A simple multi-objective optimization problem

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Introduction

Let's introduce a geometrical optimization problem, named **cones problem**, with the following characteristics:

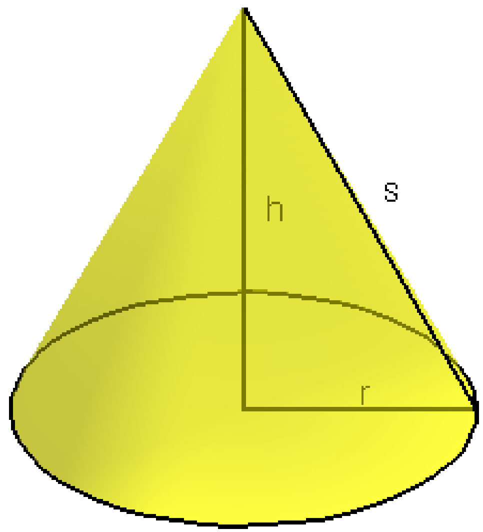
- **multi-objective** problem (two objective functions): the solution is not a single optimum design, but instead it is represented by the set of designs belonging to the *Pareto frontier*
- **simple** mathematical formulation: easy and quick implementation from scratch of the relevant modeFRONTIER project
- **constrained** problem: objectives space and designs space present *feasible* and *unfeasible* regions

Problem definition

Right circular cone:

r = base radius
h = height
s = slant height

V = volume
B = base area
S = lateral surface area
T = total area



$$s = \sqrt{r^2 + h^2}$$

$$V = \frac{\pi}{3} r^2 h$$

$$B = \pi r^2$$

$$S = \pi r s$$

$$T = B + S = \pi r (r + s)$$

Cones problem

- two input variables: r, h

$$r \in [0, 10] \text{ cm} \, , \quad h \in [0, 20] \text{ cm}$$

The cone shape (i.e. the design) is defined univocally when both *r* and *h* are given.

- two objectives:

$$\min S$$
$$\min T$$

We want to minimize both the lateral surface area and the total surface area

- one constraint:

$$V > 200 \text{ cm}^3$$

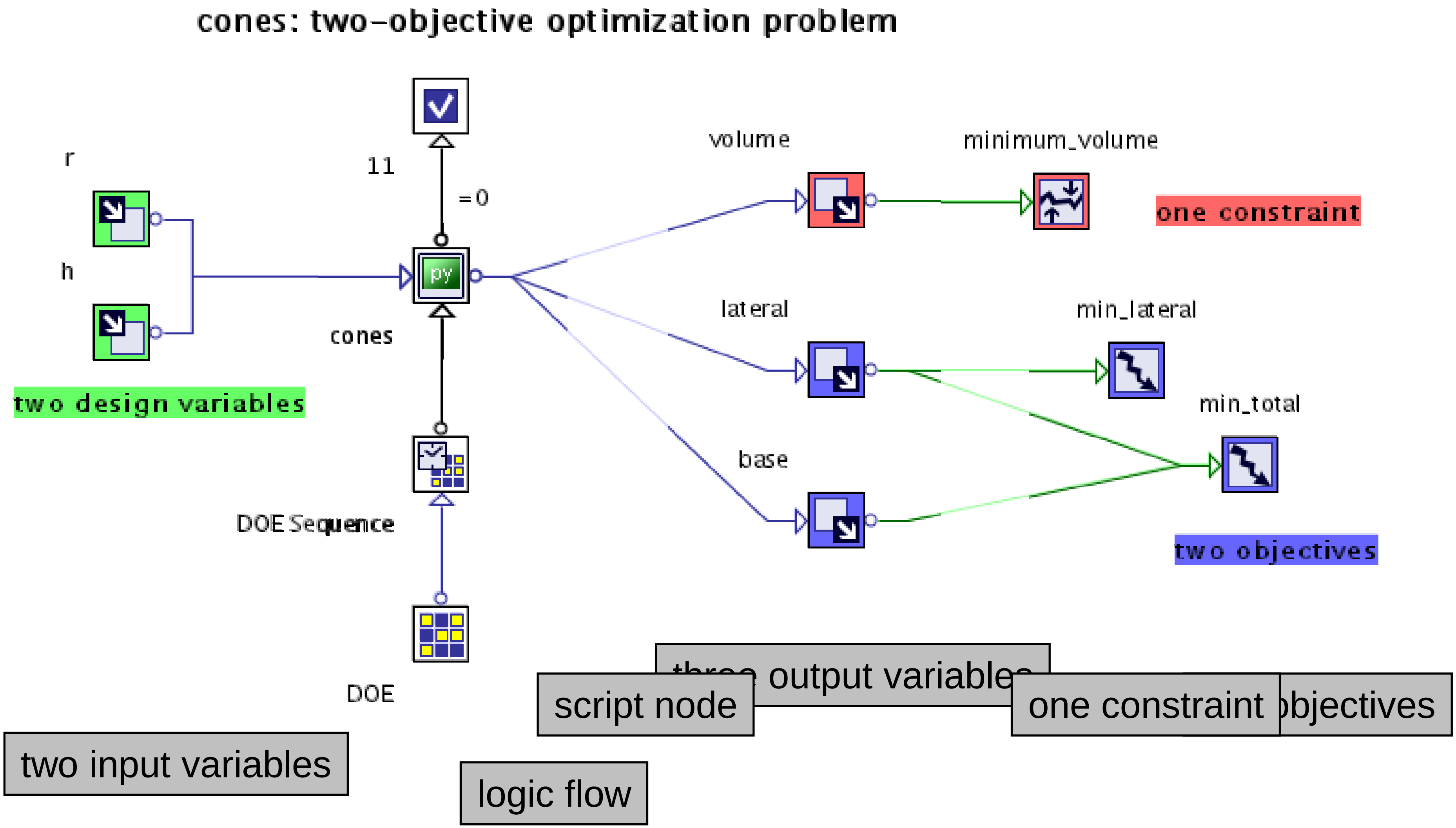
A constraint for the cone volume is given, in order to guarantee a minimum volume.

Project building

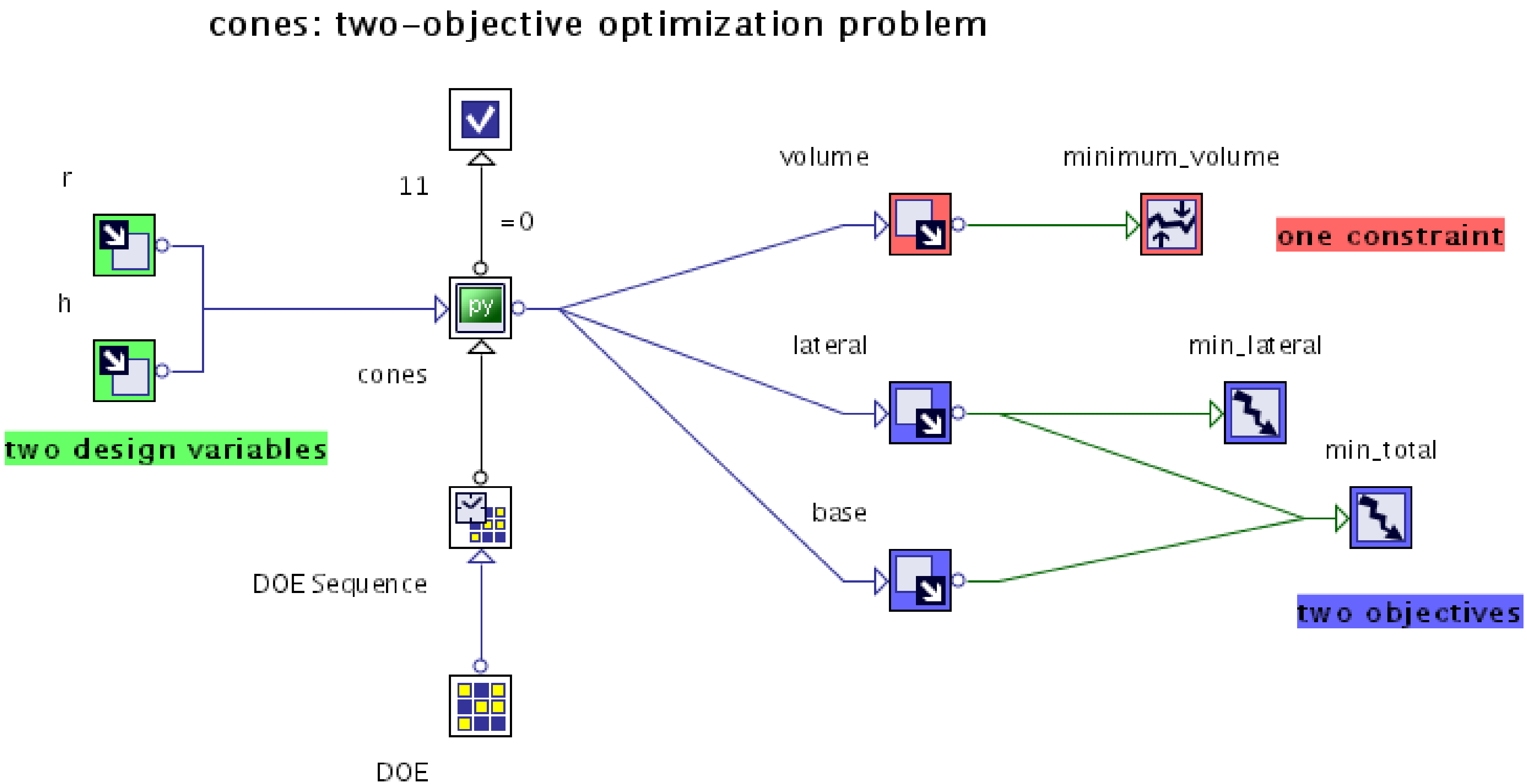
Let’s build from scratch the pertinent modeFRONTIER project:

1. Work Flow setup: fill the work canvas with the project’s building blocks
2. Script Node setup: use your favourite math tool
   - Jython script
   - Matlab node
   - Excel Workbook node
   - OpenOffice Spreadsheet node

Work Flow setup



Work Flow setup



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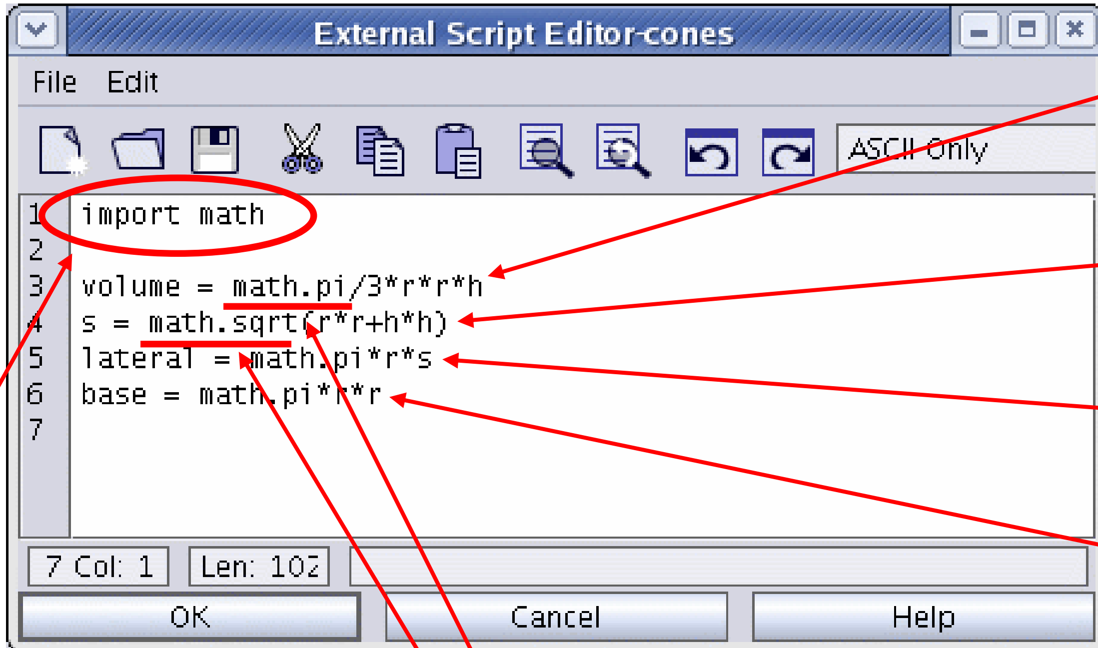
Script node: Jython

Jython (Python) script case:

Write down the formulae



$$V = \frac{\pi}{3} r^2 h$$

$$s = \sqrt{r^2 + h^2}$$

$$S = \pi r s$$

$$B = \pi r^2$$

Load math module

Note the syntax of mathematical functions and constants

Script node: Matlab

Matlab case:

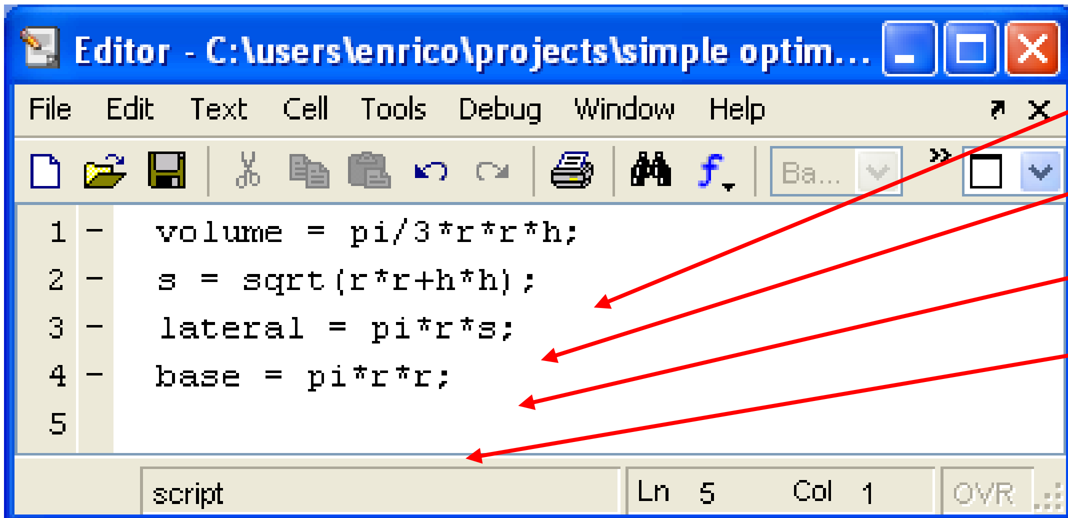
Write down the formulae

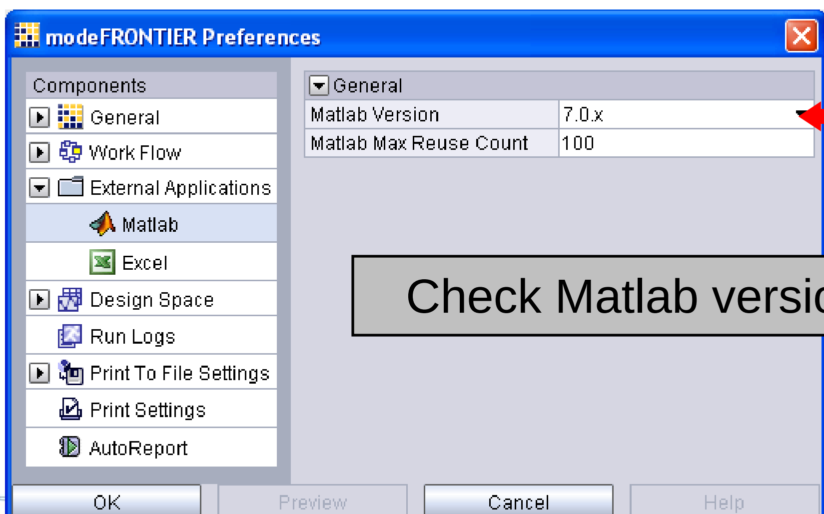
$$V = \frac{\pi}{3} r^2 h$$

$$s = \sqrt{r^2 + h^2}$$

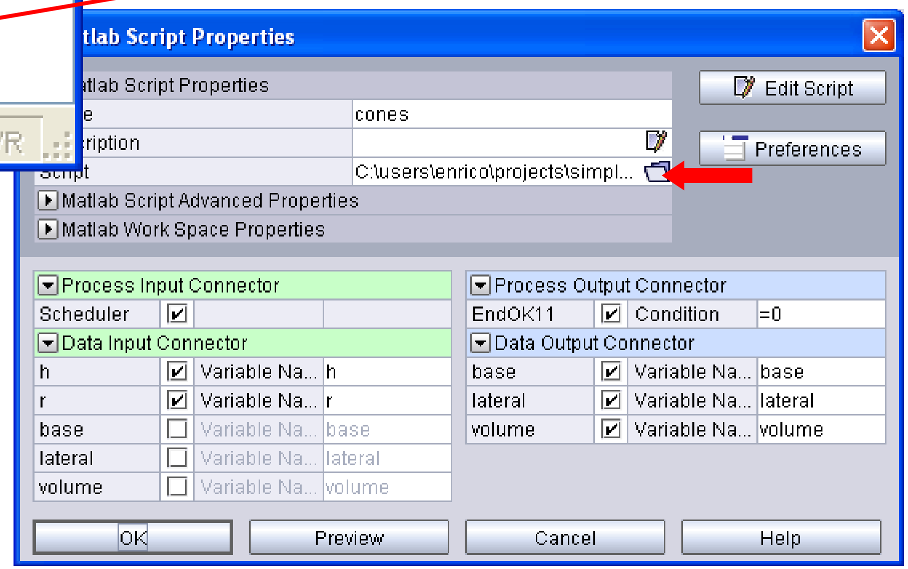
$$S = \pi r s$$

$$B = \pi r^2$$





Check Matlab version



Load the matlab file

Script node: Excel

Excel Workbook case:

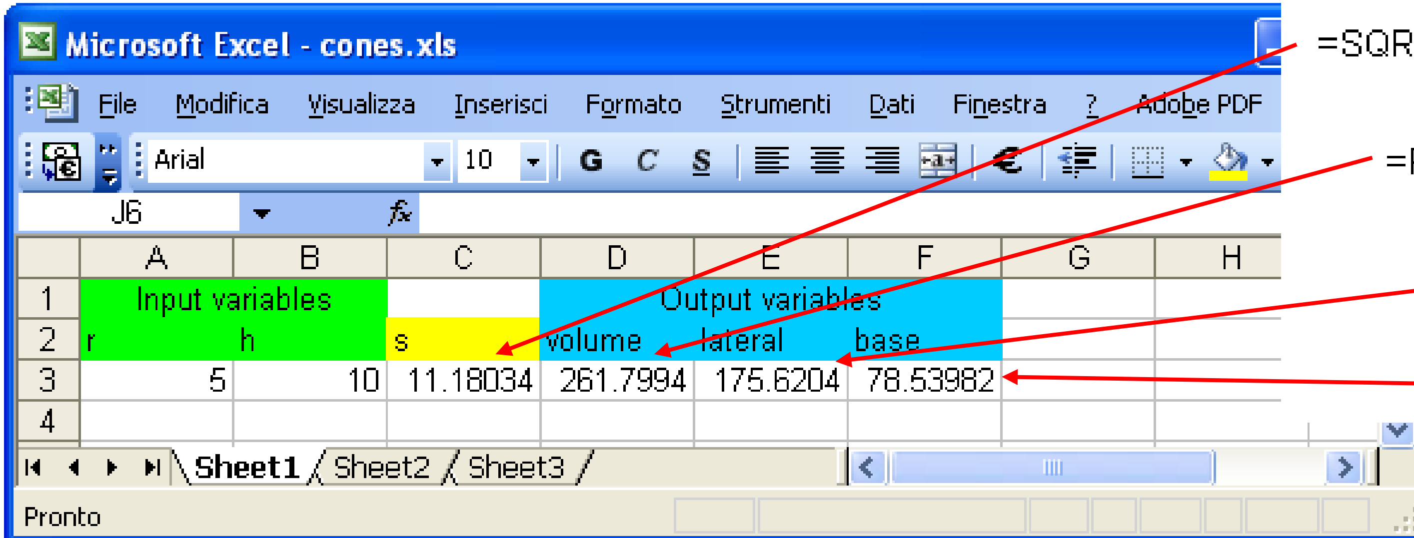
Insert the formulae

=SQRT(A1*A1+B1*B1) $s = \sqrt{r^2 + h^2}$

=PI()/3*A1*A1*B1 $V = \frac{\pi}{3} r^2 h$

=PI()*A1*C1 $S = \pi r s$

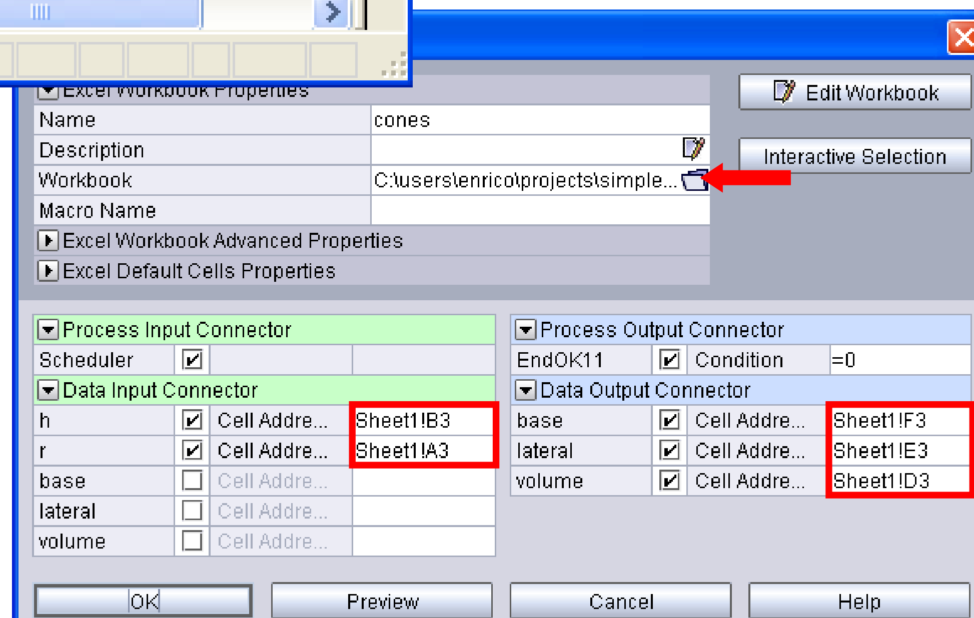
=PI()*A1*A1 $B = \pi r^2$



Build the spreadsheet

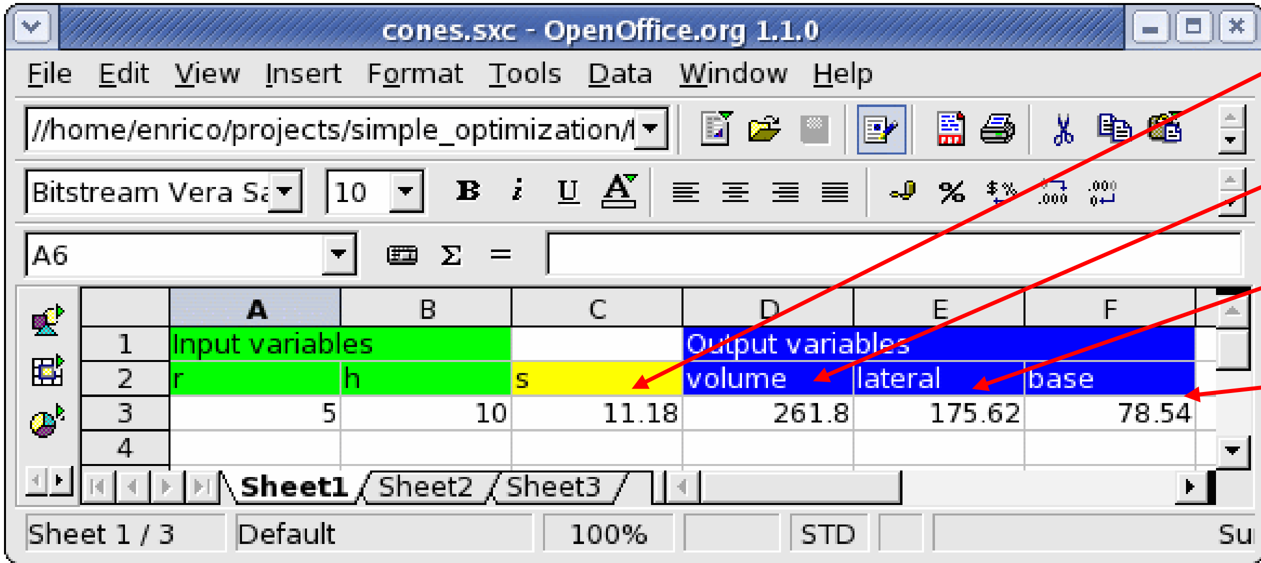
Load the xls file

Link variables to cells



Script node: OpenOffice

OpenOffice Spreadsheet case:



Insert the formulae

=SQRT(A3*A3+B3*B3) $s = \sqrt{r^2 + h^2}$

=PI()/3*A3*A3*B3 $V = \frac{\pi}{3} r^2 h$

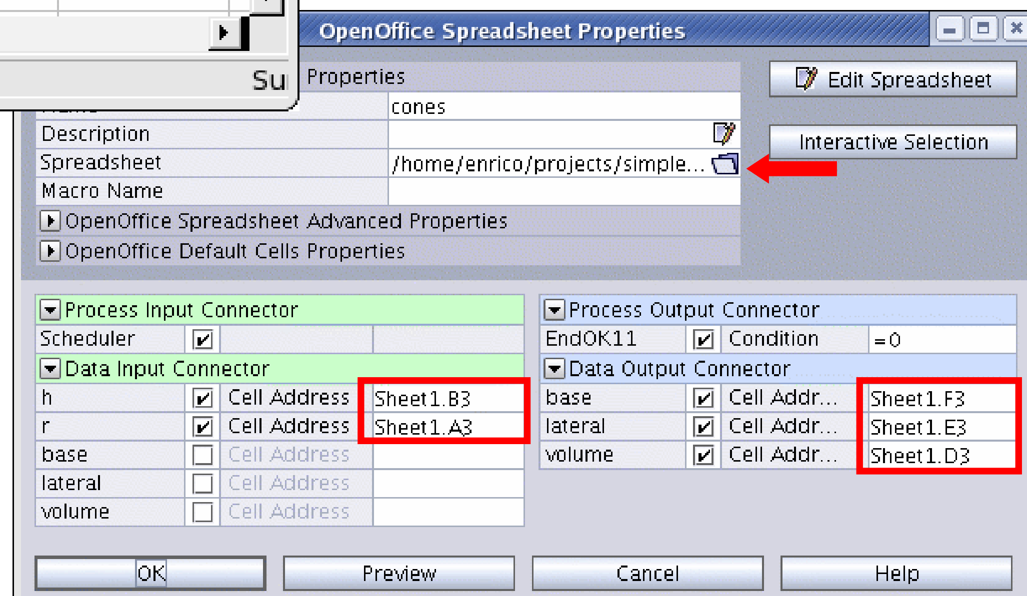
=PI()*A3*C3 $S = \pi \, r \, s$

=PI()*A3*A3 $B = \pi \, r^2$

Build the spreadsheet

Load the sxc file

Link variables to cells



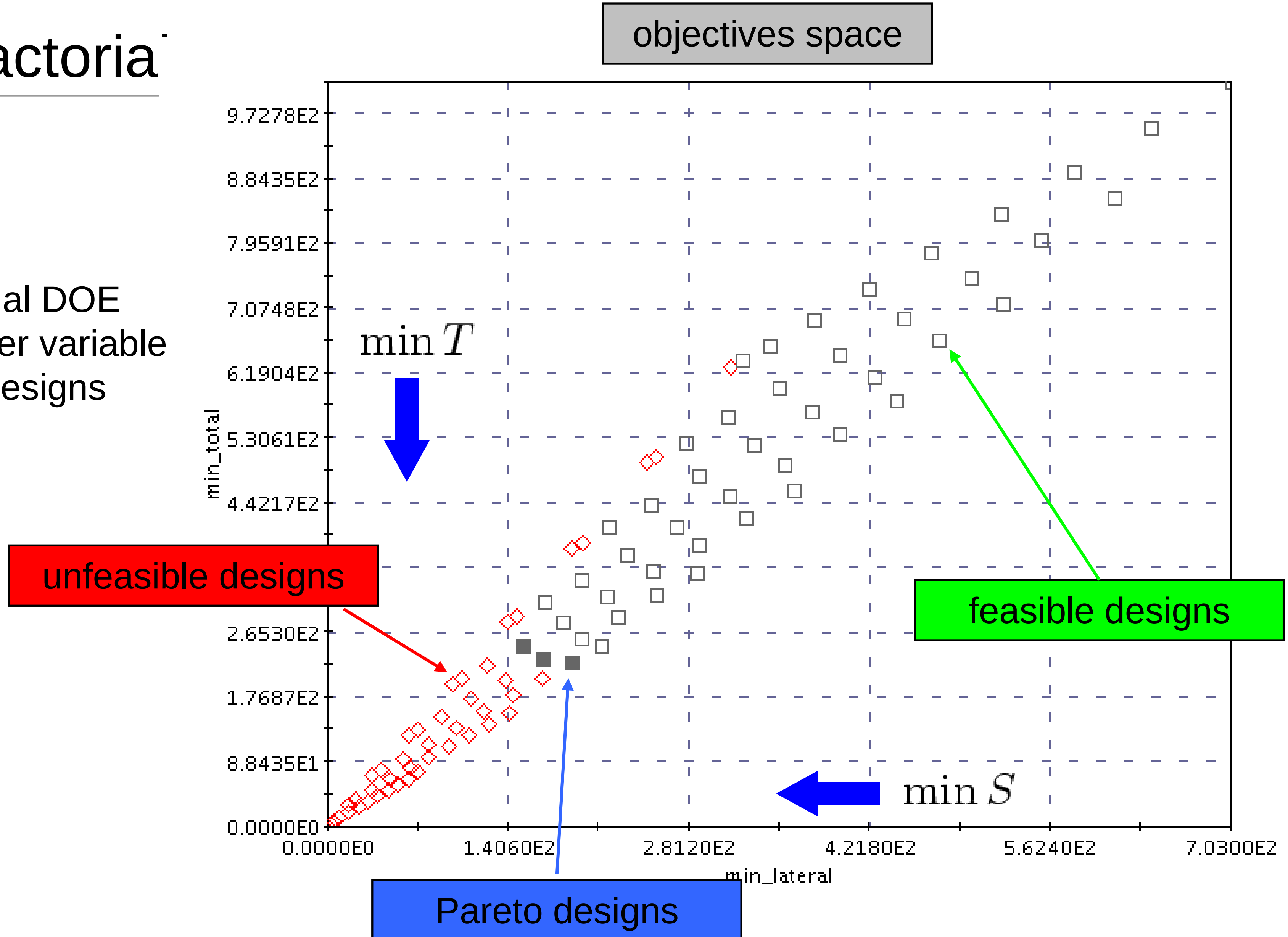
Runs examples

Let’s see some examples of runs with different DOEs and/or schedulers:

- **Full Factorial** DOE
- random samplings: **Random Sequence** and **Sobol** DOEs
- genetic algorithms: **MOGA-II**, **NSGA-II**
- **MOSA**
- **NBI-NLPQLP**

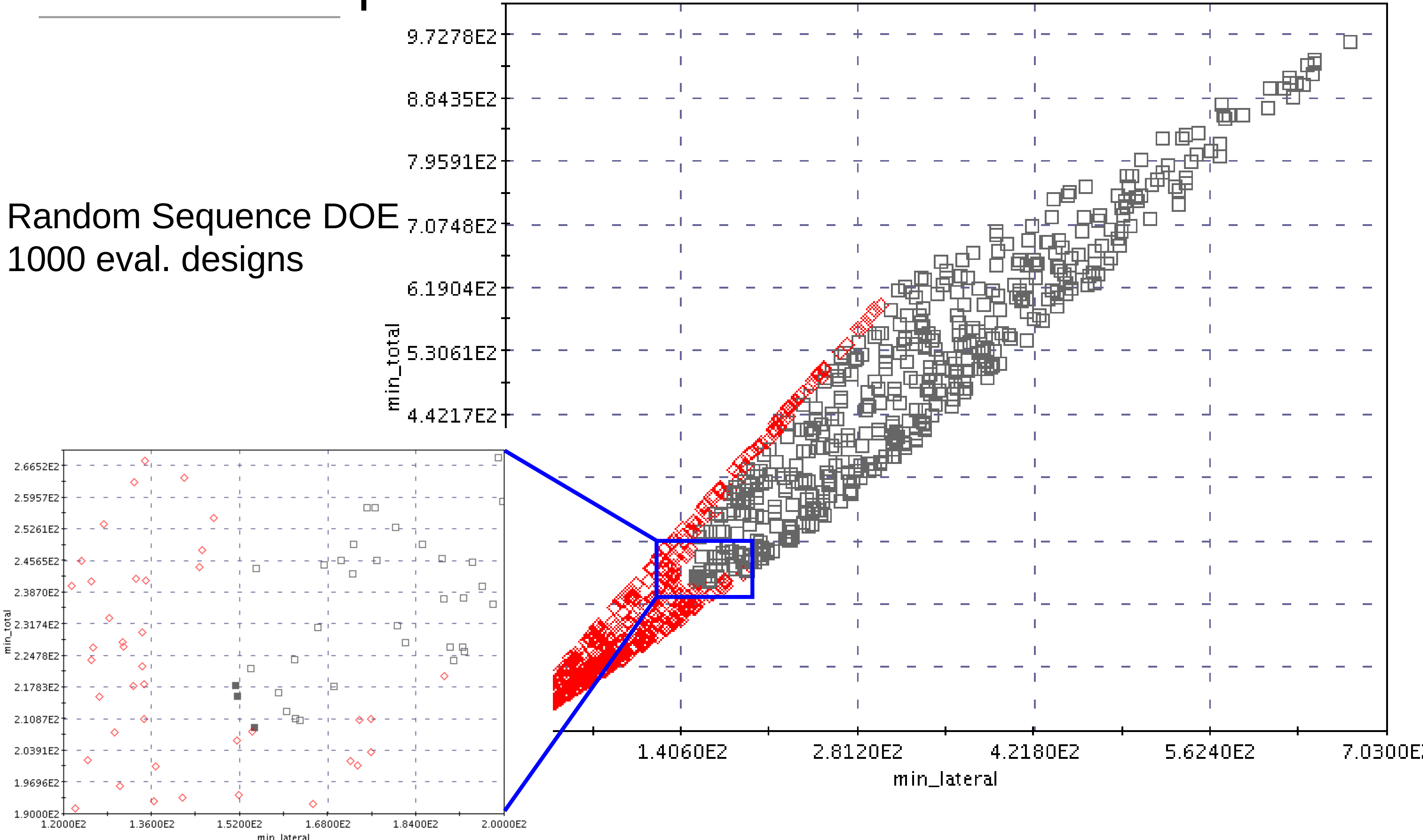
Full Factorial

Full Factorial DOE
10 levels per variable
100 eval. designs



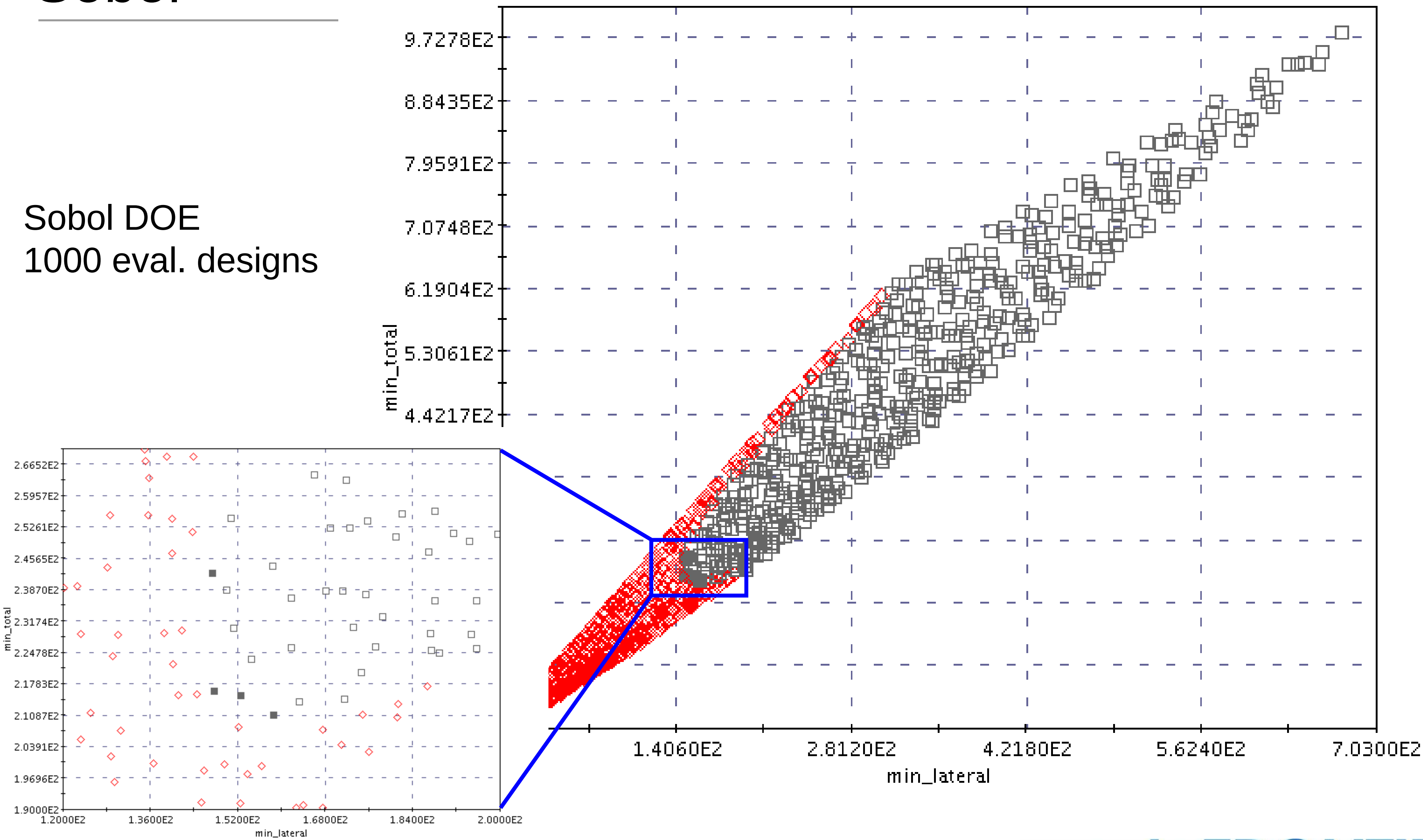
Random Sequence

Random Sequence DOE
1000 eval. designs



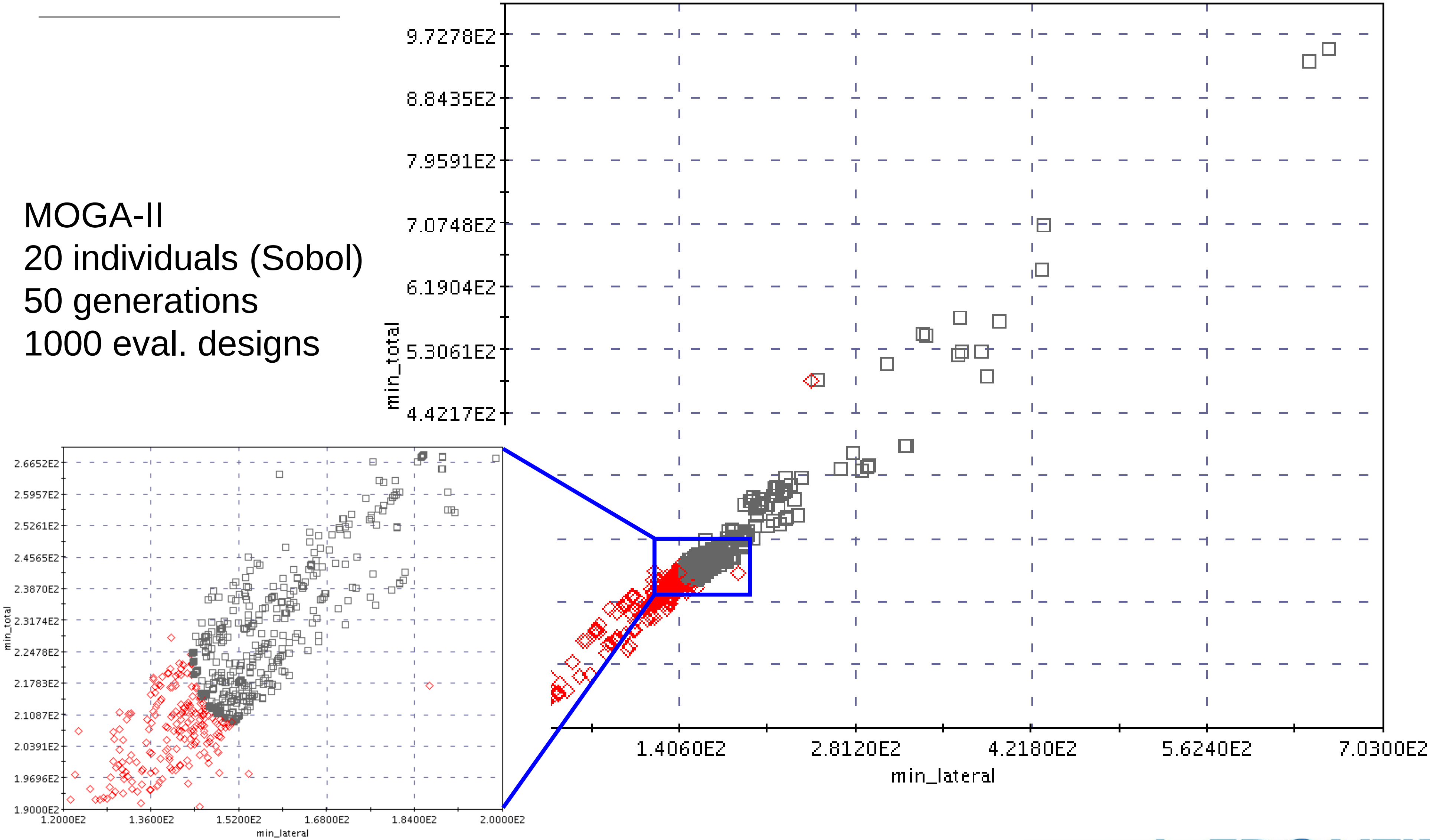
Sobol

Sobol DOE
1000 eval. designs



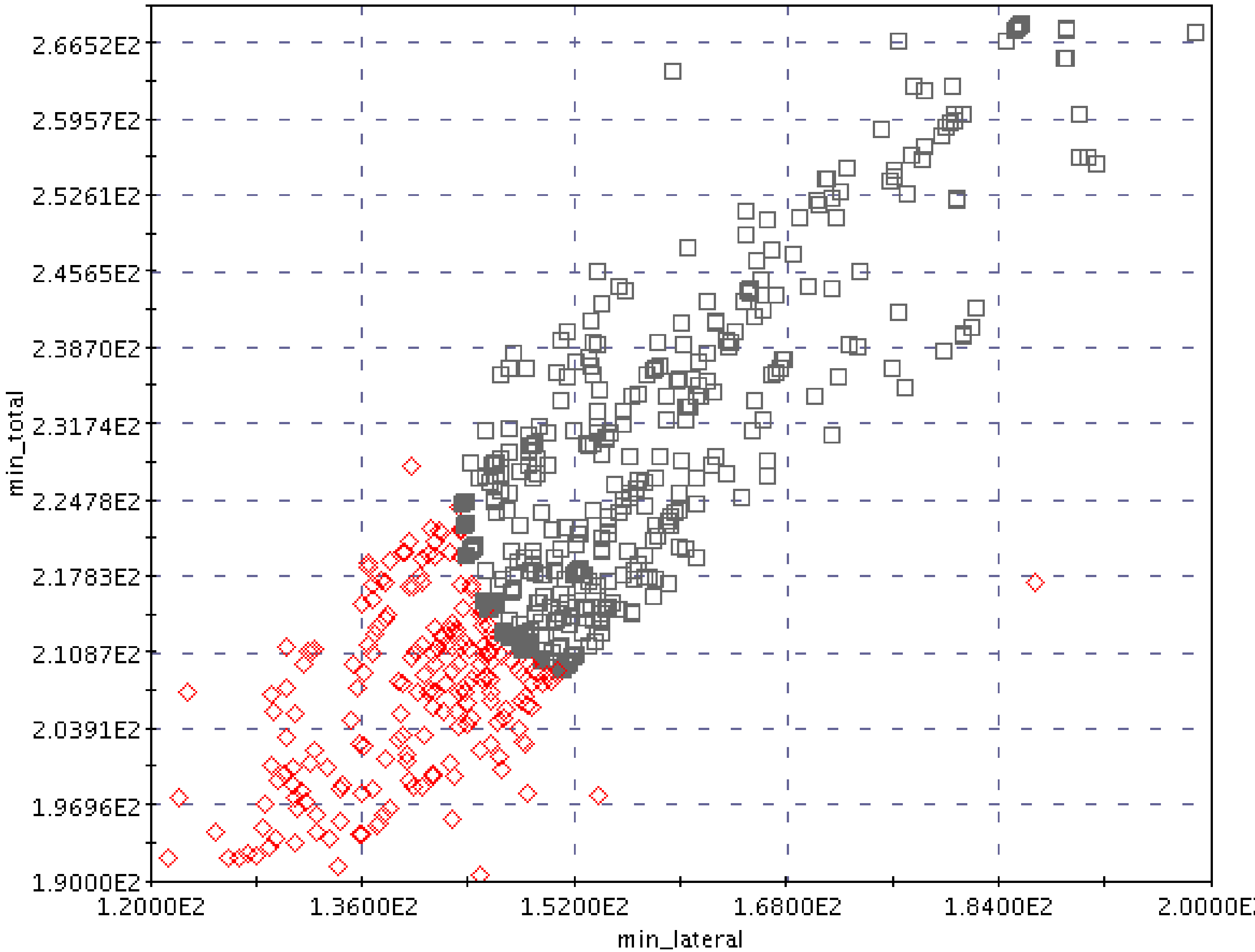
MOGA-II

MOGA-II
20 individuals (Sobol)
50 generations
1000 eval. designs



MOGA-II

MOGA-II
20 individuals (Sobol)
50 generations
1000 eval. designs



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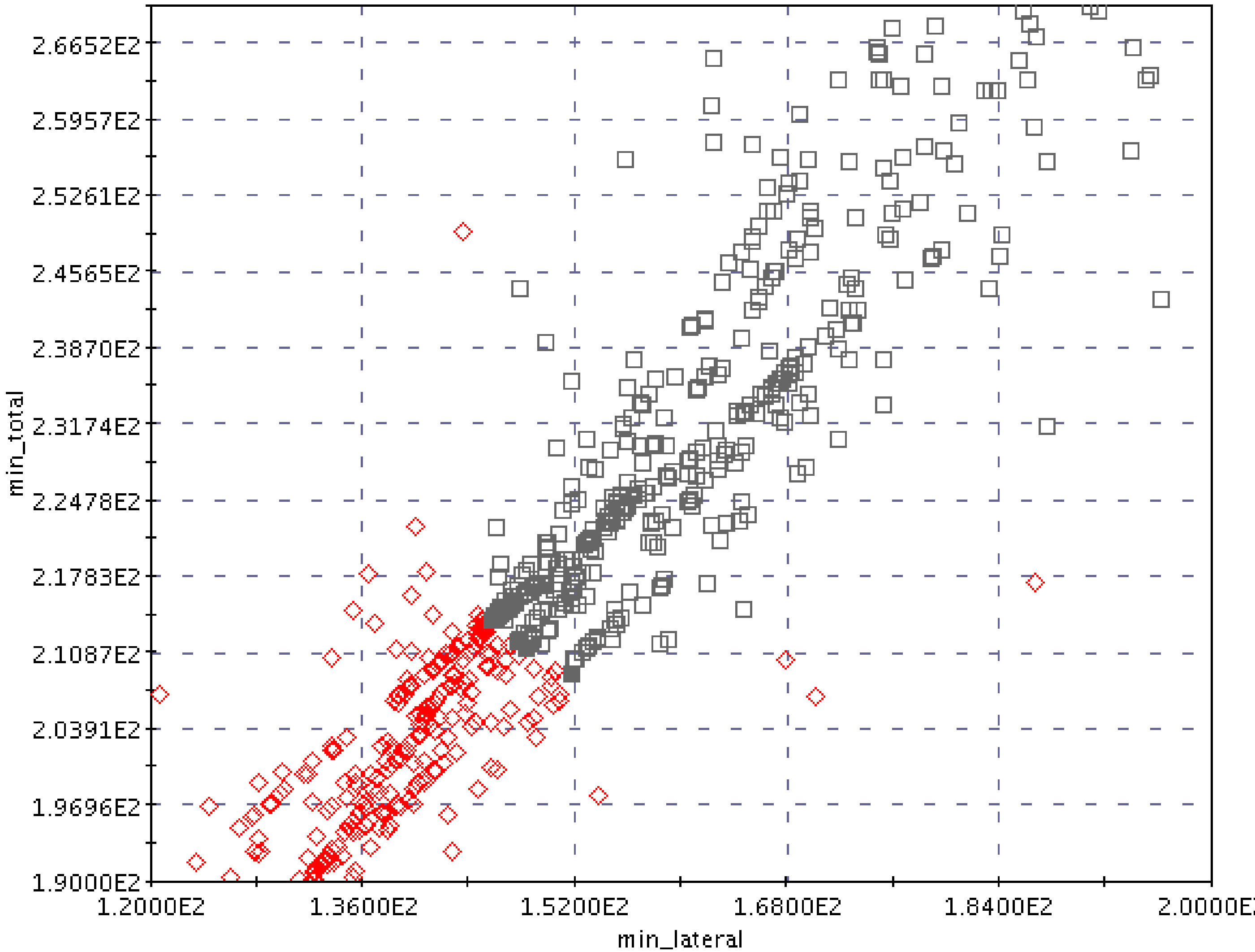
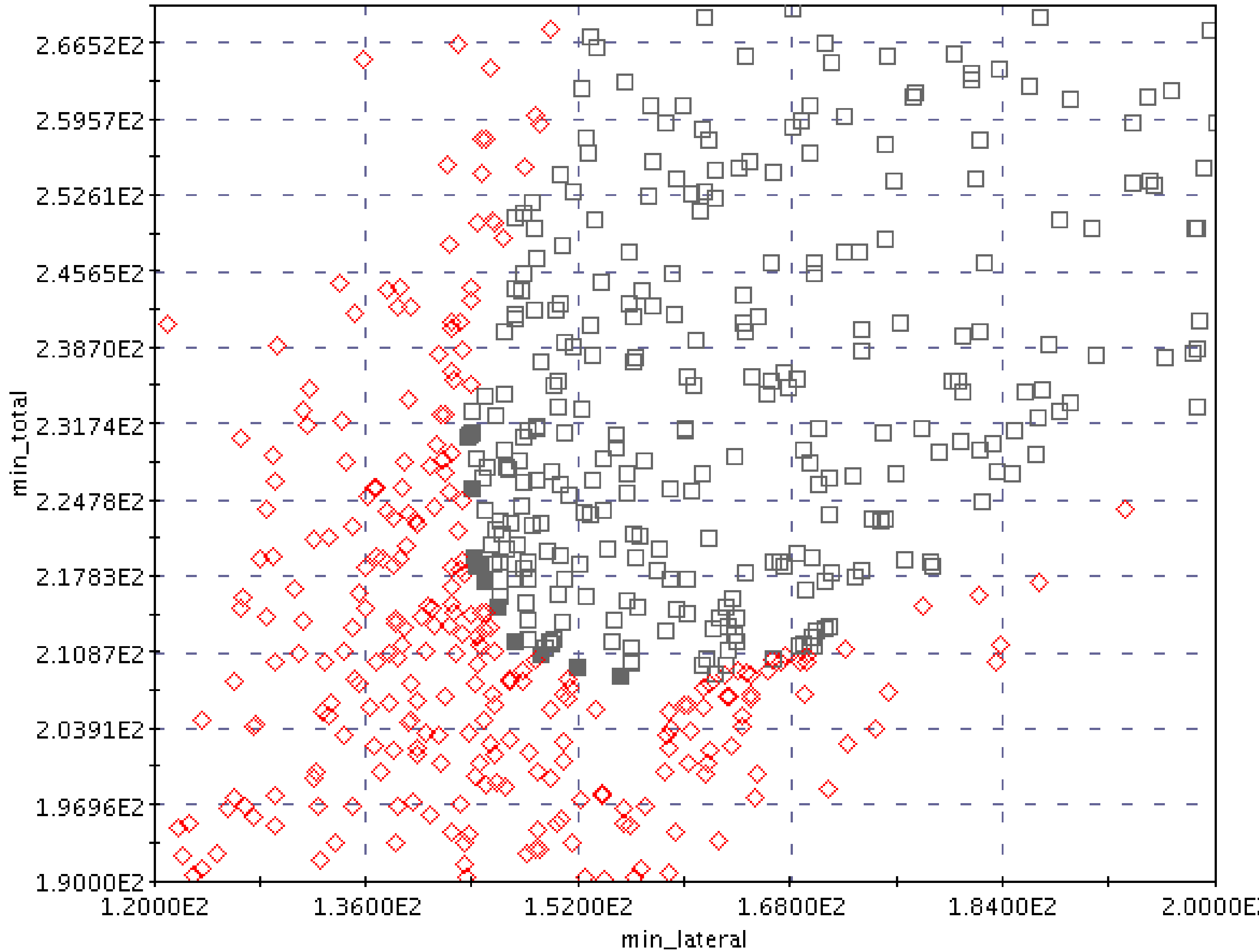
NSGA-II



NSGA-II
20 individuals (Sobol)
50 generations
1000 eval. designs

MOSA

MOSA
10 points (Sobol)
100 iterations
1000 eval. designs



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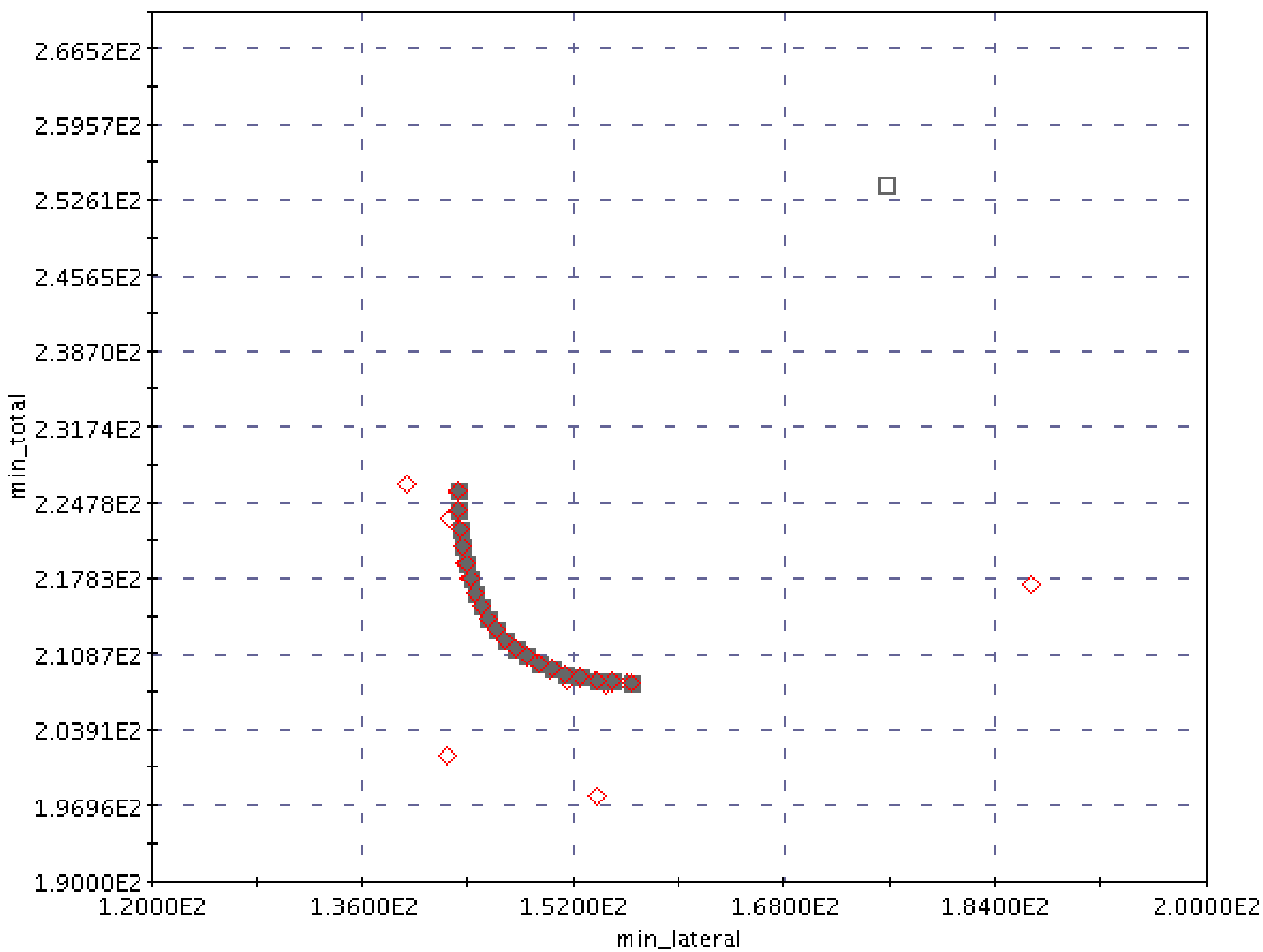
modeFRONTIER
the multi-objective optimization and design environment

NBI-NLPQLP

NBI-NLPQLP
(DOE: 10 Sobol)
20 NBI-subproblems
346 eval. designs

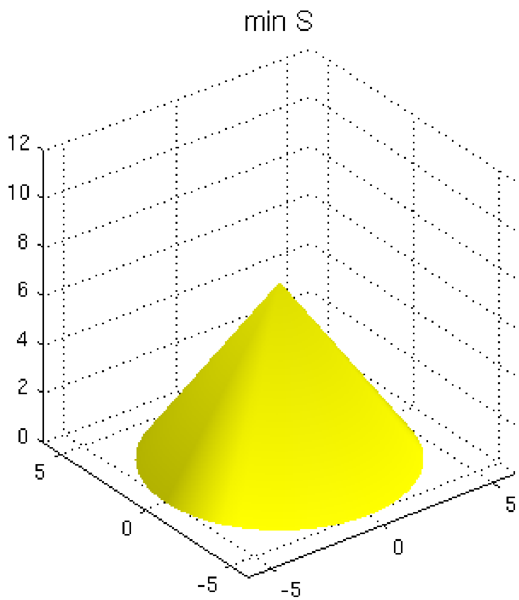


Final considerations

Let's consider the difference between

- single-objective problem solutions: two different minima
- multi-objective problem solutions: the Pareto frontier

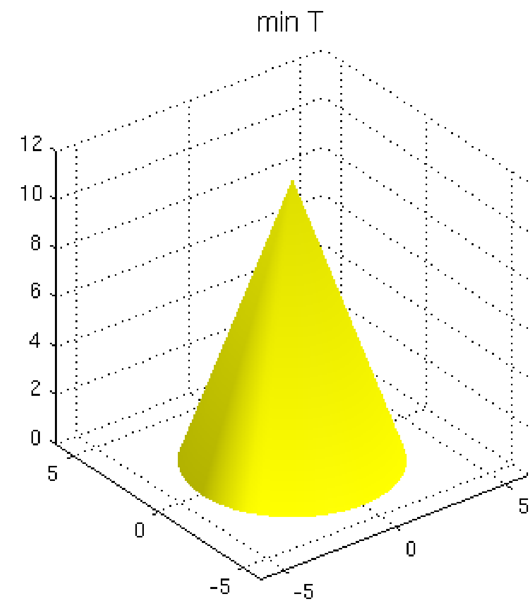
Single-objectives minima



Each design represents the optimum solution for its corresponding single-objective problem.

?

...but what about the in between designs?



$\min S$
$r = 5.131$ cm
$h = 7.256$ cm
$V = 200$ cm^3
$S = 143.23$ cm^2
$T = 225.92$ cm^2

...we would like to get a compromise solution. A trade-off of the two objectives...

$\min T$
$r = 4.072$ cm
$h = 11.518$ cm
$V = 200$ cm^3
$S = 156.28$ cm^2
$T = 208.38$ cm^2

What we want is the **Pareto frontier**!

The Pareto frontier