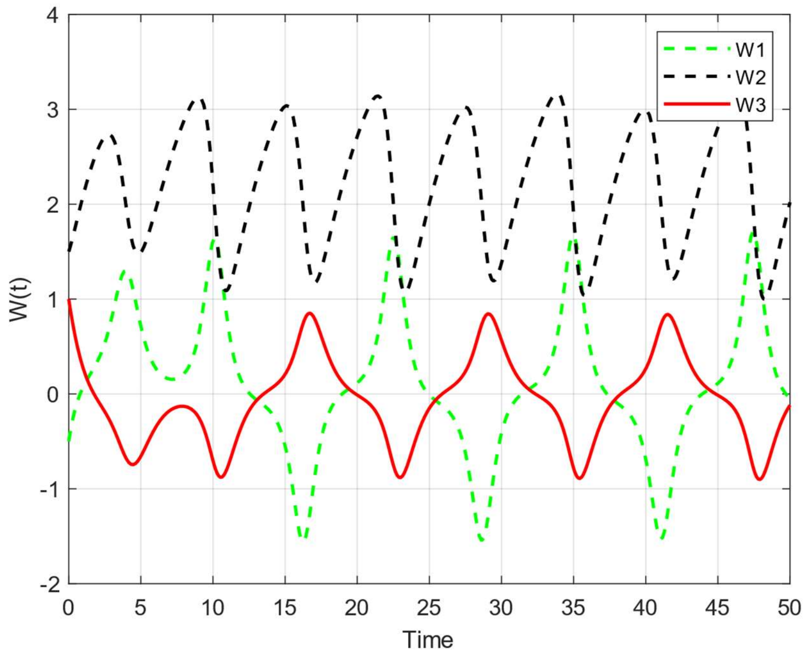
- All parameters are fixed except $n_1$. We vary $n_1$ at each step to observe its effect on the behavior of the system. If we choose $n_1 > 4.35$, then we have one fixed point. [0, 5, 0]



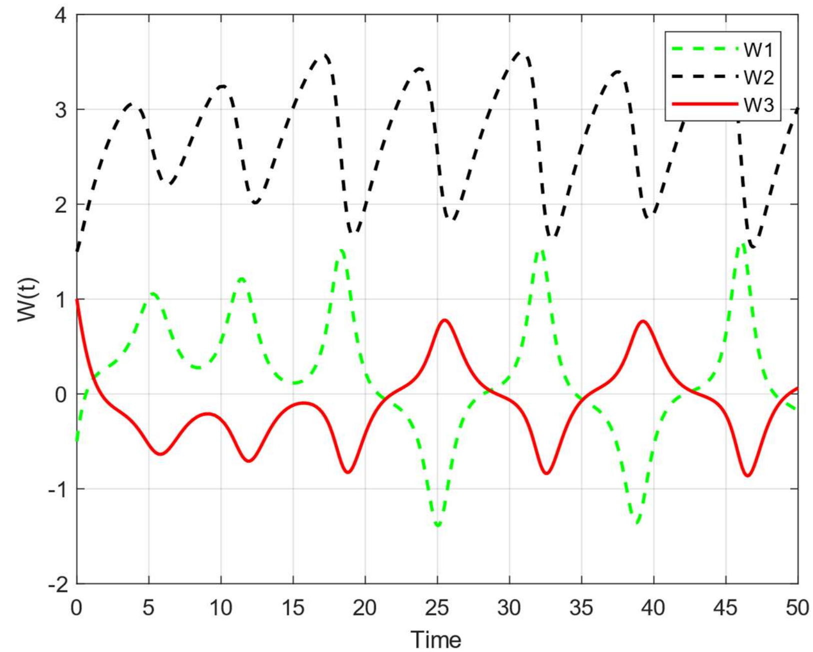

$n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 1.5$



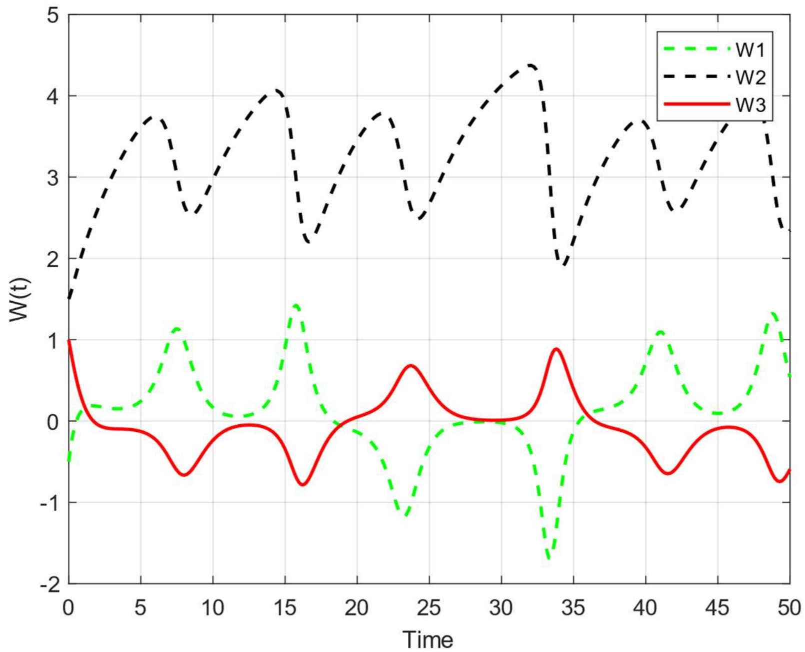

$n_1 = 1$, $n_2 = 0.2$, $n_3 = 1.5$

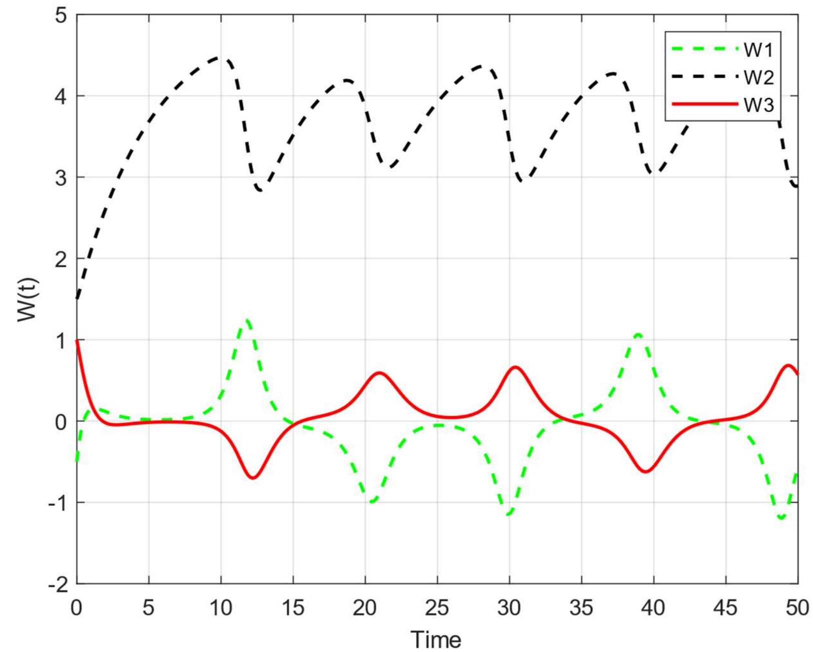



$n_1 = 1.5$, $n_2 = 0.2$, $n_3 = 1.5$



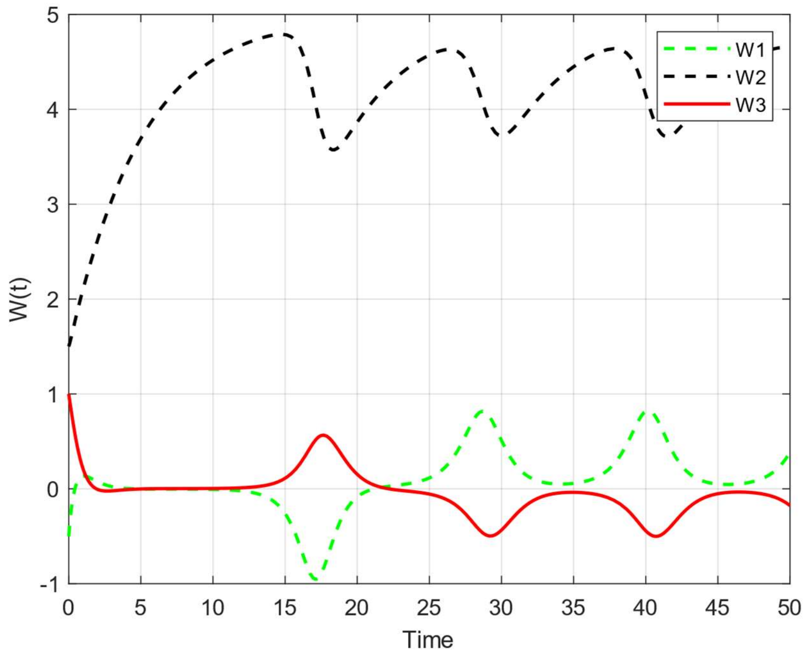

$n_1 = 2$, $n_2 = 0.2$, $n_3 = 1.5$



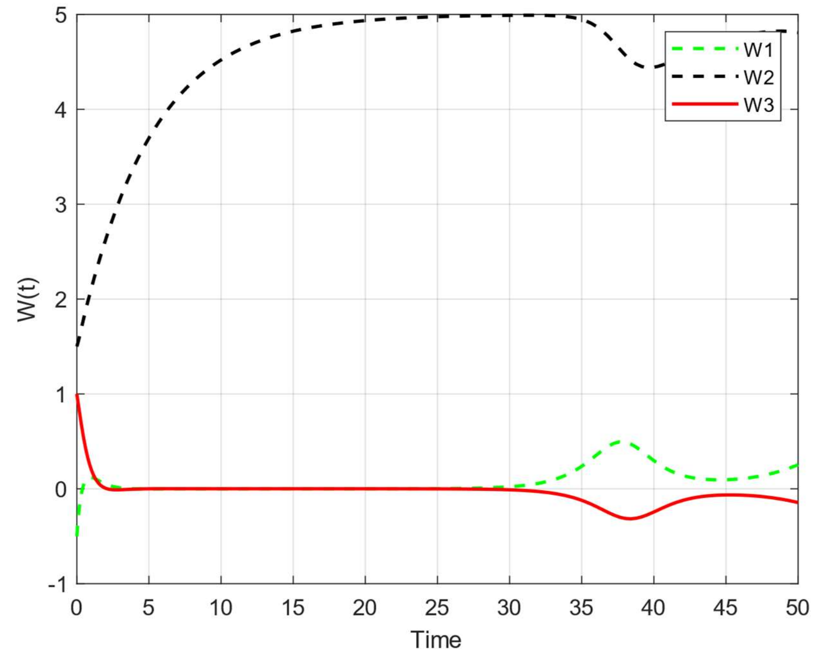




$n_1 = 2.5$, $n_2 = 0.2$, $n_3 = 1.5$

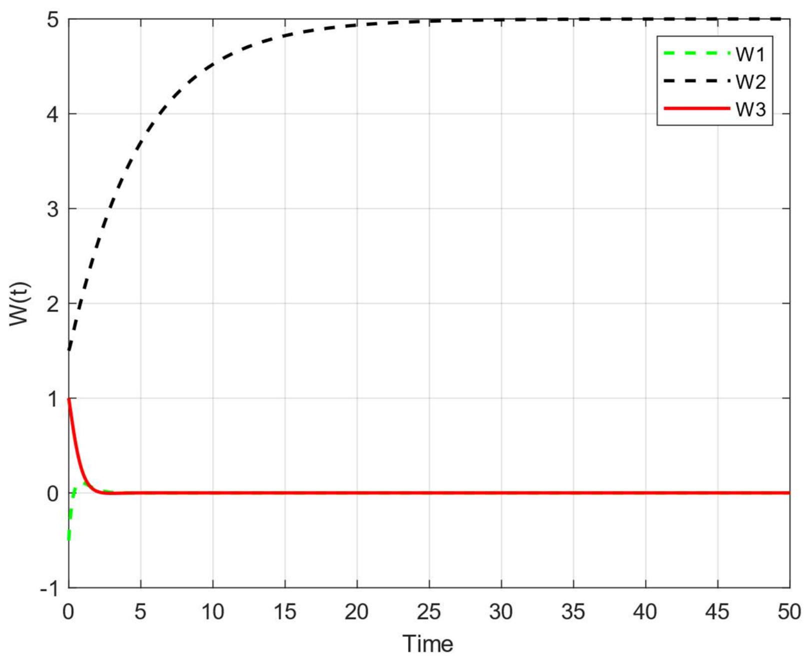
$n_1 = 3$, $n_2 = 0.2$, $n_3 = 1.5$



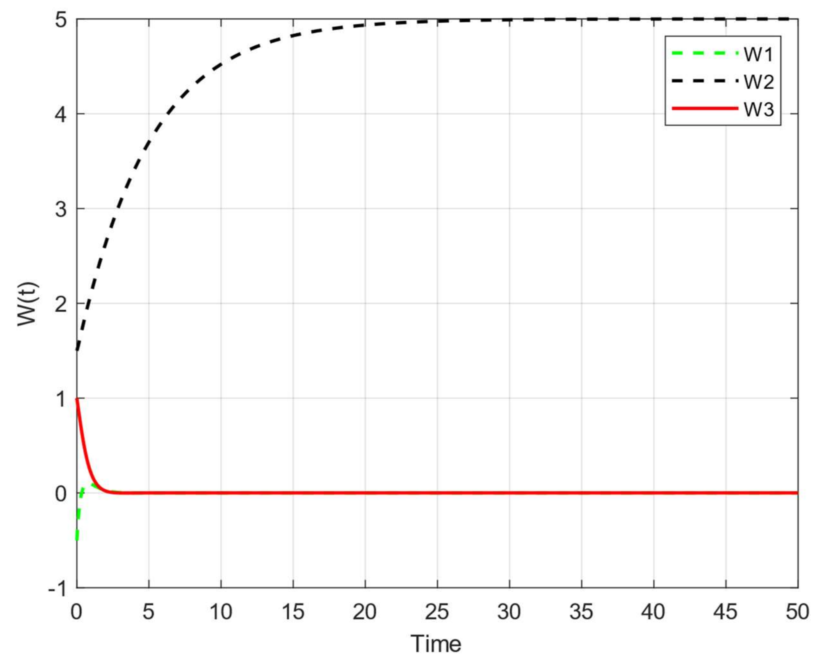

$n_1 = 3.5$, $n_2 = 0.2$, $n_3 = 1.5$



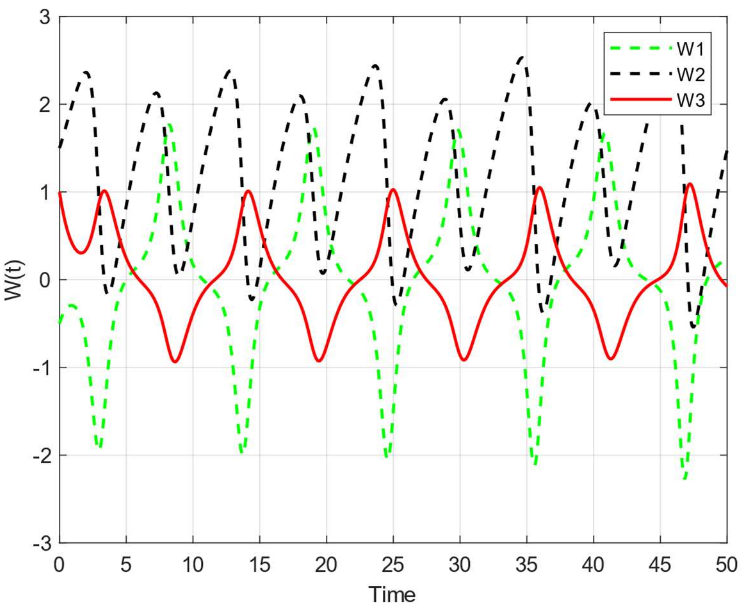

$n_1 = 4$, $n_2 = 0.2$, $n_3 = 1.5$

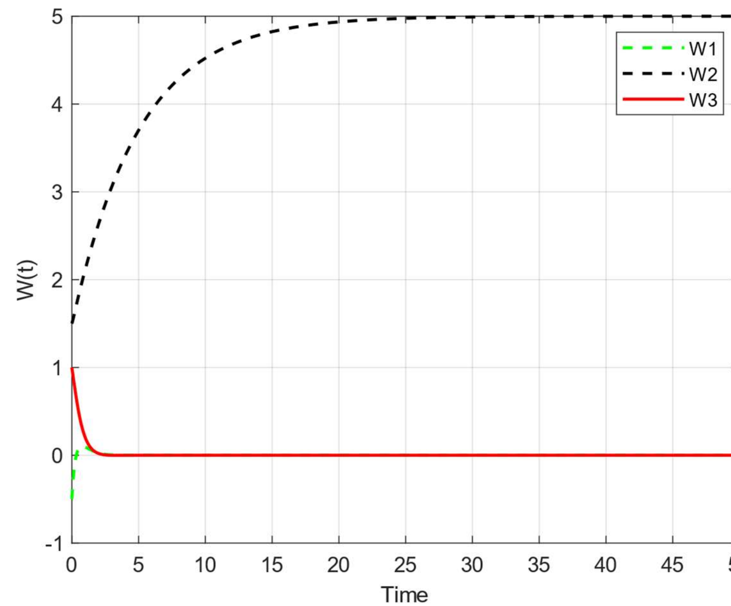



$n_1 = 4.5$, $n_2 = 0.2$, $n_3 = 1.5$




$n_1 = 5$, $n_2 = 0.2$, $n_3 = 1.5$




As it can be seen, the system has a limit cycle at small $n_1$, but as we increase $n_1$, the range of changes decreases and from some point the system becomes damped (n1 = 3.4) and the more we increase it The rate of damping increases and gradually tends to the fixed point [0, 5, 0], because as we increase $n_1$, the term under the radical becomes smaller in the other 2 fixed points, and finally, when $n_1$ is more From 4.37, since the radical becomes negative, these 2 fixed points are removed and only one fixed point [0, 5, 0] remains, which is an absorbing point.

# Impact of the parameter $n_2$

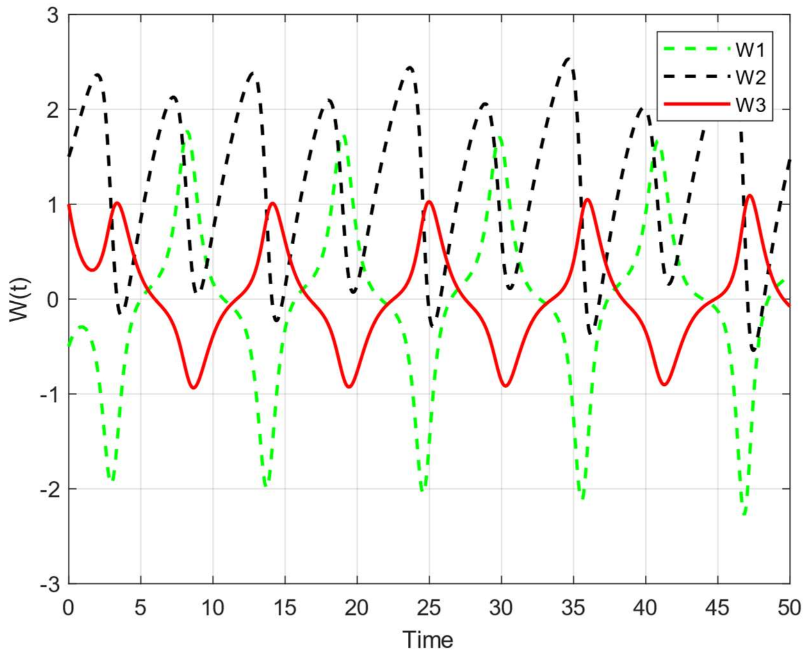
- Initial conditions:

  $w_1(0) = -0.5$, $W_2(0) = 1.5$, $W_3(0) = 1$;

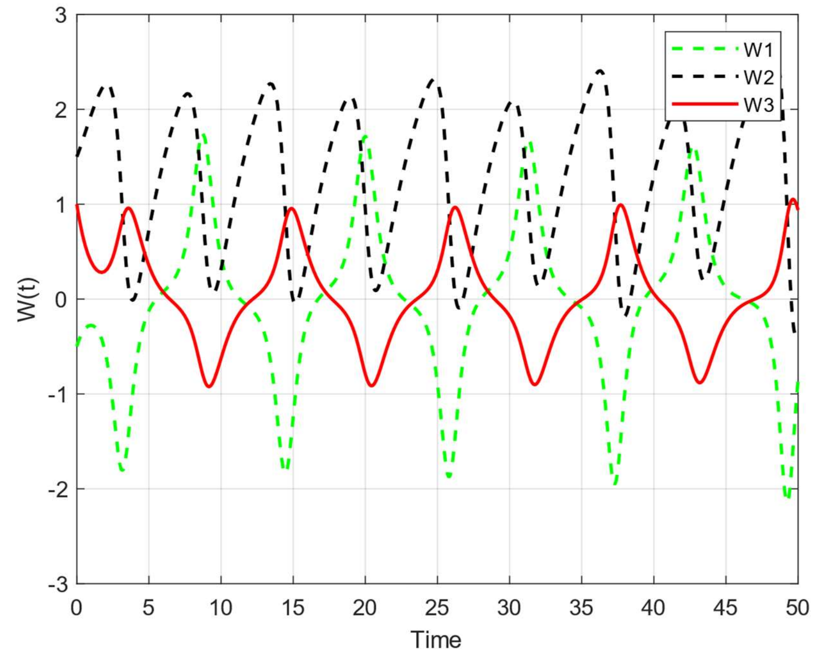
- $n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 1.5$

- Fixed points =
- $[0, 1/n_2, 0]$ ,
- $[\mathrm{sqrt}(1 - 0.5n_2 - n_2/1.5), 0.1667, -0.6667\mathrm{sqrt}(1 - 0.5n_2 - n_2/1.5)]$,
- $[-\mathrm{sqrt}(1 - 0.5n_2 - n_2/1.5), 0.1667, 0.6667\mathrm{sqrt}(1 - 0.5n_2 - n_2/1.5)]$

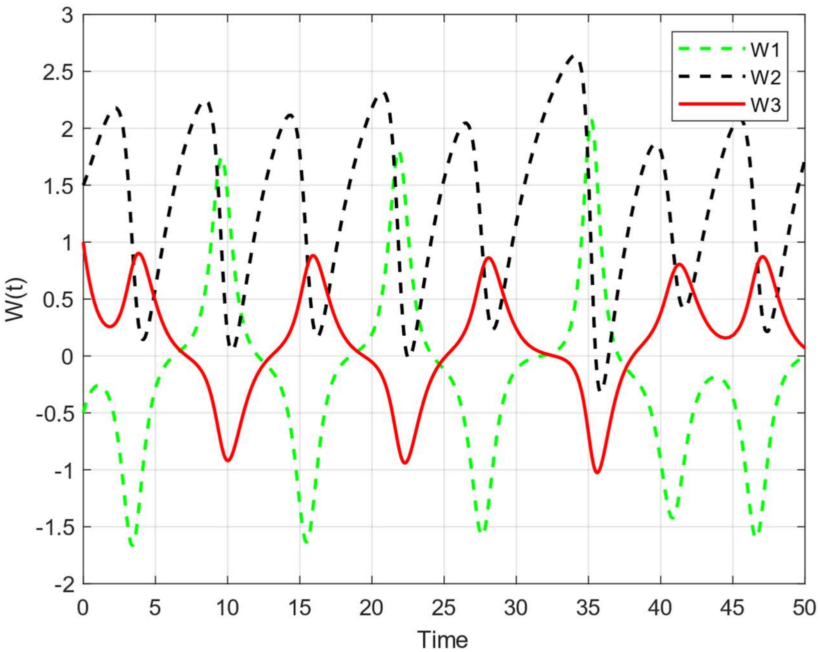
- All parameters are fixed except $n_2$. We vary $n_2$ at each step to observe its effect on the behavior of the system. If we choose $n_2 > 0.858$, then we have one fixed point. $[0, 1/n_2, 0]$



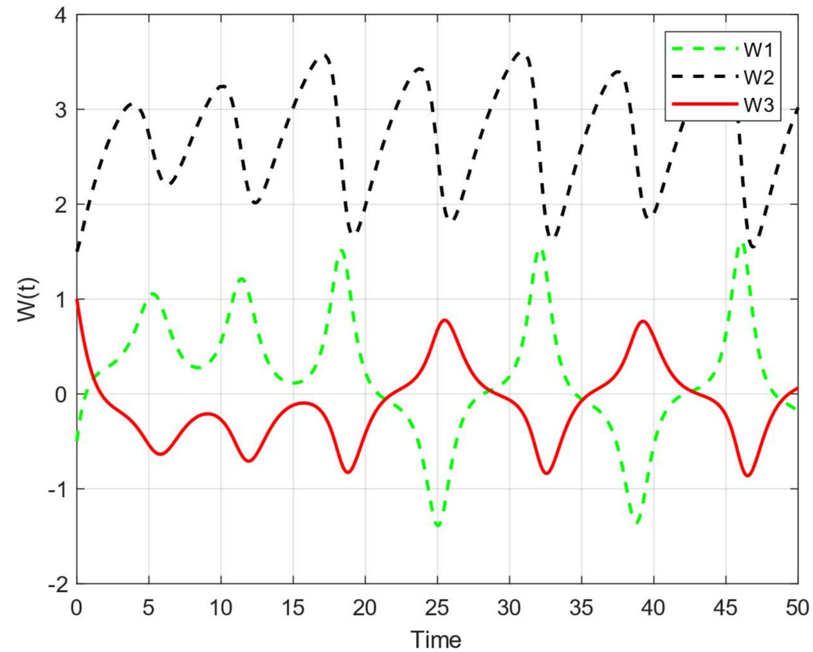

$n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 1.5$



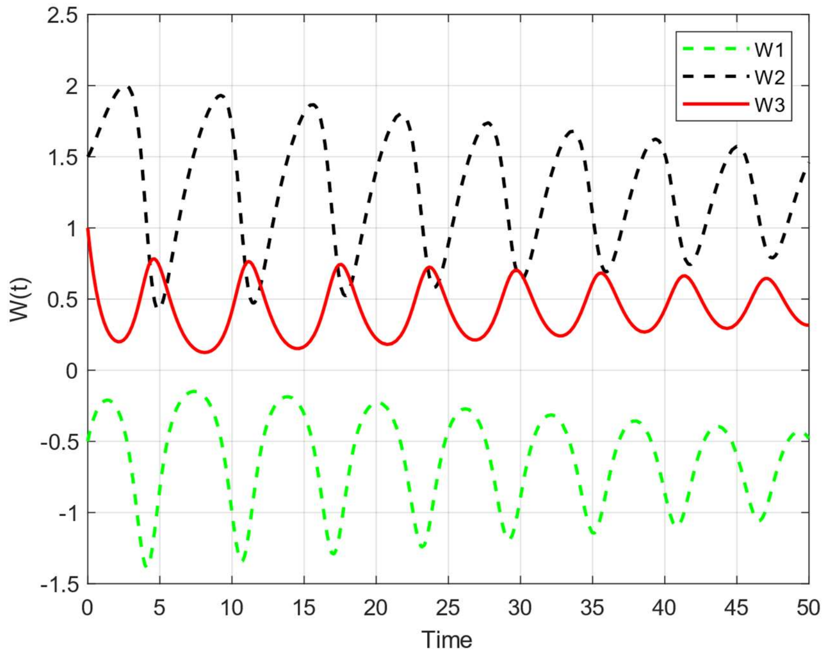

$n_1 = 0.5$, $n_2 = 0.25$, $n_3 = 1.5$



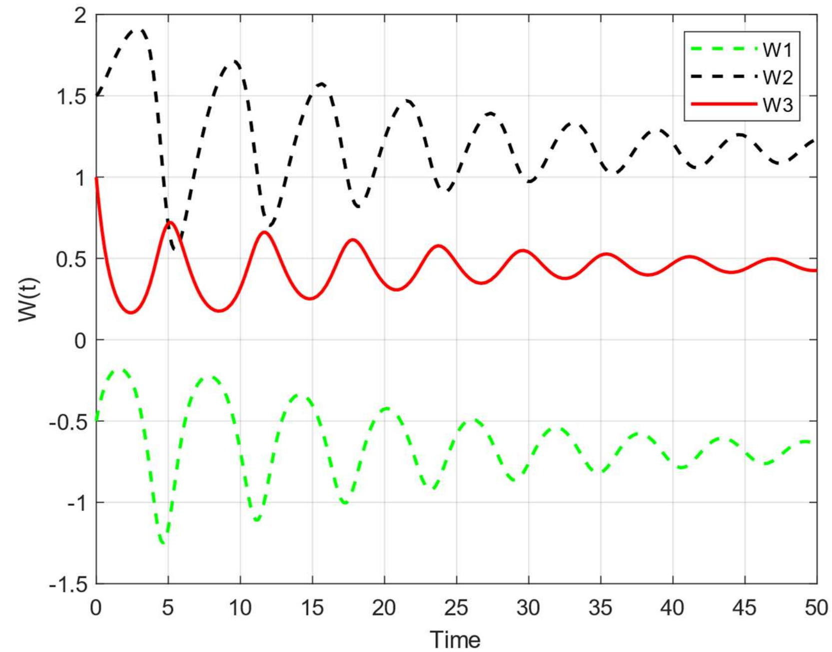

$n_1 = 0.5$, $n_2 = 0.3$, $n_3 = 1.5$



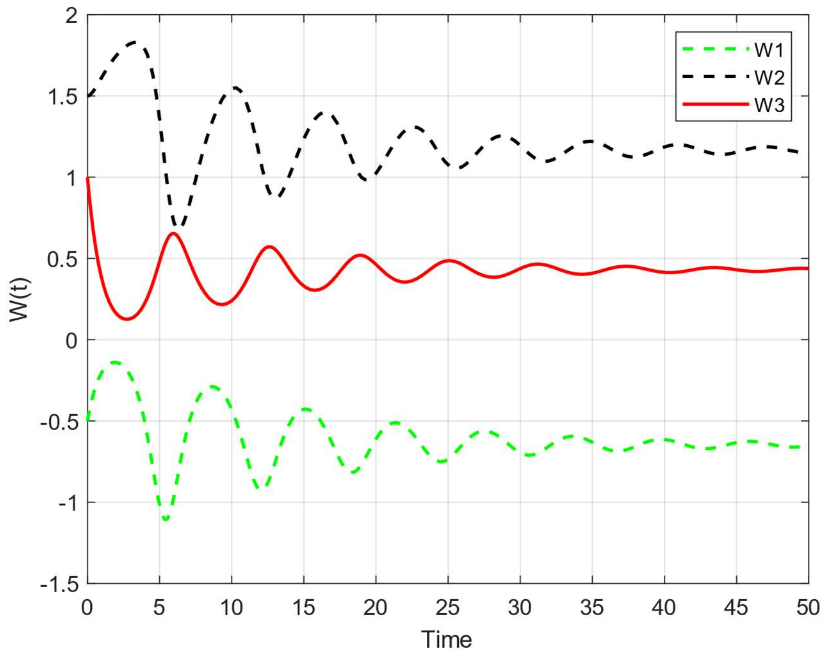

$n_1 = 0.5$, $n_2 = 0.35$, $n_3 = 1.5$



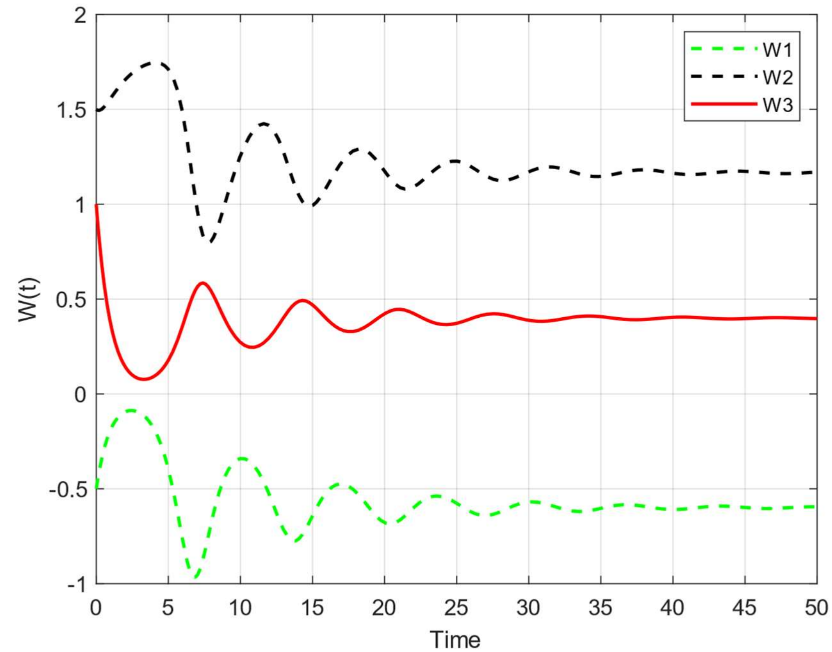

$n_1 = 0.5$, $n_2 = 0.4$, $n_3 = 1.5$



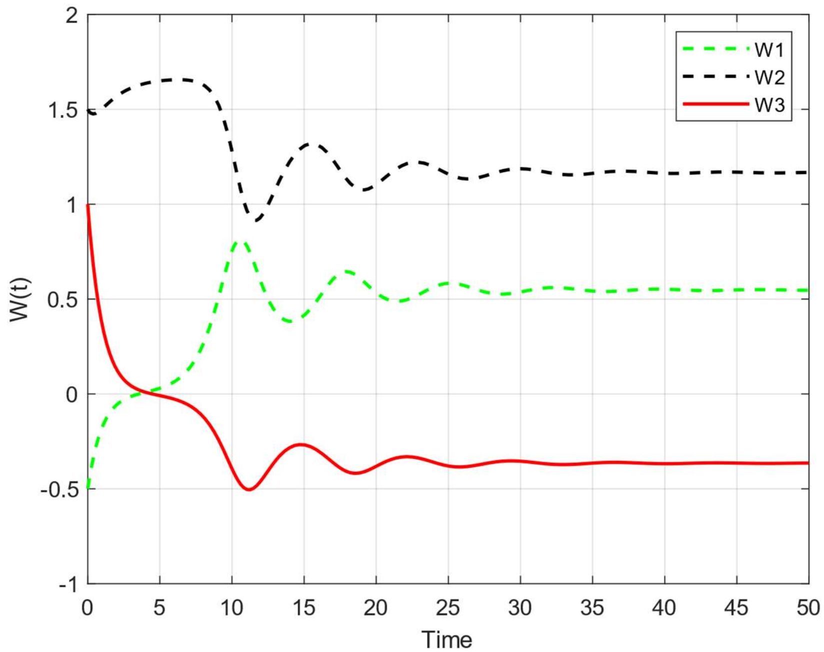

$n_1 = 0.5$, $n_2 = 0.45$, $n_3 = 1.5$



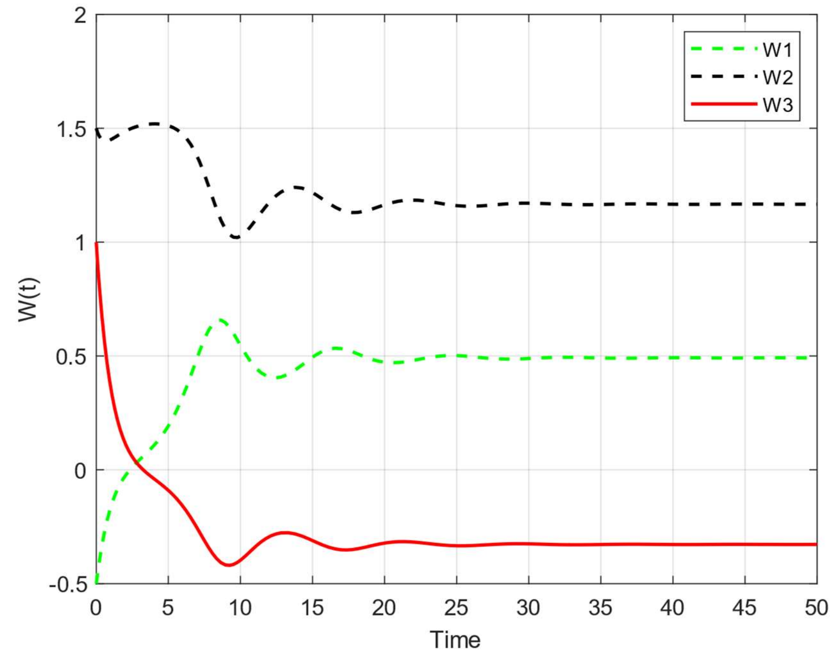

$n_1 = 0.5$, $n_2 = 0.5$, $n_3 = 1.5$



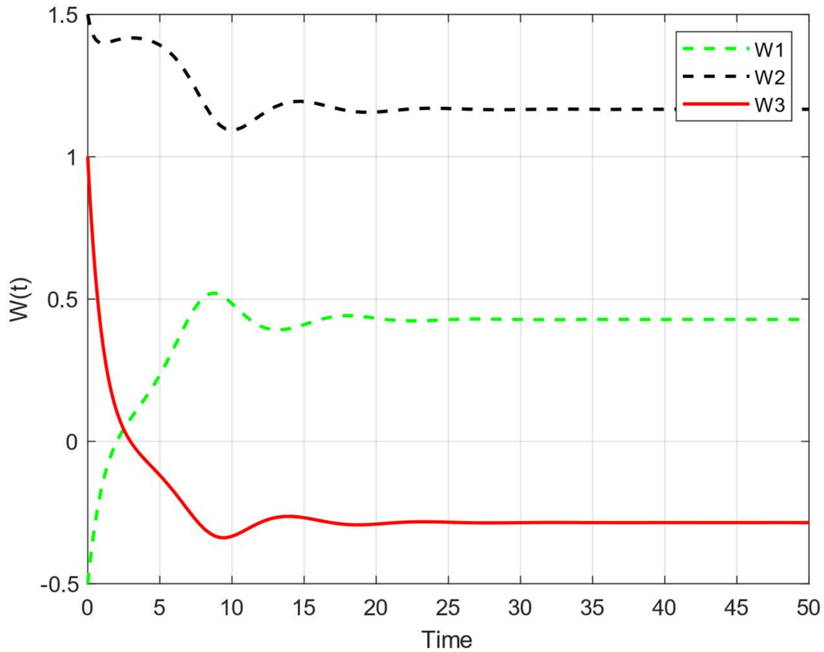

$n_1 = 0.5$, $n_2 = 0.55$, $n_3 = 1.5$



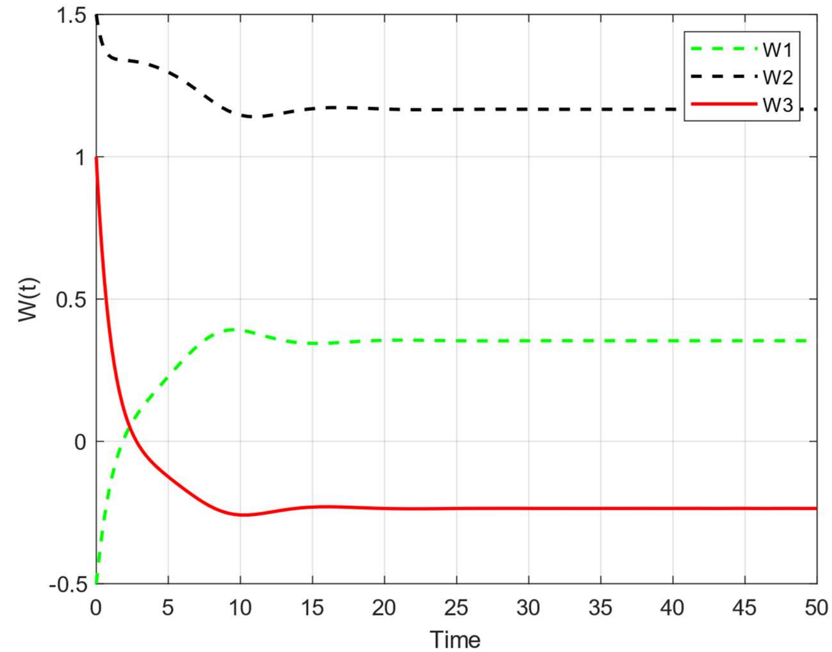

$n_1 = 0.5$, $n_2 = 0.6$, $n_3 = 1.5$



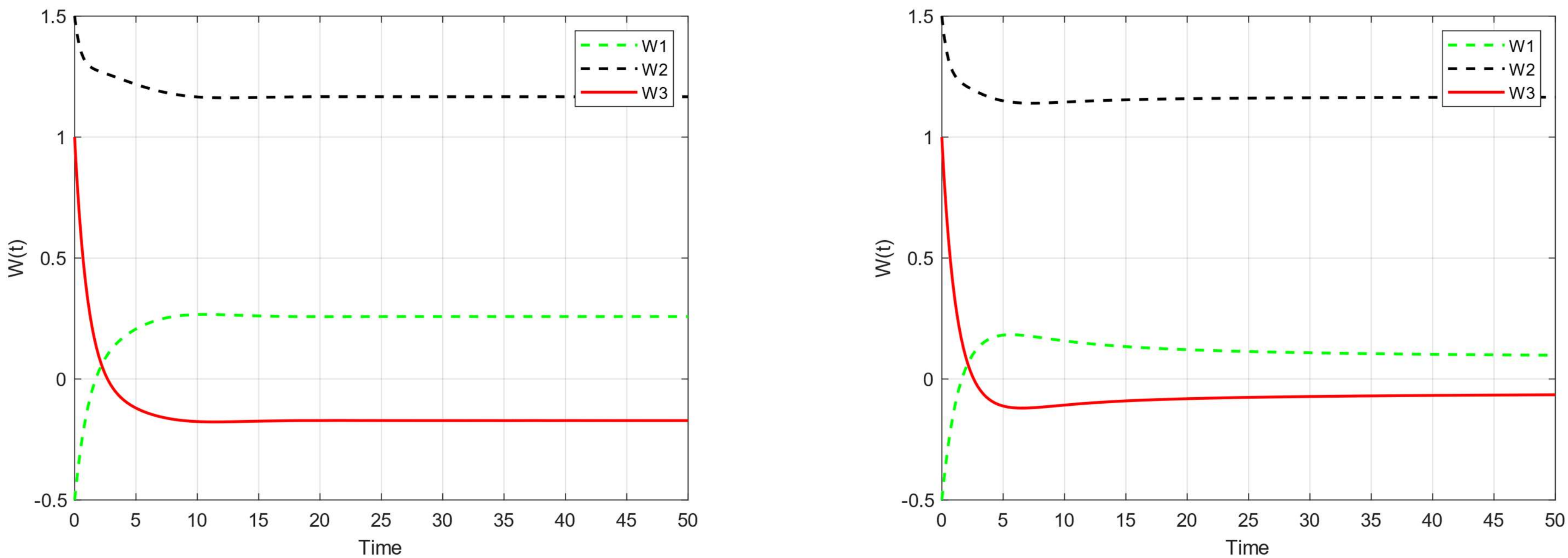

$n_1 = 0.5$, $n_2 = 0.65$, $n_3 = 1.5$




$n_1 = 0.5$, $n_2 = 0.7$, $n_3 = 1.5$

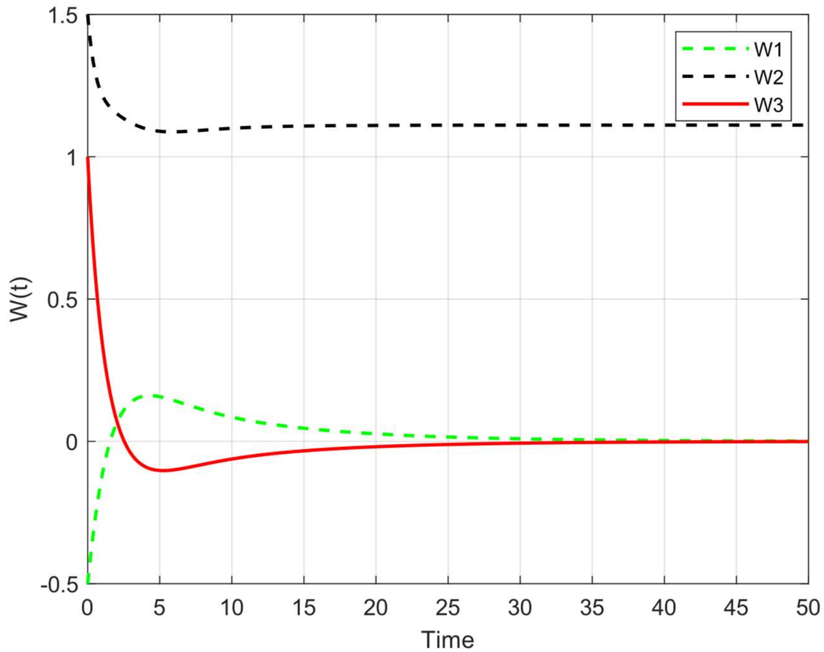



$n_1 = 0.5$, $n_2 = 0.75$, $n_3 = 1.5$

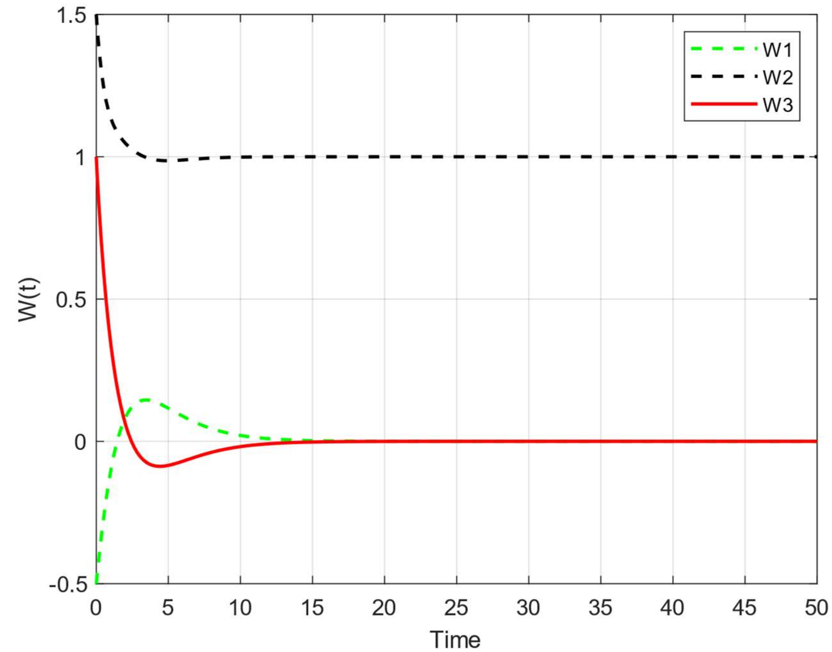



**$n_1 = 0.5$, $n_2 = 0.8$, $n_3 = 1.5$**

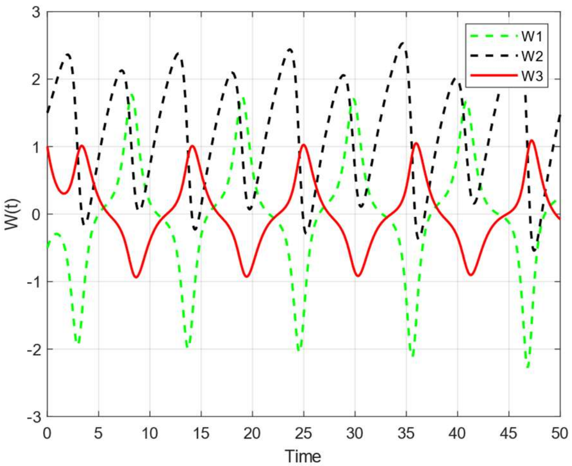
**$n_1 = 0.5$, $n_2 = 0.85$, $n_3 = 1.5$**



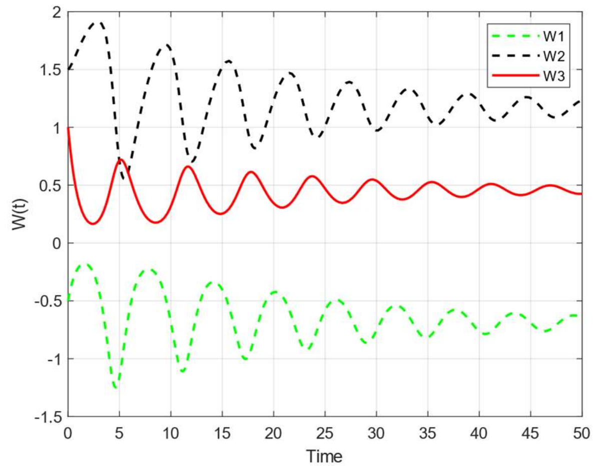

$n_1 = 0.5$, $n_2 = 0.9$, $n_3 = 1.5$



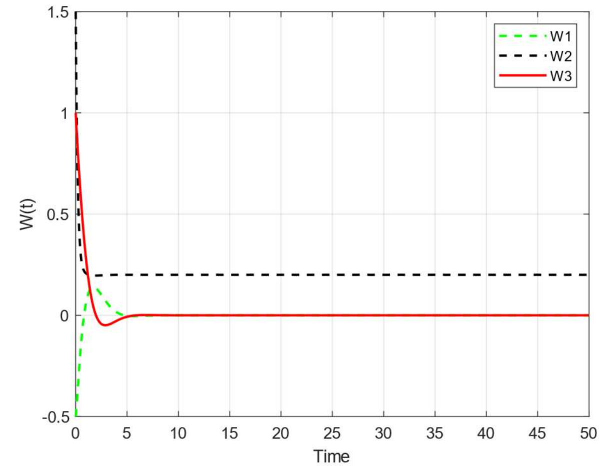

$n_1 = 0.5$, $n_2 = 1$, $n_3 = 1.5$




**$n_1$ = 0.5, $n_2$ = 0.2, $n_3$ = 1.5**




**$n_1$ = 0.5, $n_2$ = 0.45, $n_3$ = 1.5**




**$n_1$ = 0.5, $n_2$ = 5, $n_3$ = 1.5**

As it is clear from the graphs, for small $n_2$, the system has a limit cycle and when its value is greater than a certain value (0.372 here according to the rest of the parameters), the system is damped and its phase diagram is It spirals. If we make n2 bigger, the damping speed will increase, and if we set it higher than 0.858, due to the negation of the term under the radical, 2 fixed points will be removed and we will have only one absorbing point. Of course, we should also consider that our fixed points change with the change of n2 value. If $n_2$ goes to infinity, the system will have an absorbing point in [0, 0, 0].

# Impact of the parameter $n_3$

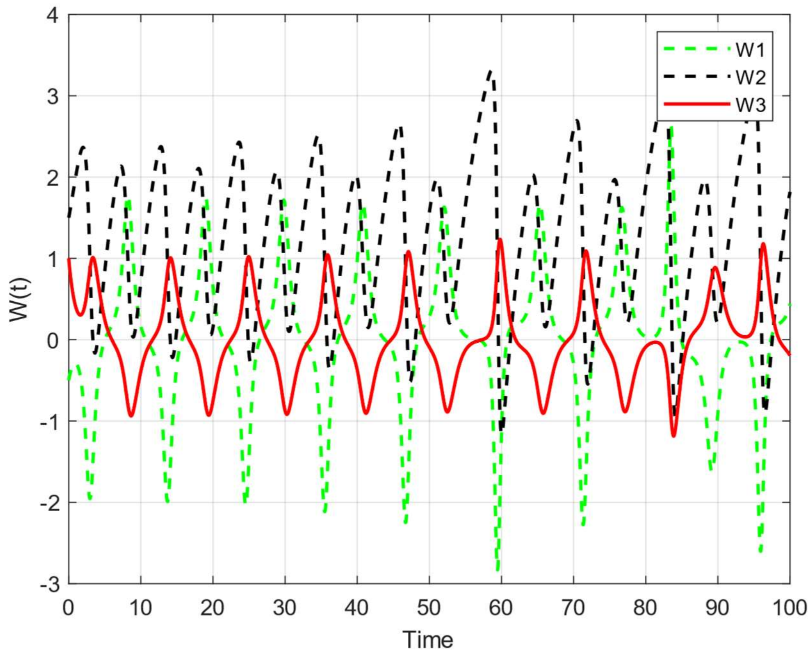
- Initial conditions:

  $w_1(0) = -0.5$, $W_2(0) = 1.5$, $W_3(0) = 1$;

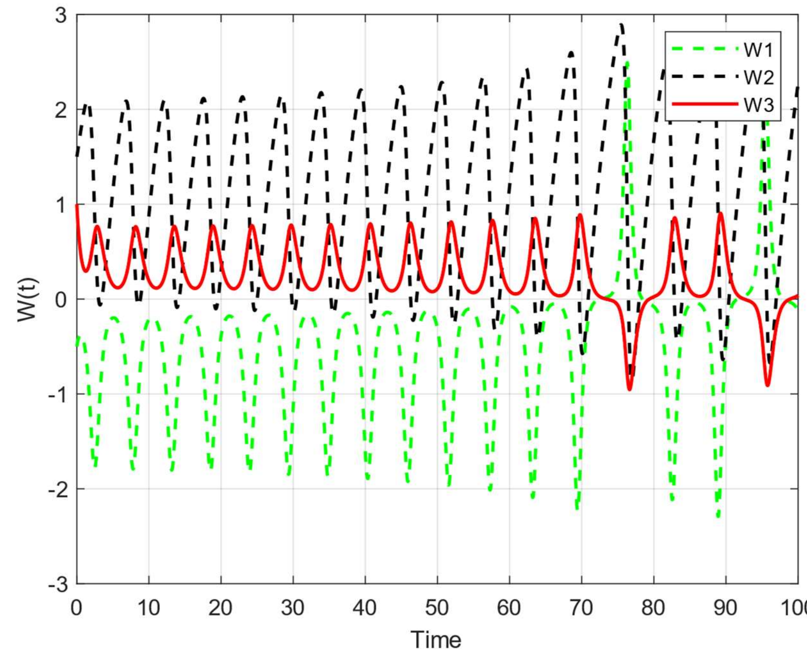
- $n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 1.5$

- Fixed points =
- [0, 5, 0] ,
- [sqrt($0.9 - 0.2/n_3$), $0.5 + 1/n_3$ , - $1/n_3$ sqrt($0.9 - 0.2/n_3$)],
- [-sqrt($0.9 - 0.2/n_3$), $0.5 + 1/n_3$ , $1/n_3$ sqrt($0.9 - 0.2/n_3$)]

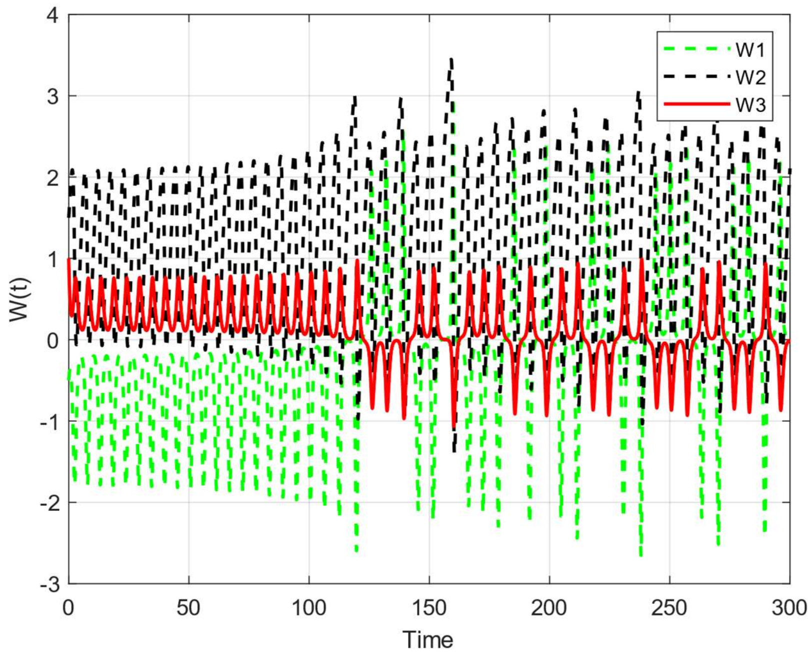
- All parameters are fixed except $n_3$. We vary $n_2$ at each step to observe its effect on the behavior of the system.



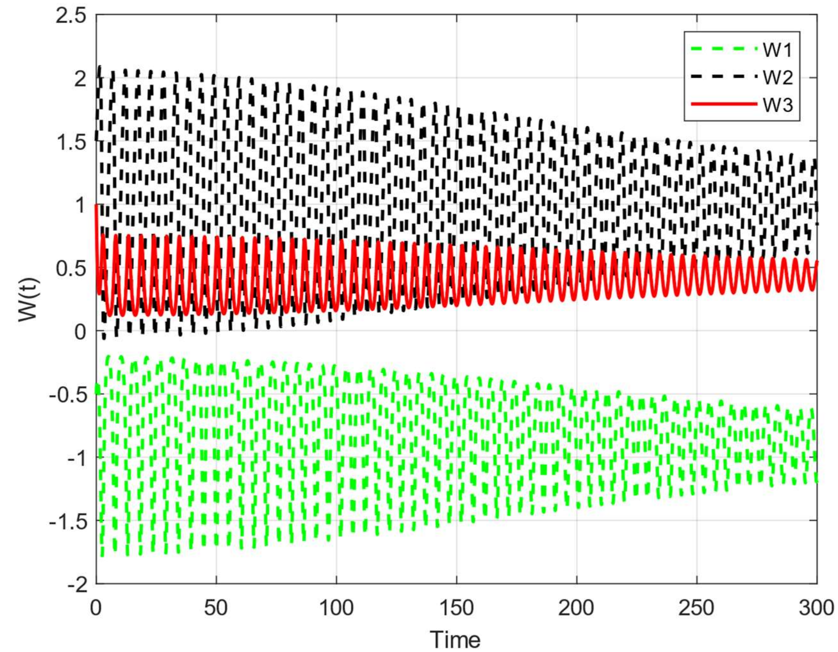

$n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 1.5$



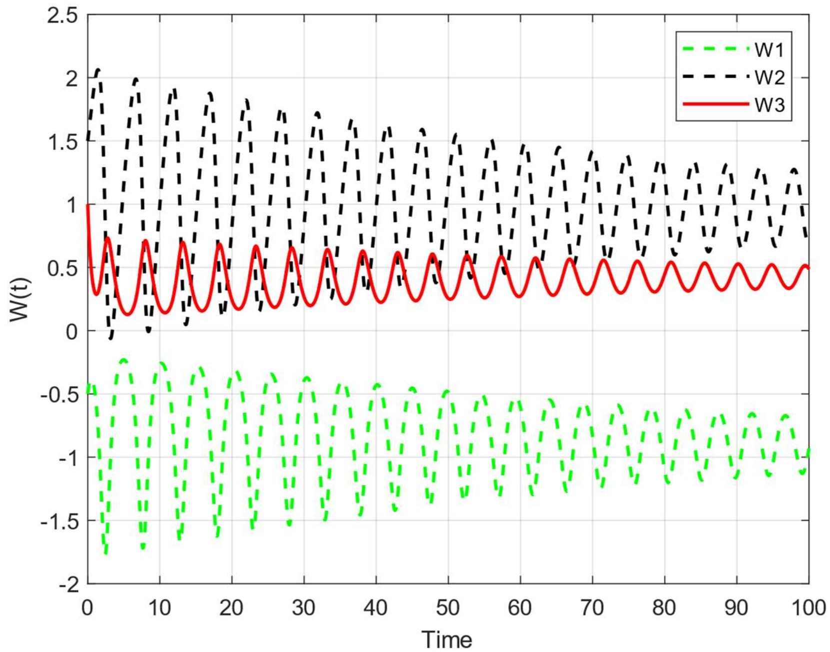

$n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 2$



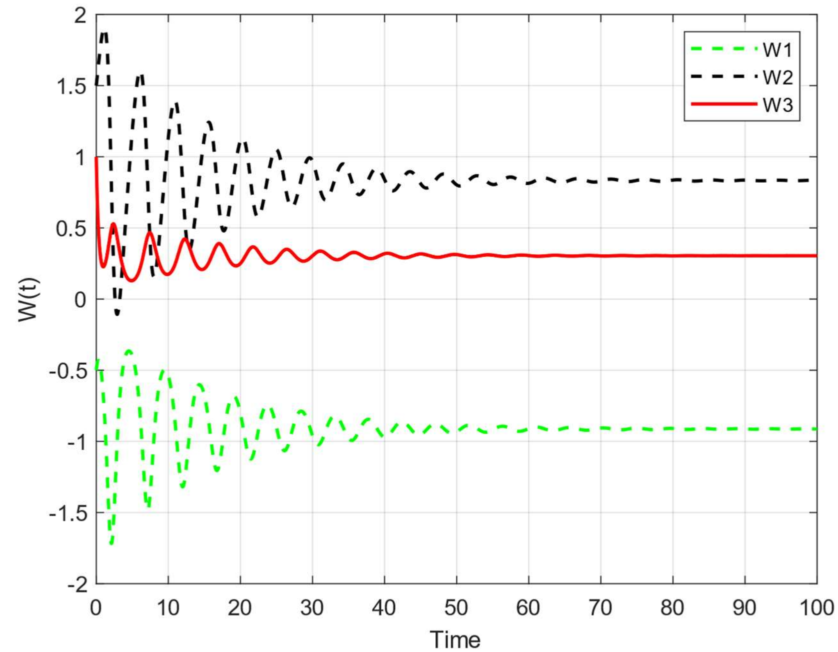

$n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 2.01$



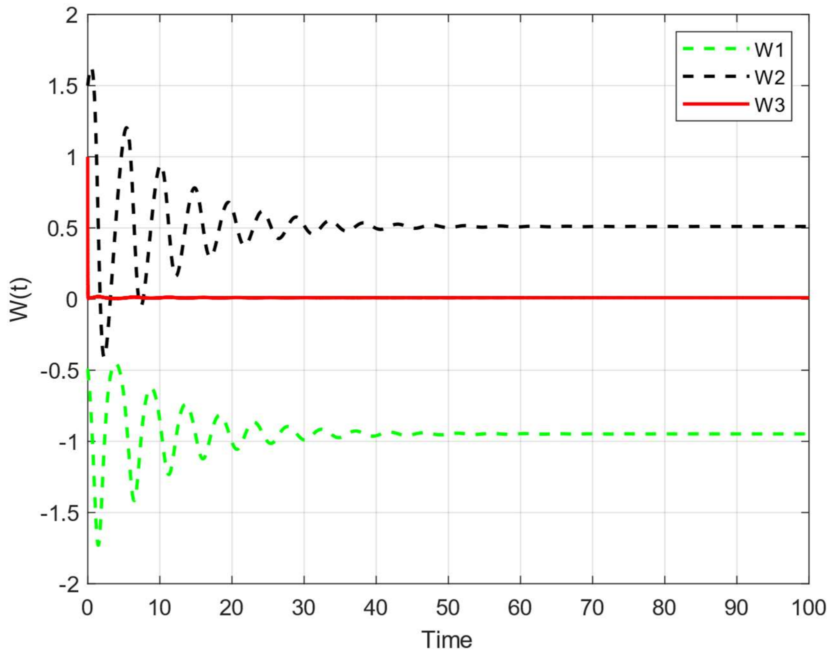

$n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 2.02$



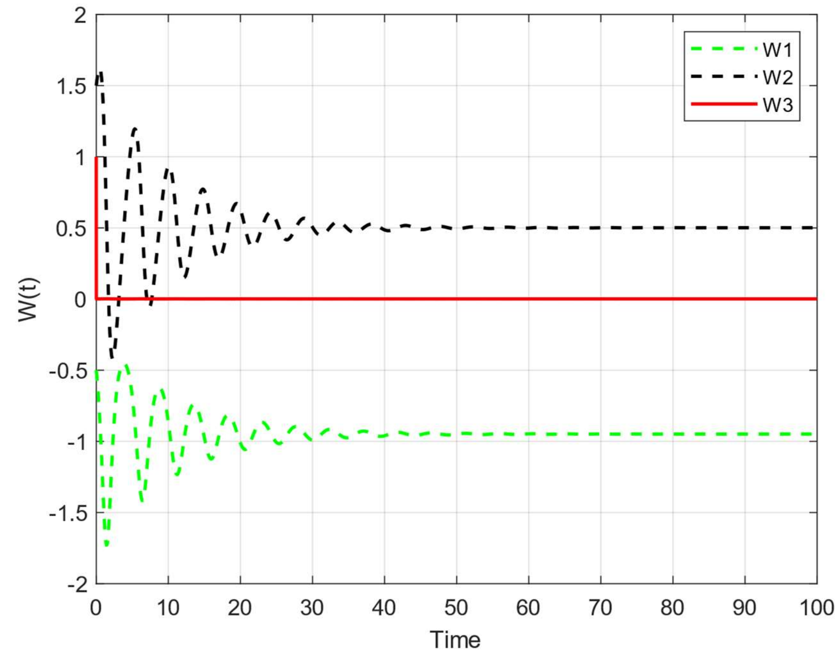

$n_1 = 0.5, n_2 = 0.2, n_3 = 2.1$



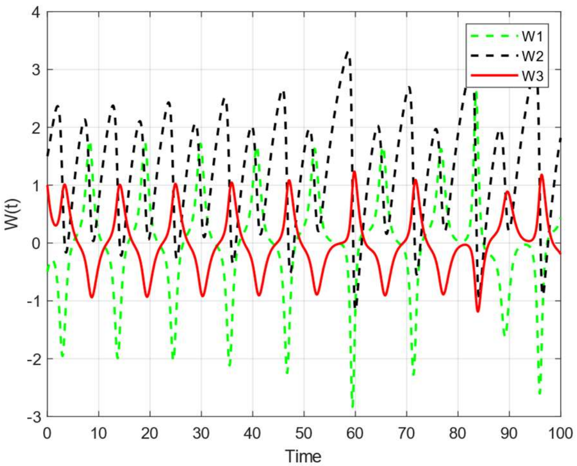

$n_1 = 0.5, n_2 = 0.2, n_3 = 3$



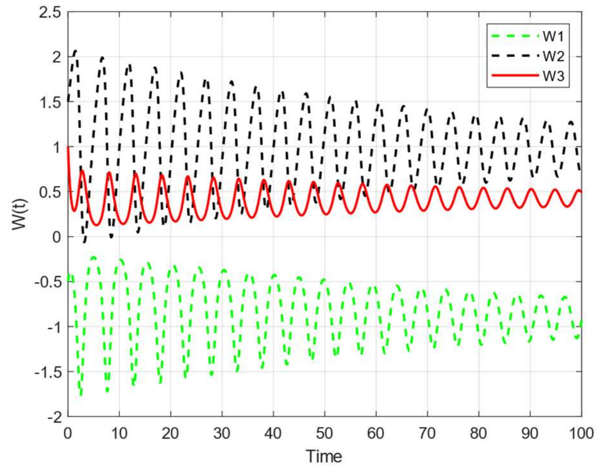

$n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 100$



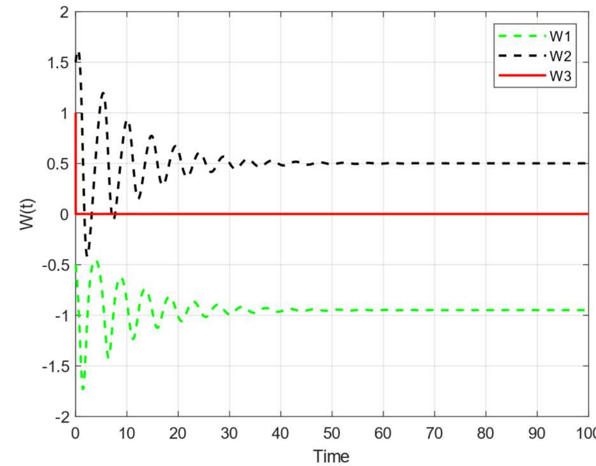

$n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 1000$




**$n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 1.5$**




**$n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 2.1$**




**$n_1 = 0.5$, $n_2 = 0.2$, $n_3 = 1000$**

As it is clear from the graphs, for small $n_3$, the system has a limit cycle and when its value is greater than a certain value (here according to the other parameters 2.01), the system is damped and its phase diagram is It spirals. If we increase $n_3$, the damping speed increases. Finally, if we take $n_3$ to infinity, the time response diagram of the system remains constant and looks almost like the figure on the right. Unlike the two parameters $n_2$ and $n_1$, increasing the $n_3$ parameter does not remove those 2 fixed points, and our system has 3 fixed points [0, 5, 0], [0.95, 0.5, 0] when $n_3$ goes to infinity. , [-0.95, 0.5, 0] where the points [-0.95, 0.5, 0], [0.95, 0.5, 0] are absorbers.

**Proposed System**

When exogenous disturbances $\Lambda_i(t)$ and control efforts $u_i(t)$ act on the CFM (1), then (12) represents the closed-loop dynamics.

$$\begin{cases} \dot{\omega}_1(t) = [\omega_2(t) - \eta_1]\omega_1(t) + \omega_3(t) + \Lambda_1(t) + u_1(t) \\ \dot{\omega}_2(t) = 1 - \eta_2\omega_2(t) - \omega_1^2(t) + \Lambda_2(t) + u_2(t) \\ \dot{\omega}_3(t) = -\eta_3\omega_3(t) - \omega_1(t) + \Lambda_3(t) + u_3(t) \end{cases} \tag{12}$$

In this scenario, it is desired to design a feedback controller for synthesizing control effort u $\in R^{3*1}$ that forces the state of the closed-loop system (12) to the origin in finite time.

# Proposed finite-time controller

Eq. (14) introduces a new FTC design that stabilizes the closed-loop (12) at the origin in finite time.

$$\mathbf{u}(t) = \begin{cases} u_i(t) = -\alpha_i \psi_i(t)\omega_i(t) - (\Omega_i(t) + \varphi_i)\operatorname{sgn}(\omega_i(t)), i = 1, 3 \\ u_2(t) = -\alpha_2 \psi_2(t)\omega_2(t) - (\Omega_2(t) + \varphi_2)\operatorname{sgn}(\omega_2(t)) - 1, \end{cases} \tag{14}$$

where $\Omega(t) = diag[\gamma_i \psi_i(t) + \beta_i]$, where $\psi_i(t) = \mathrm{e}^{-\sigma|\omega_i(t)|}$.

If $\omega_i(t) > 0$, then $\operatorname{sgn}(\omega_i(t)) = 1$, if $\omega_i(t) < 0$, then $\operatorname{sgn}(\omega_i(t)) = -1$, and if $\omega_i(t) \neq 0$, then $\operatorname{sgn}(\omega_i(t)) = \frac{\omega_i(t)}{|\omega_i(t)|}$ for $i = 1, 2, 3$.

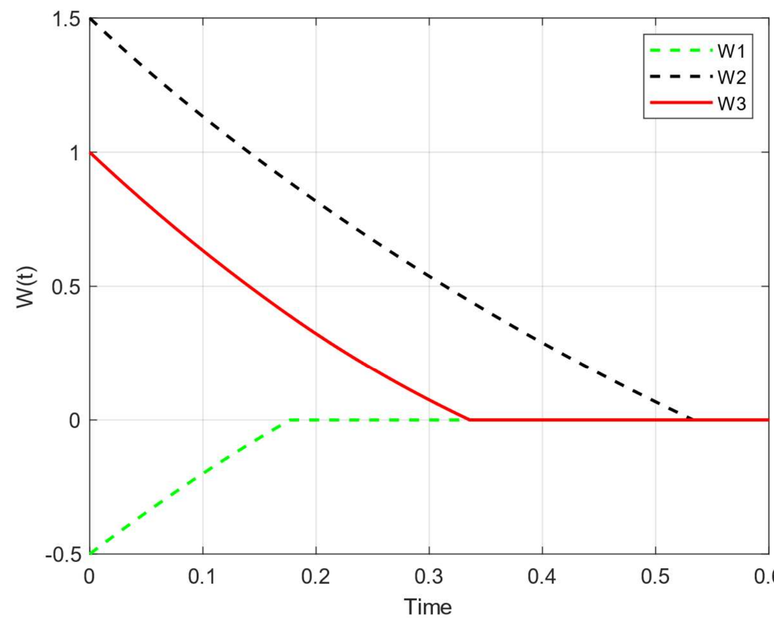
Items (i)-(iii) summarize the role of distinct components of the control input (14).

(i) $\alpha_i\psi_i(t)\omega_i(t)$ makes the closed-loop globally stable; it assures the converges of the state variables to the origin.
(ii) $(\gamma_i\psi_i(t) + \beta_i)\mathrm{sgn}(\omega_i(t))$ realizes oscillation free, smooth, and rapid convergence of the state variables to zero and establishes finite-time stabilization. The parameter $\sigma$ regulates the decay rate. The simulation results depicted in Fig. 6 validates it.
(iii) $\varphi_i\mathrm{sgn}(\omega_i(t))$ eradicates the effects of time-varying exogenous disturbances.

**Theorem 1.** *FTC (14) computes control effort $\mathbf{u}(t)$. It is an input signal to the CFM (12). Application of $\mathbf{u}(t)$ to the CFM (12) establishes convergence of the state variables vector $\omega(t)$ to the origin in finite-time $\tau$ as given in (15).*

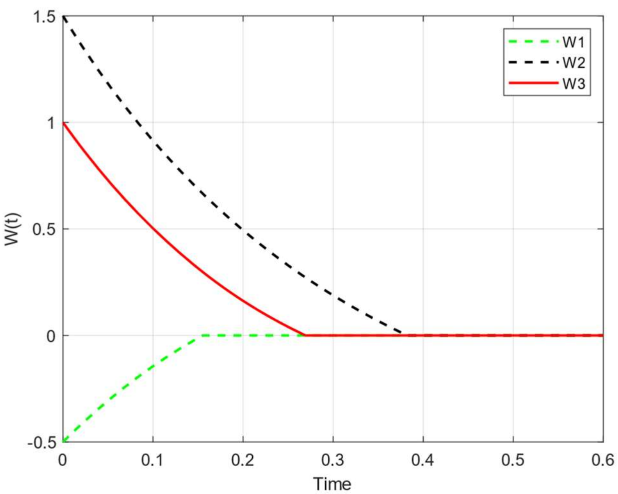
$$\tau \leq \tau_1 = \frac{\|\omega(0)\|_2}{\beta_m} \tag{15}$$

where $\beta_m = Minimum(\beta_i)$, and $0 < \tau \leq \tau_1 \in R^+$ represents the finite time. $\tau$ depends on the initial conditions $\omega(0) = [\omega_1(0), \omega_2(0), \omega_3(0)]^T$ and the controller parameter $\beta_m$.

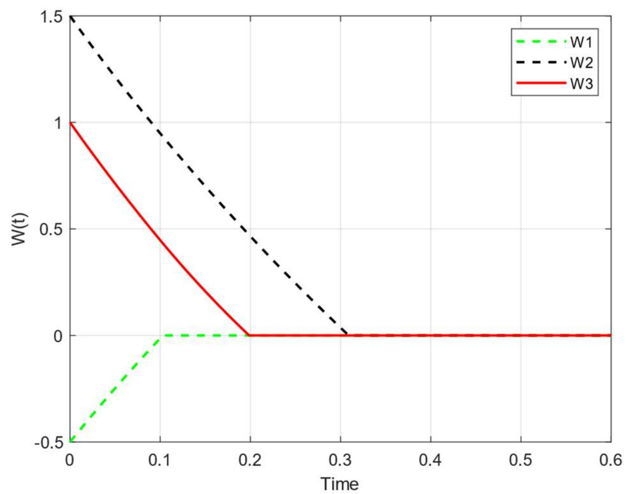



Initial conditions and parameters for simulations.

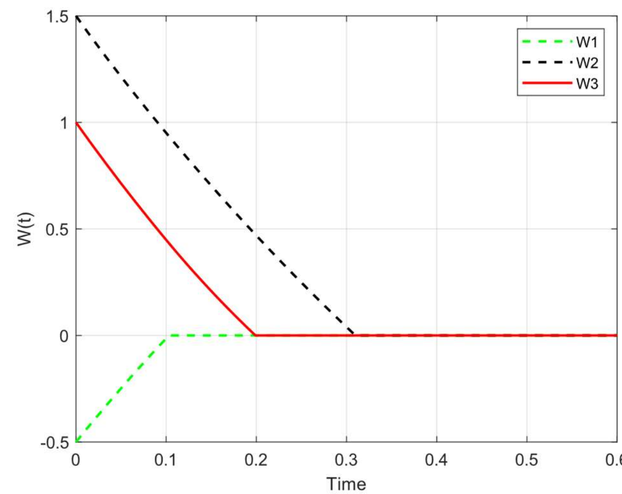
| Initial conditions | CFM parameters | Controller parameters |
|---|---|---|
| $\omega_1(0) = -0.5$ | $\eta_1 = 0.9$ | $\alpha_1 = \beta_1 = \gamma_1 = 1,\ \sigma = 0.01$ |
| $\omega_2(0) = 1.5$ | $\eta_2 = 0.2$ | $\alpha_2 = \beta_2 = \gamma_2 = 1,\ \sigma = 0.01$ |
| $\omega_3(0) = 1$ | $\eta_3 = 1.5$ | $\alpha_3 = \beta_3 = \gamma_3 = 1,\ \sigma = 0.01$ |



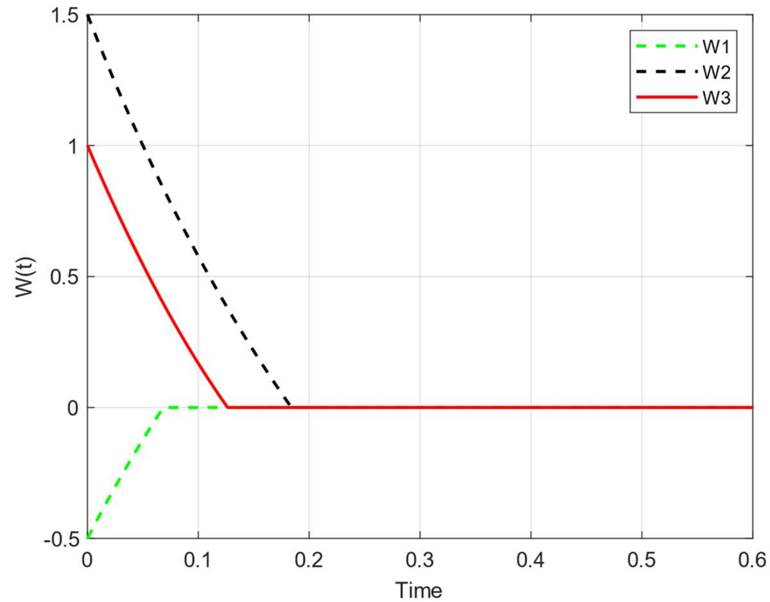

$$\boldsymbol{\beta_i = \gamma_i = 1}$$
$$\boldsymbol{\alpha_i = 3}$$




$$\boldsymbol{\alpha_i = \gamma_i = 1}$$
$$\boldsymbol{\beta_i = 3}$$




$$\boldsymbol{\alpha_i = \beta_i = 1}$$
$$\boldsymbol{\gamma_i = 3}$$




$$\boldsymbol{\alpha_i = \beta_i = \gamma_i = 3}$$
$$\boldsymbol{\Lambda = \varphi = 0}$$

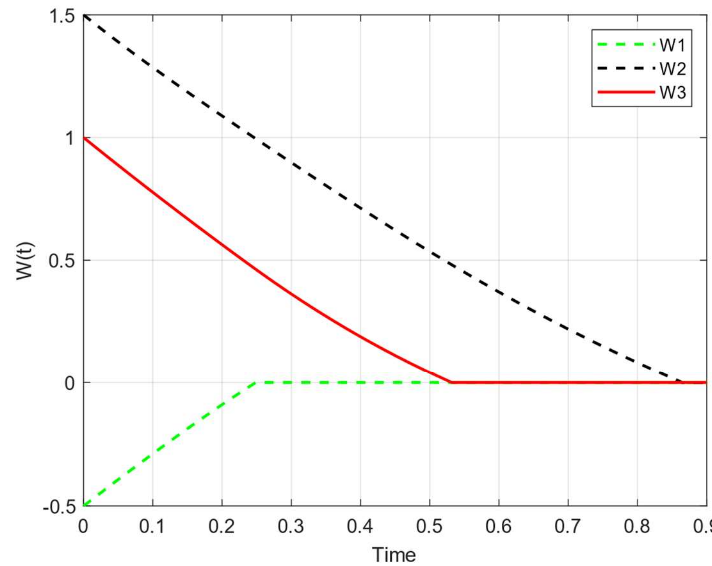
By increasing each of the parameters $\alpha_i = \beta_i = \gamma_i$ we see that the speed of convergence decreases. Meanwhile, here the parameters $\boldsymbol{\Lambda}$ and $\boldsymbol{\varphi}$ are considered zero.

The influence of the economic factors such as supply-demand, fluctuations in the prices, and cost of demand; the positive parameters $\eta_1$, $\eta_2$, and $\eta_3$ of the CFM (1) vary due to the variations in the financial market activities. Similarly, the exogenous disturbances caused by environmental interference influence the financial and economic chaotic models [34]; it may destabilize systems, resulting in undesirable behavior. Therefore, to analyze the robustness of the proposed FTC (14), this subsection discusses the convergence behavior of the state variable trajectories to the origin when the CFM (1) parameters change smoothly and expose to the various exogenous disturbances acting on the system.

**Example 6.** This example considers parameters variations $\eta_i(t)$ and exogenous disturbances $\Lambda_i(t)$, which are given in (25a) and (25b), respectively.

$$(i)\quad \eta_1 = 0.9 - 0.2\cos 5t, \eta_2 = 0.2 - 0.2\cos 5t, \text{ and } \eta_3 = 1.2 - 0.2\cos 5t \tag{25a}$$

$$(ii)\quad \Lambda_i(t) = 0.3\sin 0.5\frac{\pi}{6}t, i = 1, 2, 3 \tag{25b}$$




$$\varphi = 0.3$$
$$\alpha_i = \beta_i = \gamma_i = 0.5$$

**Assumption 1.** The exogenous disturbances caused by environmental interference influence the financial and economic chaotic models [34]; it may destabilize systems, resulting in undesirable behavior. This paper assumes that the bounded exogenous disturbances $\Lambda_i(t)$ act upon CFM (12). Therefore, there exists an upper bound $\varphi_i \in R^+$ of the disturbances such that:

$$|\Lambda_i(t)| \le \varphi_i, \quad i \in (1, 2, 3) \tag{13}$$