

KCL($AI = 0$)

Node 0: $-i_{v1} + i_{s1} + i_{c1} + i_{R1} = 0$
 Node 1: $i_{v1} - i_{L1} = 0$
 Node 2: $i_{L1} - i_{S1} - i_{S2} = 0$
 Node 3: $i_{S2} - i_{C1} - i_{R1} = 0$

$$\begin{bmatrix} -1 & 0 & 1 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} i_{v1} \\ i_{L1} \\ i_{S1} \\ i_{S2} \\ i_{C1} \\ i_{R1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(Node 0 Can be Used to Verify a Closed Circuit)

State Variables:

$$x = \begin{bmatrix} i_{L1} \\ v_{C1} \end{bmatrix}$$

KVL($V - A^T e = 0$)

$$\begin{bmatrix} v_{v1} \\ v_{L1} \\ v_{S1} \\ v_{S2} \\ v_{C1} \\ v_{R1} \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

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* SPICE Netlist for Boost Topology
* component node_n node_p value
V1 0 1 5
L1 1 2 10u
S1 2 0 100m 10G
S2 2 3 100m 10G
C1 3 0 10u
R1 3 0 100
.end
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Component VCR ($K_i I + K_v V = S$)

V1: $v_{v1} = 5$
 L1: $v_{L1} = -10u \times \frac{di_{L1}}{dt}$
 S1: $i_{S1} = -\frac{1}{R_{S1}} \times v_{S1}$
 S2: $i_{S2} = -\frac{1}{R_{S2}} \times v_{S2}$
 C1: $i_{C1} = -10u \times \frac{dv_{C1}}{dt}$
 R1: $i_{R1} = -\frac{1}{100} \times v_{R1}$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{v1} \\ i_{L1} \\ i_{S1} \\ i_{S2} \\ i_{C1} \\ i_{R1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{R_{S1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_{S2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -10u & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{100} \end{bmatrix} \begin{bmatrix} v_{v1} \\ v_{L1} \\ v_{S1} \\ v_{S2} \\ v_{C1} \\ v_{R1} \end{bmatrix} = \begin{bmatrix} 5 \\ -10u \times \frac{di_{L1}}{dt} \\ 0 \\ 0 \\ -10u \times \frac{dv_{C1}}{dt} \\ 0 \end{bmatrix}$$

Matrix Assembly

On State: $R_{S1} = 0.01$ $R_{S2} = 100Meg$. Off State: $R_{S1} = 100Meg$ $R_{S2} = 0.01$

$$\begin{bmatrix} K_i & K_v & 0 \\ 0 & I & -A^T \\ A & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ V \\ e \end{bmatrix} = \begin{bmatrix} S \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{R_{s1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_{s2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{100} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{v1} \\ i_{L1} \\ i_{S1} \\ i_{S2} \\ i_{C1} \\ i_{R1} \\ v_{v1} \\ v_{L1} \\ v_{S1} \\ v_{S2} \\ v_{C1} \\ v_{R1} \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -10u \times \frac{di_{L1}}{dt} \\ 0 \\ 0 \\ -10u \times \frac{dv_{C1}}{dt} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

State Variables:

$$x = \begin{bmatrix} i_{L1} \\ v_{C1} \end{bmatrix}$$

Now how can we get $\frac{dx}{dt} = Ax + Bu$ from the following equations?

$$x = \begin{bmatrix} i_{L1} \\ v_{c1} \end{bmatrix}, u = 5$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{R_{s1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_{s2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{100} \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{v1} \\ i_{L1} \\ i_{S1} \\ i_{S2} \\ i_{C1} \\ i_{R1} \\ v_{v1} \\ v_{L1} \\ v_{S1} \\ v_{S2} \\ v_{c1} \\ v_{R1} \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -10u \times \frac{di_{L1}}{dt} \\ 0 \\ 0 \\ -10u \times \frac{dv_{c1}}{dt} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Maybe we can try selection matrix

$$x = \begin{bmatrix} i_{L1} \\ v_{c1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{v1} \\ i_{L1} \\ i_{S1} \\ i_{S2} \\ i_{C1} \\ i_{R1} \\ v_{v1} \\ v_{L1} \\ v_{S1} \\ v_{S2} \\ v_{C1} \\ v_{R1} \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} C_I^{L1} & 0 & 0 \\ 0 & C_V^{C1} & 0 \end{bmatrix} \begin{bmatrix} I \\ V \\ e \end{bmatrix}$$

$$\equiv K_x \begin{bmatrix} I \\ V \\ e \end{bmatrix} = K_x \begin{bmatrix} K_i & K_v & 0 \\ 0 & I & -A^T \\ A & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} S \\ 0 \\ 0 \end{bmatrix}$$

Then we can decompose $\begin{bmatrix} S \\ 0 \\ 0 \end{bmatrix}$ as $\begin{bmatrix} S \\ 0 \\ 0 \end{bmatrix} = S_D \frac{dx}{dt} + S_u u$

We want $\frac{dx}{dt} = \textcolor{red}{A}x + \textcolor{blue}{B}u$, we have $\begin{bmatrix} S \\ 0 \\ 0 \end{bmatrix} = S_D \frac{dx}{dt} + S_u u$, $x = K_x \begin{bmatrix} K_i & K_v & 0 \\ 0 & I & -A^T \\ A & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} S \\ 0 \\ 0 \end{bmatrix}$

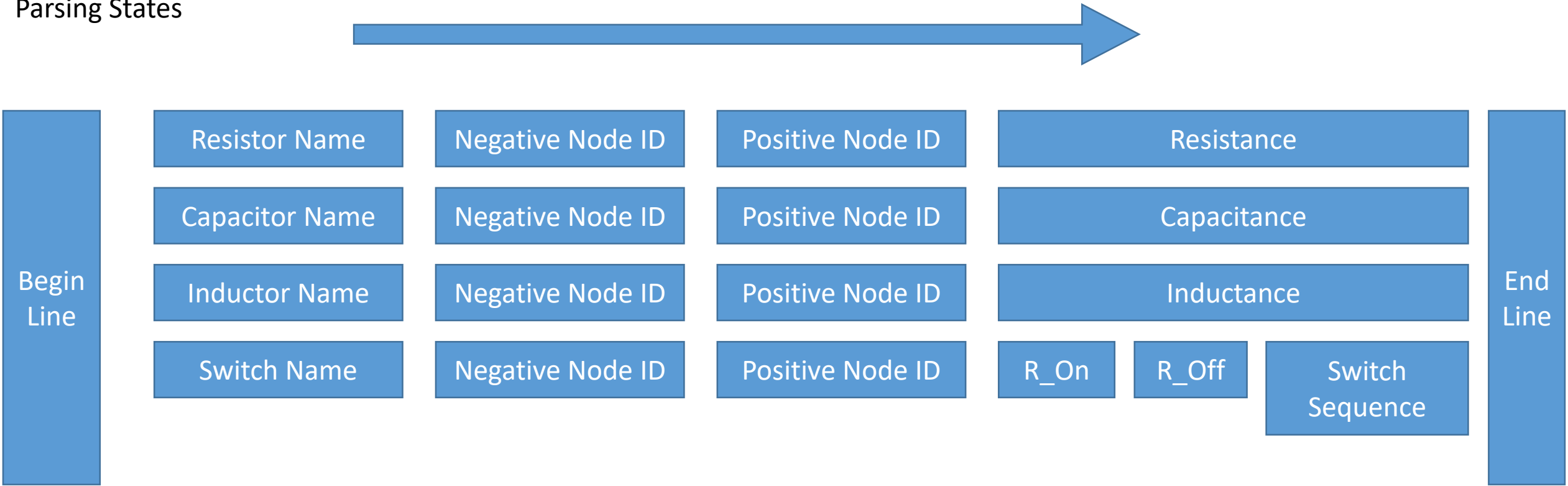
Then, $x = K_x \begin{bmatrix} K_i & K_v & 0 \\ 0 & I & -A^T \\ A & 0 & 0 \end{bmatrix}^{-1} \left(S_D \frac{dx}{dt} + S_u u \right) = K_x \begin{bmatrix} K_i & K_v & 0 \\ 0 & I & -A^T \\ A & 0 & 0 \end{bmatrix}^{-1} S_D \frac{dx}{dt} + K_x \begin{bmatrix} K_i & K_v & 0 \\ 0 & I & -A^T \\ A & 0 & 0 \end{bmatrix}^{-1} S_u u$

$K_x \begin{bmatrix} K_i & K_v & 0 \\ 0 & I & -A^T \\ A & 0 & 0 \end{bmatrix}^{-1} S_D \frac{dx}{dt} = x - K_x \begin{bmatrix} K_i & K_v & 0 \\ 0 & I & -A^T \\ A & 0 & 0 \end{bmatrix}^{-1} S_u u$

$\frac{dx}{dt} = \left(\textcolor{red}{K}_x \begin{bmatrix} \textcolor{red}{K}_i & \textcolor{red}{K}_v & 0 \\ 0 & I & -\textcolor{red}{A}^T \\ \textcolor{red}{A} & 0 & 0 \end{bmatrix}^{-1} S_D \right)^{-1} x - \left(\textcolor{blue}{K}_x \begin{bmatrix} \textcolor{blue}{K}_i & \textcolor{blue}{K}_v & 0 \\ 0 & I & -\textcolor{blue}{A}^T \\ \textcolor{blue}{A} & 0 & 0 \end{bmatrix}^{-1} S_D \right)^{-1} \textcolor{blue}{K}_x \begin{bmatrix} \textcolor{blue}{K}_i & \textcolor{blue}{K}_v & 0 \\ 0 & I & -\textcolor{blue}{A}^T \\ \textcolor{blue}{A} & 0 & 0 \end{bmatrix}^{-1} S_u u$

$y \equiv K_y \begin{bmatrix} V \\ I \\ e \end{bmatrix} = K_y \begin{bmatrix} K_i & K_v & 0 \\ 0 & I & -A^T \\ A & 0 & 0 \end{bmatrix}^{-1} \left(S_D \frac{dx}{dt} + S_u u \right) = (K_y L^{-1} S_D \textcolor{red}{A}_{\text{system}}) \cdot x + (K_y L^{-1} (S_D \textcolor{blue}{B}_{\text{system}} + S_u)) \cdot u$

Parsing States



We can build the following data structures in order to assemble SPICE Tableau matrix:

- 1. Branch List
- 2. Node List
- 3. Switch State Sequence

We also need the following Data Structures to assemble the system matrix ABCD

- 1. A List of Which Branch Contains Variables
- 2. A List of Interested Observation Variables from Branches and Nodes