Actuarial Statistics

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Basics

What is actuarial Science?

Actuarial Science is a discipline that applies

- mathematical and statistical methods to assess risk in
- insurance, finance, pensions, investment, and other industries.

Actuaries are professionals who use these methods to solve real-world problems related to risk and uncertainty.

Actuarial Science is the discipline that uses

- mathematics, probability, statistics, and financial theory to study uncertain future events, especially those related to
- insurance, pensions, investments, and other financial risks.

It helps in evaluating the likelihood of events and designing strategies to reduce the financial impact of those events.

Basics

Core Areas of Actuarial Science

Mathematics & Statistics

- Probability theory, statistics, calculus, and linear algebra are the foundation.
- Used to model uncertain future events (e.g., mortality, accidents, market behavior).

Finance & Economics

- Understanding of financial markets, interest rates, and economic principles is crucial.
- Helps in pricing insurance policies, valuing pensions, and managing investments.

Basics

Insurance & Risk Management

- Actuaries calculate premiums, reserves, and predict future claims.
- Analyze risks for life insurance, health insurance, property and casualty insurance, etc.

Data Science & Modeling

- Increasing use of programming and data tools (like R, Python, Excel, SAS).
- Predictive modeling, machine learning, and simulations are becoming standard tools.

Basics

Why It Matters?

- Actuarial science helps institutions make informed decisions by predicting risks and financial outcomes.
- It plays a key role in ensuring the stability and fairness of financial systems.

Basics

Relationship Between Statistics and Actuarial Science

Statistics is one of the core foundations of **actuarial science**. Their relationship is strong and interdependent, as actuarial science heavily relies on statistical methods to make informed, data-driven decisions.

How They Are Related?

Risk Assessment and Prediction

- Actuarial science uses **statistical techniques** to estimate the probability of future events like death, illness, accidents, or financial loss.
- For example, actuaries use survival analysis (a statistical method) to estimate life expectancy.

Basics

Model Building

- Actuaries develop mathematical **models using statistics** to predict claim frequency, insurance premium pricing, and reserve requirements.
- Regression analysis, time series, and Bayesian methods are commonly used.

Data Analysis

- **Statistical analysis** helps actuaries understand patterns, trends, and anomalies in large datasets (e.g., health data, financial records, or customer behavior).

Uncertainty Measurement

- Statistics provides tools (like standard deviation, confidence intervals, and hypothesis testing) to **measure and manage uncertainty**, which is central to actuarial work.

Basics

Decision Making

- By using statistical evidence, actuaries make **reliable decisions** under uncertainty, such as setting insurance premiums or evaluating investment risks.

Statistics provides the tools, and actuarial science applies them to real-world financial and risk-related problems.

Basics

What is insurance?

Insurance is a risk management tool that provides financial protection against unexpected events like accidents, illnesses, or property damage.

Example:

If you buy health insurance, the insurer will help cover medical expenses if you fall ill or get injured, depending on the policy terms.

Basics

Role of insurance in the economy

Risk Management and Protection

Primary Role: Insurance helps individuals and businesses transfer risk of financial loss (e.g., due to accidents, illness, natural disasters) to insurers.

Impact: This reduces uncertainty and enables people and businesses to take productive risks (e.g., starting a business, investing in assets).

Promotes Financial Stability

Insurance acts as a **financial shock absorber** for households and enterprises, especially in crises like floods, fires, or medical emergencies.

This stability helps prevent bankruptcy and poverty due to unexpected losses.

Basics

Encourages Investment and Savings

- Life insurance and pension products help people save over the long term.
- These savings are **pooled and invested** by insurers in capital markets, infrastructure, and government bonds—stimulating overall economic growth.

Supports Credit and Lending

- Banks often require insurance (e.g., on property or life) before issuing loans.
- Insurance **reduces lending risk** for banks, making them more willing to lend, which boosts business activities and home ownership.

Basics

Generates Employment and Income

The insurance sector itself is a major employer: agents, underwriters, actuaries, claim adjusters, etc.

It also supports other industries such as healthcare, automotive, and construction.

Mobilizes Long-Term Capital

Premiums collected by insurers are used for **long-term investments**, often in infrastructure projects, which are essential for sustainable economic development.

Contributes to Tax Revenue

Insurance companies pay **taxes** on their earnings and support economic activity that contributes to a government's revenue base.

Basics

Promotes Innovation and Entrepreneurship

- By managing risks, insurance enables innovators and startups to operate without the fear of devastating losses, fostering economic dynamism.

Basics

What is an Actuary?

An actuary is an expert who assesses financial risks, especially in the fields of insurance and finance, by analyzing data and using models to predict future outcomes.

An **actuary** is a professional who uses **mathematics, statistics, and financial theory** to analyze and manage **risk and uncertainty**, especially in the **insurance**, **pension**, and **finance** industries.

Key Roles of an Actuary:

Risk Assessment and Management

- Actuaries **evaluate the likelihood** of future events (e.g., death, illness, accidents, natural disasters).
- They **quantify risk** and help design strategies to **reduce financial uncertainty**.

Basics

Designing Insurance Products

- Determine **premium rates**, **policy terms**, and **coverage limits** based on statistical analysis.
- Ensure that insurance products are **profitable yet affordable** and comply with regulations.

Pension and Retirement Planning

- Help design and evaluate **pension plans** to ensure **long-term sustainability**.
- Calculate **contributions** and **future liabilities** to make sure funds will be available when needed.

Basics

Financial Forecasting

- Predict future **claim costs**, **investment returns**, and **expenses**.
- Create financial models for **budgeting**, **reserving**, and **capital planning**.

Compliance and Regulation

- Ensure that companies comply with **regulatory standards** for solvency and reporting.
- Actuaries often certify that insurance reserves and risk models meet **legal and professional standards**.

Data Analysis and Decision Support

- Analyze **large datasets** to identify trends and patterns.
- Provide advice to executives on **pricing**, **investments**, and **business strategy**.

Basics

Enterprise Risk Management (ERM)

- Help organizations manage **overall business risks**, not just insurance-related.
- Work in banks, investment firms, and corporations to assess risks from interest rates, credit, market volatility, etc.

Sectors Where Actuaries Work:

- Insurance (Life, Health, General)
- Pensions and Retirement Funds
- Banking and Finance
- Government and Public Policy
- Consulting Firms
- Investment Companies

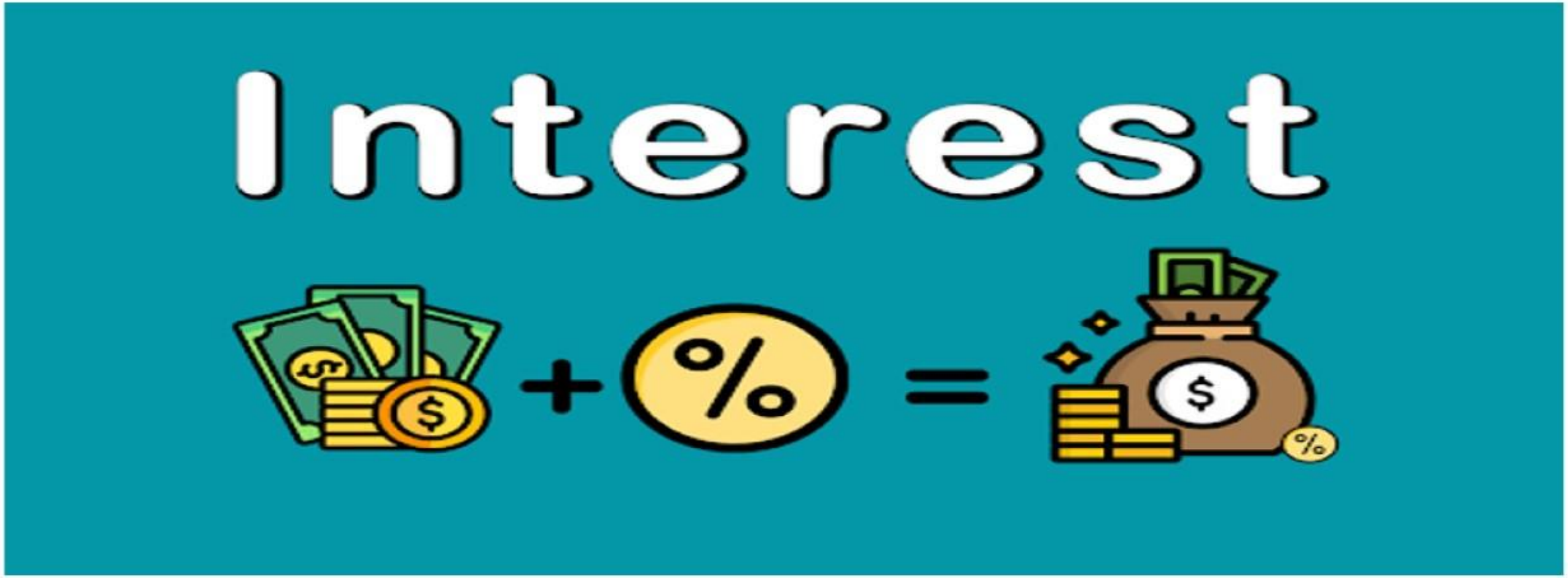
Basics

Technical terms

- Interest(simple, compound)
- Accumulated amount
- Present value
- Effective and nominal rate of interest
- Discount

Fundamental of theory of interest

Interest



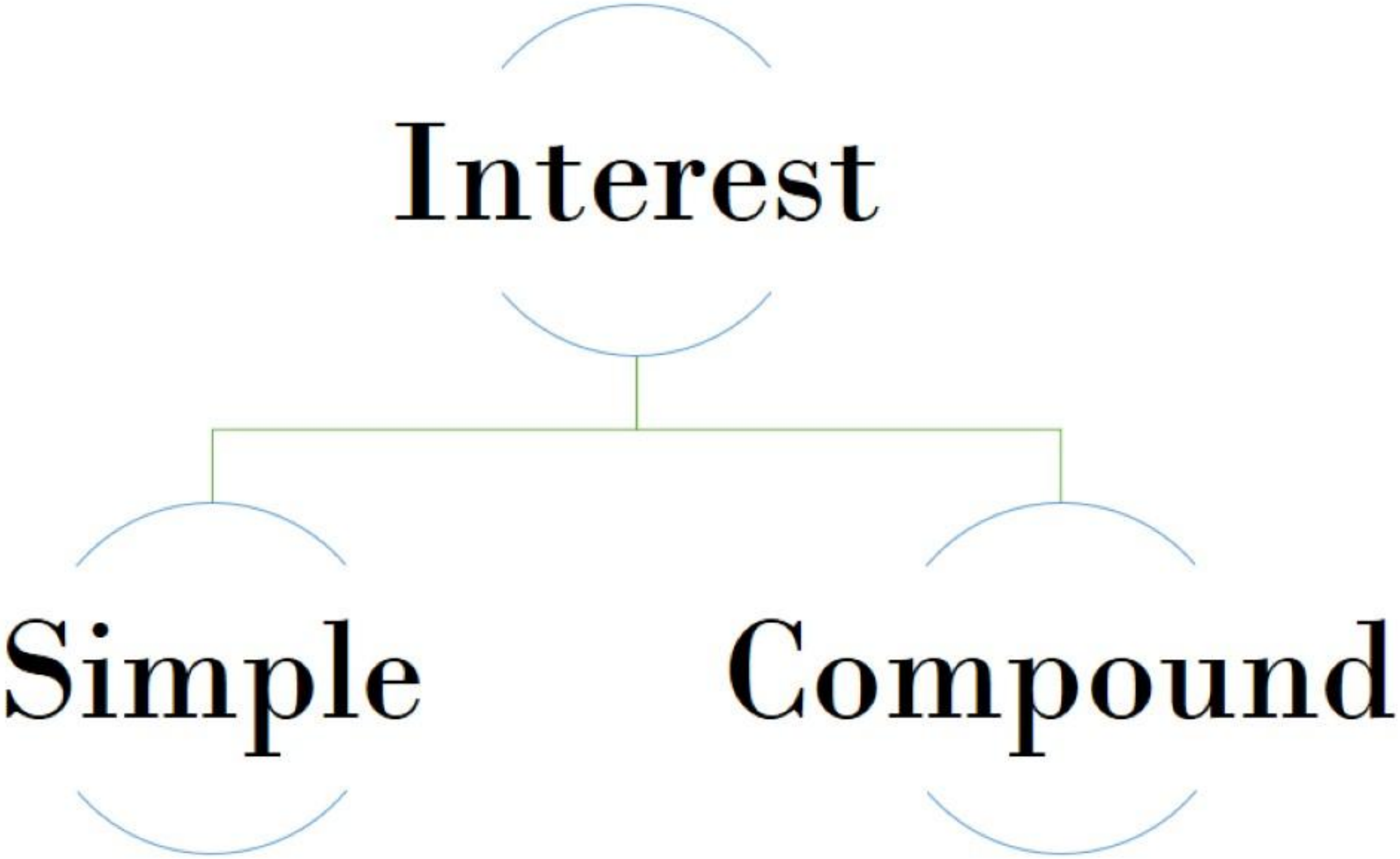
Interest is the **cost of using someone else's money**. When you borrow money, you pay interest; when you lend or invest money, you earn interest. It is usually expressed as a **percentage** of the principal over a certain time period.

Fundamental of theory of interest

Rate of Interest

- The rate of interest is the percentage charged or earned on a principal amount of money over a specific period of time, usually per year.
- It determines how much extra money will be paid (in case of a loan) or earned (in case of investment).

Fundamental of theory of interest



Fundamental of theory of interest

Simple interest (SI)

Interest is calculated only on the original principal.

Formula:

SI= Principal × Rate × Time

$= P \times r \times t$

Interest = the amount of money earned or paid

Principal (P) = the original amount of money invested or borrowed

Rate (r) = annual interest rate (in decimal or percent form)

Time (t) = time in years

Fundamental of theory of interest

Compound Interest (CI)

Interest is calculated on the principal **and** also on accumulated interest.

- A=$P(1+\frac{r}{n})^{nt}$

If compounded annually (n=1)

- A=$P(1+r)^{t}$

CI=A-P

Where:

A= Accumulated amount (total after interest)

P = Principal (initial amount)

r= Annual interest rate (in decimal)

n= Number of times interest is compounded per year

t= Time in years

Fundamental of theory of interest

Accumulated Amount(A)

The **accumulated amount** (also called **future value**) is the **total amount of money** you will have after a certain time, including **both the principal and interest.**

It depends on whether the interest is **simple** or **compound.**

With Simple Interest

- A=P(1+rt) or A=P+SI

With compound interest

- $A=P(1+\frac{r}{n})^{nt}$

Fundamental of theory of interest

Example of Simple interest (SI):

Let's say you invest **$1,000** at an interest rate of **5% per year** for **3 years**. Find out simple interest and accumulated amount.

Solution:

Simple interest:

$SI = P \times r \times t$

$= 1000 \times 0.05 \times 3$

$= \$150$

You will get $150 as interest.

Accumulated amount:

$A = P + SI = \$1000 + \$150 = \$1150$ or

$A = P(1+rt) = \$1000(1+0.05*3) = = \$1150$

Fundamental of theory of interest

Compound Interest (CI)

You invest **$1,000** at **5%** annual interest, compounded **annually** for **3 years**. Find compound interest and accumulated amount.

Solution:

$$A = P(1+\frac{r}{n})^{nt}$$

$$= 1000(1+\frac{0.05}{1})^{1*3}$$

$$= \$1157.63$$

<u>Compound interest:</u>

$$CI = A - P$$

$$= \$1157.63 - \$1000$$

$$= \$157.63$$

<u>Accumulated amount:</u>

$A = P + CI$

$= \$1000 + \$157.63$

$= \$1157.63$

OR

$A = P(1+\frac{r}{n})^{nt}$

$= 1000(1+\frac{0.05}{1})^{1*3}$

$= \$1157.63$

Fundamental of theory of interest

You invest **\$1,000** at **5%** annual interest, compounded **half-yearly** for **3 years**. Find compound interest and accumulated amount.

Solution:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$= 1000\left(1+\frac{0.05}{2}\right)^{2*3}$$

$$= \$1159.69$$

<u>Compound interest:</u>

$$CI = A - P$$

$$= \$1159.69 - \$1000$$

$$= \$159.69$$

Fundamental of theory of interest

Uses of Simple Interest

Short-term loans and advances

Simple interest is often used for short-term borrowing where the interest calculation is straightforward and easy to understand (e.g., personal loans, car loans).

Bank savings accounts with fixed interest

Some savings accounts or fixed deposits calculate interest simply when the period is short.

Commercial transactions

In business, simple interest is used for trade credit or short-term financing between companies.

Easy financial planning

Because of its simplicity, it helps individuals and businesses easily estimate interest without compounding complexities.

Fundamental of theory of interest

Uses of Compound Interest

Long-term investments and savings

Compound interest is used in savings accounts, fixed deposits, retirement funds, and investment portfolios to maximize returns over time.

Loans and mortgages

Most banks and financial institutions use compound interest for home loans, education loans, and credit card balances.

Growth of funds

Compound interest helps in wealth accumulation by reinvesting earned interest, leading to exponential growth.

Valuation of financial products

It is essential in calculating present and future values for bonds, annuities, and other financial instruments.

Fundamental of theory of interest

Present Value

Present Value is the current worth of a future sum of money, discounted at a specific interest rate.

It answers the question:

"How much should I invest today to get a certain amount in the future?"

Present value for simple interest

$$PV = \frac{A}{1 + rt}$$

Where:

PV = Present Value

A = Future (accumulated) amount

r = Annual interest rate (decimal)

t = Time in years

Fundamental of theory of interest

Example-You want $1,200 after 3 years, and the simple interest rate is 5% per year.

Solution

Present value for simple interest

$$PV = \frac{A}{1 + rt}$$

$$= \frac{1200}{1+0.05*3}$$

$$= \$1043.48$$

So, you need to invest $1043.48 today to get $1,200 in 3 years at 5% simple interest.

Fundamental of theory of interest

Present value for compound interest

$$PV = \frac{A}{(1+\frac{r}{n})^{nt}}$$

If compounded annually (n=1)

$$PV = \frac{A}{(1+r)^{t}}$$

PV = Present Value

A = Future (accumulated) amount

r = Annual interest rate (decimal)

t = Time in years

n = Number of compounding periods per year

Fundamental of theory of interest

Example- You want \$1,200 after 3 years, and the annual interest rate is 5%, compounded annually.

Solution:

$$PV = \frac{A}{(1+r)^t}$$

$$PV = \frac{1200}{(1+0.05)^3}$$

$$PV = \$1036.48$$

So, you need to invest \$1036.48 today to get \$1200 in 3 years at 5% interest.

Fundamental of theory of interest

Why Present Value is Important?

- Used in investment decisions and loan planning
- Helps evaluate the value of future cash flows today
- Essential in financial planning, insurance, and actuarial science

Fundamental of theory of interest

Difference between SI and CI

Feature	Simple Interest	Compound Interest
Interest Calculation	On **original principal only**	On **principal + accumulated interest**
Growth Pattern	**Linear** (constant)	**Exponential** (accelerates over time)
Formula	A=P(1+rt)	A=P(1+r)^t annual compounding)
Interest Over Time	Same every year	Increases each year

Relationship between SI and CI

$$A_{CI} > A_{SI}$$

Fundamental of theory of interest

Nominal and effective rate of interest

The nominal interest rate is the stated or advertised annual interest rate not accounting for compounding within the year.

It includes the frequency of compounding, like annually, semi-annually, quarterly, etc., but doesn't reflect the actual interest earned or paid in a year due to that compounding.

Example:

- If 5% annual interest is compounded semi-annually, the nominal rate is still:
- 5% per annum, compounded semi-annually, compounded semi-annually
- This does not mean you earn 12% total interest in a year — because monthly compounding increases the actual amount you earn (or owe).

Fundamental of theory of interest

Effective rate of interest (effective Annual Rate, EAR)

The effective interest rate shows the actual interest earned or paid in one year, after accounting for compounding.

It's especially important when comparing different loans or investments with different compounding frequencies.

$$r_{effective} = (1 + \frac{r_{nom}}{n})^{n} - 1$$

Accumulated value, A=$P(1 + \frac{r_{nom}}{n})^{nt}$

r_{nom} = nominal annual rate (in decimal)

n = number of compounding periods per year

Example:

With a 5% nominal annual interest rate, compounded semi-annually:

$r_{effective} = (1 + \frac{.05}{2})^{2} - 1$=5.0625%

Fundamental of theory of interest

Example:

You invest $1,000 at a nominal interest rate of 12% per annum, compounded at different frequencies for 1 year.

Compounding	n	Nominal Rate	Effective Rate (EAR)	Accumulated Amount ($1000)
Annually	1	12%	$(1.12)^{1} - 1 = 12.00\%$	$1,120.00
Half-yearly	2	12%	$(1.12)^{2}$-1=12.36%	$1,123.60
Quarterly	4	12%	$(1.03)^{4} - 1 = 12.55\%$	$1,125.51
Monthly	12	12%	$(1.01)^{12} - 1 = 12.68\%$	$1,126.83
Daily	365	12%	$(1.00)^{365} - 1 = 12.75$	$1,127.47

Fundamental of theory of interest

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Where:

- $P = 1000$
- $r = 0.12$
- $t = 1$
- n = compounding frequency (varies)

Annually (n = 1)

$$A = 1000\left(1+\frac{0.12}{1}\right)^{1} = 1000 \times (1.12) = \boxed{1120.00}$$

$$\text{CI} = 1120.00 - 1000 = \boxed{120.00}$$

Semi-Annually / Half-Yearly (n = 2)

$$A = 1000\left(1+\frac{0.12}{2}\right)^{2} = 1000 \times (1.06)^2 = 1000 \times 1.1236 = \boxed{1123.60}$$

$$\text{CI} = 1123.60 - 1000 = \boxed{123.60}$$

Monthly (n = 12)

$$A = 1000\left(1+\frac{0.12}{12}\right)^{12} = 1000 \times (1.01)^{12} = 1000 \times 1.1268 = \boxed{1126.83}$$

$$\text{CI} = 1126.83 - 1000 = \boxed{126.83}$$

Daily (n = 365)

$$A = 1000\left(1+\frac{0.12}{365}\right)^{365} = 1000 \times (1.0003288)^{365}$$

$$A \approx 1000 \times 1.1275 = \boxed{1127.47}$$

$$\text{CI} = 1127.47 - 1000 = \boxed{127.47}$$

Fundamental of theory of interest

Relation between nominal and effective rate of interest

$$r_{nom} = n\{(1 + r_e)^{\frac{1}{n}} - 1\}$$

Fundamental of theory of interest

Feature	Nominal Rate of Interest	Effective Rate of Interest (EAR)
🔷 Definition	Stated annual interest rate (does not account for compounding)	Actual annual interest earned/paid after compounding
🔄 Considers Compounding?	❌ No	✅ Yes
🔧 Formula	Stated as: r_{nom}	Stated as: $r_{effective} = (1 + \frac{r_{nom}}{n})^{n} - 1$
🔁 Depends on Frequency?	Yes, usually mentioned with compounding frequency (e.g., 10% compounded monthly)	Yes, increases with compounding frequency
💰 Used For	Quoting interest rates in contracts (loans, savings, etc.)	Comparing different interest offers accurately
📈 Value	Always less than or equal to effective rate	Always greater than or equal to nominal rate
🧮 Example (12% annually)	12% nominal rate, compounded monthly	Effective rate ≈ 12.68%

Fundamental of theory of interest

Effective rate of discount

The Effective Rate of Discount (denoted as d) is the true rate of reduction in value over one year based on the final amount (future value) rather than the present value.

$$d = \frac{r}{1 + r}$$

Suppose the effective rate of interest is 10% (i.e., r=.10).

Then the effective rate of discount is:

$$d = \frac{.10}{1+.10} = 9.09\%$$

Term	Meaning
10% Interest Rate	If you invest \$100 today, you will receive \$110 after 1 year.
9.09% Discount Rate	If you are to receive \$110 after 1 year, you would get \$100 today (i.e., a 9.09% discount in advance).

Fundamental of theory of interest

Relation between effective rate of interest in terms of effective rate of discount

$$r = \frac{d}{1 - d}$$

Relation between discount and present value of r payable at the end of a year

$$d = PV.r$$

Fundamental of theory of interest

Example:

Find the effective rate of discount when the effective annual interest rate is 12%.

Solution:

$$d = \frac{r}{1+r}$$

$$d = \frac{.12}{1+.12} = 10.71\%$$

If the effective interest rate is 12%, then the effective rate of discount is 10.71%.

That means:
If someone deducts 10.71% in advance from a payment of \$100 to be paid after 1 year, you would receive \$89.29 today, and it would grow to \$100 in 1 year.

Now, find the effective rate of interest

We know, $FV=PV(1+r_{eff})$

Where, $FV=100$ (Future value after 1 year)

$PV=89.29$ (present value today)

$r_{eff}=?$ (effective interest rate)

Fundamental of theory of interest

$$FV = PV(1 + r_{eff})$$

$$100 = 89.29(1 + r_{eff})$$

$$r_{eff} = 0.12 = 12\%$$

If you receive \$89.29 today and it becomes \$100 after 1 year, the effective interest rate is 12%.

Here the nominal discount rate was 10.71%, but the effective interest rate is 12%.

Fundamental of theory of interest

Relation between Nominal rates of discount and effective rate of discount

$$d = 1 - \left(1 - \frac{d^{(m)}}{m}\right)^{m}$$

The effective rate of discount in terms of nominal rate of discount.

$$d^{(m)} = m\left(1 - (1 - d)^{\frac{1}{m}}\right)$$

Nominal rate of discount in terms of effective rate of discount

Fundamental of theory of interest

Suppose the **nominal rate of discount is 12% per annum convertible quarterly.**

That means $d^{(4)} = 0.12$.

- **Step 1. Per-quarter discount**

$$\frac{d^{(4)}}{4} = \frac{0.12}{4} = 0.03$$

- **Step 2. Present value of 1 due in 1 year**

$$PV = \left(1 - \frac{0.12}{4}\right)^4 = (0.97)^4 = 0.8853$$

- **Step 3. Effective annual discount**

$$d = 1 - PV = 1 - 0.8853 = 0.1147$$

✅ So, the **effective discount rate** is **11.47%**.

Fundamental of theory of interest

Example

Suppose the **effective annual discount is** $d = 15\%$.

Find the **equivalent nominal discount rate** $d^{(4)}$ (convertible quarterly).

✅ Solution

- Effective discount: $d = 0.15$.
- Formula:

$$d^{(4)} = 4\left[1 - (1 - d)^{1/4}\right]$$

$$d^{(4)} = 4\left[1 - (0.85)^{1/4}\right]$$

$$(0.85)^{1/4} = 0.9629$$

$$d^{(4)} = 4 \times (1 - 0.9629) = 4 \times 0.0371 = 0.1484$$

✅ Final Answer

- **Effective discount = 15%**
- **Equivalent nominal discount (quarterly) = 14.84%**

Fundamental of theory of interest

Relation Between nominal rate of interest(r_{nom}) and the nominal rate of discount $d^{(m)}$

$$d^{(m)} = \frac{r_{nom}}{1 + \frac{r_{nom}}{m}}$$

The nominal rate of discount in terms of nominal rate of interest

$$r_{nom} = \frac{d^{(m)}}{1 - \frac{d^{(m)}}{m}}$$

The nominal rate of interest in terms of nominal rate of discount.

Fundamental of theory of interest

A bank quotes a **nominal interest rate** of 8% per year, compounded quarterly.

(a) Find the **nominal discount rate** of interest convertible quarterly.

(b) Verify your answer by computing the **effective annual rates** of interest and discount.

Solution:

Nominal discount rate:

$$d^{(m)} = \frac{r_{nom}}{1 + \frac{r_{nom}}{m}}$$

$$d^{(4)} = \frac{.08}{1 + \frac{.08}{4}}$$

$$=0.07843$$

$$=7.843\%$$

Nominal discount rate $d^{(4)}$=7.843%

- If you are to receive $100 after 1 year, using **7.843% nominal discount quarterly**, the **present value** today is slightly less than $100 (about $92.15).
- Each quarter, 1/4 of 7.843% (~1.9607%) is effectively deducted from the future value to get the present value.

Fundamental of theory of interest

Effective annual rates of interest,

$$r_{effective} = (1 + \frac{r_{nom}}{n})^{n} - 1$$

$$r_{effective} = (1 + \frac{.08}{4})^{4} - 1$$

$$= 0.08243$$

$$= 8.24\%$$

Effective annual rates of discount using effective interest.

$$d_{eff} = \frac{r_{effective}}{1 + r_{effective}}$$

$$d_{eff} = \frac{0.08243}{1 + 0.08243}$$

$$= 7.615\%$$

From effective interest to effective discount

Fundamental of theory of interest

Effective annual rates of discount using nominal discount.

$$d_{eff} = 1 - \left(1 - \frac{d^{(m)}}{m}\right)^{m}$$

$$d_{eff} = 1 - \left(1 - \frac{0.07843}{4}\right)^{4}$$

$$= 7.615\%$$

From nominal discount to effective discount

✅ Verification

From **effective interest** → Effective discount = 7.6155%

From **nominal discount** → Effective discount = 7.6155%

Since both give the **same effective annual discount**, our result is **verified**.

Fundamental of theory of interest

A bank quotes a **nominal discount rate** of $d^{(12)}$=9%per year, convertible monthly (m=12).

(a) Find the **nominal interest rate** $r^{(12)}$ convertible monthly that is equivalent to this nominal discount.

(b) Verify the result by computing the effective annual rates of interest and discount.

Solution:

Solution

(a) Formula and calculation

We use the relation

$$r^{(m)} = \frac{m\, d^{(m)}}{m - d^{(m)}}.$$

Substitute $m = 12$ and $d^{(12)} = 0.09$:

$$r^{(12)} = \frac{12 \times 0.09}{12 - 0.09} = \frac{1.08}{11.91} \approx 0.0906801008.$$

$$\boxed{r^{(12)} \approx 0.0906801 = 9.06801\% \text{ (nominal, monthly)}}$$

Fundamental of theory of interest

(b) Verification via effective rates

1. **Effective annual interest** from $r^{(12)}$:

$$i = \left(1 + \frac{r^{(12)}}{12}\right)^{12} - 1 = \left(1 + \frac{0.0906801008}{12}\right)^{12} - 1 \approx 0.0945454873 = 9.45455\%.$$

2. **Effective annual discount** from that i:

$$d = \frac{i}{1+i} = \frac{0.0945454873}{1.0945454873} \approx 0.0863787649 = 8.63788\%.$$

3. **Effective annual discount** computed directly from the nominal discount $d^{(12)}$:

$$d = 1 - \left(1 - \frac{d^{(12)}}{12}\right)^{12} = 1 - \left(1 - \frac{0.09}{12}\right)^{12} \approx 0.0863787649 = 8.63788\%.$$

Both methods give the same effective annual discount (≈ **8.63788%**), so the nominal interest $r^{(12)} \approx 9.06801\%$ is correctly found.

Fundamental of theory of interest