

Question 1.

Describe the following sets by the listing method:

1. $\{n : n \in \mathbb{Z} \text{ and } 5 \leq n < 8\}$
2. $\{3n : n \in \mathbb{Z} \text{ and } 5 \leq n < 8\}$
3. $\{2^n : n \in \mathbb{Z} \text{ and } 5 \leq n < 8\}$

Question 2.

Let $\Sigma = \{x, y\}$ be an alphabet. List the element of the set L_1 and L_2

1. L_1 is the language consisting of all strings over Σ of length **less or equal to 4** that are **palindromes**.
2. L_2 is the language consisting of all strings over Σ of length **less or equal to 3** in which all the x 's appear to the left of all the y 's.

Question 3.

Describe the following sets by giving a suitable universal set and rules of inclusion:

1. $\{4, 8, 12, 16, 20\}$
2. $\{0, 2, -2, 4, -4, \dots\}$
3. $\{2, 4, 8, 16, 32\}$
4. $\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}\}$

Question 4.

Let $X = \{f, g, h, i, k\}$ and $Y = \{d, g, h, k\}$ be subsets of a universal set $U = \{d, e, f, g, h, i, j, k, l\}$. Find each of the following:

1. \overline{X}
2. $X \cap Y$

3. $X \cup \overline{Y}$
4. $X - Y$
5. $X \oplus Y$
6. $\overline{(X \cap Y)}$

Question 5.

Let $A = \{2, \frac{1}{2}, \sqrt{2}\}$ and $B = \{x \in \mathbb{Q} : x \notin \mathbb{Z}\}$ be two sets. List the following sets:

$$A \cap B, \quad A - B, \quad A \cap \mathbb{R}, \quad A \cap \mathbb{Z},$$

Question 6.

Let X and Y be two sets with $X = \{f, g, h, j, k\}$ and $Y = \{f, g\}$.

1. What is cardinality of X ?
2. What is the total number of subsets of X ?
3. Put the correct sign $\in \notin \subset \subseteq$ between the following pairs:

$$f \quad X, \quad Y \quad X, \quad X \quad X, \quad \emptyset \quad X, \quad \text{and} \quad h \quad Y$$

Question 7.

Let X and Y be two sets of the universal set U .

1. Use Venn diagram to show to show that $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$.
2. Use membership tables to prove that $\overline{X \cap Y} = \overline{X} \cup \overline{Y}$.

Question 8.

Let A and B and C be subsets of a universal set U .

1. Draw a three binary digit labelled Venn diagram depicting A, B, C in such a way that they divide U into 8 disjoint regions.
2. The subset $X \subseteq U$ is defined by the following membership table:

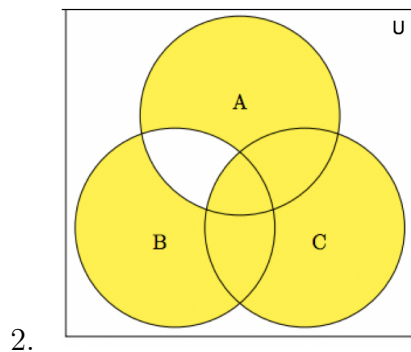
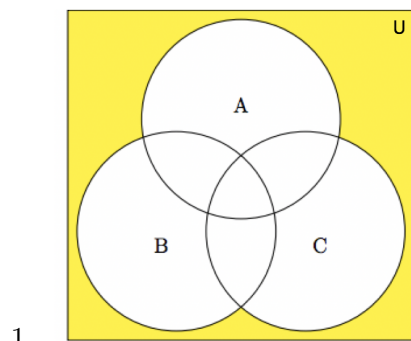
A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Identify the region X on your diagram. Describe the region you have identified in set notation as simply as you can.

- let Y be the set represented by the region 000, 011, 101, 110, and 111. Describe the set Y using the set notation,

Question 9.

Given three sets A , B and C , subsets of the universal set U . For each of each of the following Venn diagram write, in terms of A , B and C , the set representing the area coloured in yellow:



Question 10.

Let $A = \{t, u, v, w\}$ and let S_1 be the set of all subsets of A that do not contain w and S_2 be the set of all subsets of A that contain w .

- Find S_1 and S_2 .
- Are S_1 and S_2 disjoint?
- Find $S_1 \cup S_2$.
- What is the relation between $S_1 \cup S_2$ and $\mathcal{P}(A)$?

Question 11.

Let $A = \{1, 2\}$ and let $B = \{2, 3\}$. Find each of the following:

1. $\mathcal{P}(A \cap B)$
2. $\mathcal{P}(A \cup B)$
3. $\mathcal{P}(A \times B)$

Question 12.

Given three sets A, B and C . Prove that the expression $\overline{\overline{(A \cup B) \cap C} \cup \overline{B}}$ is equivalent to $B \cap C$ by re-writing the expression using algebraic laws, state the name of each law used.

Question 13.

Given three sets A, B and C . Using set identities, prove that:

$$(A \cup B) - (C - A) = A \cup (B - C).$$

Question 14.

Given two sets A and B . Simplify $\overline{(\overline{A} \cup \overline{B})} - A$.

Question 15.

Show that for all sets A and B , $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

Question 16.

Let A, B and C be three sets. Prove that if $C \subseteq (B - A)$ then $A \cap C = \emptyset$

End of questions