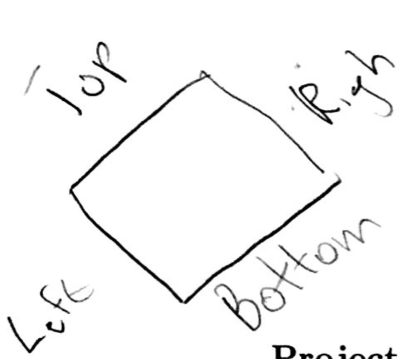

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APc2-2

MECE 5397

Project A – Poisson Equation

Solving for u

Write a computer code to solve the two-dimensional Poisson equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -F(x, y) \tag{8}$$

The domain of interest is the rectangle

$$a_x < x < b_x\,, \qquad a_y < y < b_y \tag{9}$$

and the boundary conditions

Top, Bottom

$$u(x, y = b_y) = f_a(x)\,, \qquad u(x, y = a_y) = g_a(x)\,, \tag{10}$$

Left, right

Dirichlet BC

$$\frac{\partial u}{\partial x}\bigg|_{x=a_x} = 0\,, \qquad u(x = b_x, y) = g_a(b_x) + \frac{y - a_y}{b_y - a_y}[f_a(b_x) - g_a(b_x)] \tag{11}$$

Domain

$$a_x = a_y = -\pi\,, \qquad b_x = b_y = \pi \tag{12}$$

Boundary Function

$$g_a(x) = (x - a_x)^2 \cos\frac{\pi x}{a_x}\,, \qquad f_a(x) = x(x - a_x)^2 \tag{13}$$

analytical Solution

$$F(x, y) = \sin\left[\pi\frac{x - a_x}{b_x - a_x}\right] \cos\left[\frac{\pi}{2}\left(2\frac{y - a_y}{b_y - a_y} + 1\right)\right] \tag{14}$$

Use ghost node(s) for Neumann condition(s).
After carrying out all the simulations needed for the report, run one last simulation with $F = 0$ and include the results in the report.

Input — Gauss $\downarrow$ Relaxation

$$\frac{du}{dx}\bigg|_{x=a_x} = 0 = \frac{u_j^{n+1} - u_j^{n}}{\Delta t} \;{}_{2} \Rightarrow$$