MECE 5397

Assignment 1

Due Wednesday January 23 at class time

We have seen in class that the derivative of a function $f(x)$ can be approximated in a number of different ways:

- Forward difference (1st order accuracy)

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h}$$

- Backward difference (1st oder)

$$f'(x) \simeq \frac{f(x) - f(x-h)}{h}$$

- Centered difference (2nd order)

$$f'(x) \simeq \frac{f(x+h) - f(x-h)}{2h}$$

- One-sided forward difference (2nd order)

$$f'(x) \simeq \frac{4f(x+h) - 3f(x) - f(x+2h)}{2h}$$

- One-sided backward difference (2nd order)

$$f'(x) \simeq \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$$

For the second derivative we saw the centered difference formula

$$f''(x) \simeq \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

The point of this assignement is to see how these formulae work in practice in some cases. For this purpose consider the family of functions

$$f_k(x) = \sin kx \,, \qquad 0 \leq x \leq 1$$

where k is a positive integer. Divide the interval $0 \leq x \leq 1$ in N equal parts, so that the h to be used in the previous formuale is $h = 1/N$. In comparing the exact derivatives to their approximate values we will use the following definition of the error (this is known as the L_1 norm):

$$E'_k = \frac{1}{N+1} \sum_{j=0}^{N} |(f'_k)_{exact}(x_j) - (f'_k)_{approx}(x_j)|$$

where $x_j = jh$ so that $x_0 = 0$, $x_1 = h$, ..., $x_N = Nh$; $(f'_k)_{exact}(x_j)$ is the exact analytic value of f'_k evaluated for $x = x_j$ and $(f'_k)_{approx}(x_j)$ is the approximate values given by the previous formulae. The error E''_k for the second derivative is defined in a similar way. Using MATLAB (or any other language of your choice) write a computer program to calculate the error. Consider the following cases:

1. Take $N = 10$ and calculate the error incurred by the preceding formulae for $k = 1$;
2. Repeat by taking $k = 5$, 10, 20 with $N = 10$;
3. Repeat by taking $k = 20$ and $N = 100$ and $N = 500$;
4. Repeat by taking $k = 1$ and $N = 100$, 10^3, $10^4, 10^6$, 10^9; for this question consider only the first derivative f' calculated according to the 1st-order forward formula and to the 2nd-order centered difference formula.

Show your results for E'_k and E''_k in tabular form separating those of the 1st-order formulae from those of the 2nd-order formulae. Comment on the results: what happens to the error when you increase k keeping h constant? Why? How do the 1st-order errors compare with the 2nd-order ones? What happens when you go from $N = 100$ to $N = 500$? Why? In the last question, what happens as N is increased all the way to 10^9? Why? For the last question show also graphs of the error as a function of $|\log h|$.