

Introduction :

The code in the solution 1 and 2 count the number of divisors for a given number

- 1) For the solution 1 we wrote a function that takes a parameter n and count a number of divisors for a given number by checking the $n\%d=0$ (if this number is divide d so d is a divisor of n) after that we calculate the sum of d all these under the condition $d\leq n$.
- 2) For the solution 2 we count the number of divisor's by doing the same thing in the solution 1 but in the different condition which ($d*d\leq n$) .
- 3) & 4) In general we get the follow estimation :

```
%%timeit
def count_divisors(n):
    count = 0
    d=1
    while d <= n :
        if n%d==0 :
            count+=1
        d+=1
    return count
```

140 ns \pm 0.278 ns per loop (mean \pm std. dev. of 7 runs, 10,000,000 loops each)

```
%%timeit
def count_divisors(n):
    count = 0
    d=1
    while d * d <= n :
        if n%d==0 :
            count+=1 if n/d == d else 2
        d+=1
    return count
```

138 ns \pm 1.32 ns per loop (mean \pm std. dev. of 7 runs, 10,000,000 loops each)

Wich means in general the solution 2 is faster than the first one .

For $n = 10000$:

```
%%timeit
def count_divisors(n):
    count = 0
    d=1
    while d <= n :
        if n%d==0 :
            count+=1
        d+=1
    return count
count_divisors(10000)
```

2.64 ms \pm 103 μ s per loop (mean \pm std. dev. of 7 runs, 100 loops each)

```
%%timeit
def count_divisors(n):
    count = 0
    d=1
    while d * d <= n :
        if n%d==0 :
            count+=1 if n/d == d else 2
        d+=1
    return count
count_divisors(10000)
```

33 μ s \pm 1.49 μ s per loop (mean \pm std. dev. of 7 runs, 10,000 loops each)

For n = 500000 :

```
%%timeit
def count_divisors(n):
    count = 0
    d=1
    while d <= n :
        if n%d==0 :
            count+=1
        d+=1
    return count
count_divisors(500000)
```

135 ms \pm 2.79 ms per loop (mean \pm std. dev. of 7 runs, 10 loops each)

```
%%timeit
def count_divisors(n):
    count = 0
    d=1
    while d * d <= n :
        if n%d==0 :
            count+=1 if n/d == d else 2
        d+=1
    return count
count_divisors(500000)
```

234 μ s \pm 6.31 μ s per loop (mean \pm std. dev. of 7 runs, 1,000 loops each)

Big-O notation:

1) $T(n) = 3n^3 + 2n^2 + \frac{1}{2}n + 7$

1) Méthode 1

We have:

$$T(n) = 3n^3 + 2n^2 + \frac{1}{2}n + 7$$

if $n \geq 1$

since: $2n^2 \leq 2n^3$

$$\frac{1}{2}n \leq \frac{1}{2}n^3 \text{ and } 7 \leq 7n^3$$

So:

$$T(n) \leq 3n^3 + 2n^3 + \frac{1}{2}n^3 + 7n^3$$
$$\leq \frac{25}{2}n^3$$

$$\text{So } \lambda = \frac{25}{2}$$

$$\Rightarrow T(n) = \Theta(n^3)$$

Méthode 2

We have:

$$\Theta(f(n)) \leq f(n)$$
$$\lim_{n \rightarrow \infty} \frac{T(n)}{f(n)} = \lim_{n \rightarrow \infty} \frac{3n^3 + 2n^2 + \frac{1}{2}n + 7}{n^3}$$

$$= 3$$

$$\lambda = 3$$

$$\text{So } T(n) = \Theta(n^3)$$

2)

To show that n^k is not $O(n^{k-1})$, we need to show that there does not exist a positive constant C such that $n^k \leq Y \cdot n^{k-1}$ for all n greater than some n_0 .

Assume that n^k is $O(n^{k-1})$, then by definition of big-O notation, there exists a constant $C > 0$ and a positive integer n_0 such that:

$$n^k \leq Y \cdot n^{k-1} \text{ for all } n \geq n_0$$

Dividing both sides by n^{k-1} , we get:

$$n \leq Y$$

However, this is a contradiction because n is an unbounded variable that can grow arbitrarily large. Therefore, there is no constant C that can satisfy the above inequality for all $n \geq n_0$, and we have shown that n^k is not $O(n^{k-1})$.

Multiplication of matrices :

With C

```
#include<stdio.h>
```

```
void multiply(int r1, int c1, int r2, int c2);
```

```
int main()
```

```
{
```

```
int i,j,k,r1,c1,r2,c2;
```

```
printf("Enter row and column of first matrix\n");

scanf("%d%d", &r1, &c1);

printf("Enter row and column of second matrix\n");

scanf("%d%d", &r2, &c2);

multiply(r1,c1,r2,c2);

return 0;

}
```

```
void multiply(int r1, int c1, int r2, int c2)

{

int i,j,k;

float a[10][10], b[10][10], mul[10][10];

if(c1==r2)

{

printf("Enter elements of first matrix:\n");

for(i=0;i< r1;i++)

{

for(j=0;j< c1;j++)

{

printf("a[%d][%d]=",i,j);

scanf("%f", &a[i][j]);

}

}

printf("Enter elements of second matrix:\n");
```

```
for(i=0;i< r2;i++)

{

for(j=0;j< c2;j++)

{

printf("b[%d][%d]=",i,j);

scanf("%f", &b[i][j]);

}

}

for(i=0;i< r1;i++)

{

for(j=0;j< c2;j++)

{

mul[i][j] = 0;

for(k=0;k< r2;k++)

{

mul[i][j] = mul[i][j] + a[i][k]*b[k][j];

}

}

}

printf("Multiplied matrix is:\n");

for(i=0;i< r1;i++)

{

for(j=0;j< c2;j++)

{
```

```
    printf("%f\t", mul[i][j]);  
  
    }  
  
    printf("\n");  
  
    }  
  
    }  
  
else  
  
{  
  
    printf("Dimension do not match for multiplication.");  
  
    }  
  
}
```

QUIZ

1) The time complexity for the following fragment is :

A) $O(n)$

2) The time complexity for the following fragment is :

D) $O(\log_k(n))$

3) The time complexity for the following fragment is :

C) $O(n*m)$