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Assignment 2 (individual)
Backpropagation Neural Network

a) Calculation of the output $y_i(p)$ at neuron 3,4 and. subsequently, Calculation of the error e:

$$y_i(p) = \text{sigmoid}\left[\sum_{i=1}^{n} x_i(p) \cdot w_{ij}(p) - \theta_j\right] \Rightarrow s(t) = \frac{1}{1+e^{-t}}$$

$$y_3 = \text{sigmoid}\left(((x_1 w_{13}) + (x_2 w_{23})) - (-0.2 \times \cancel{1})\right)$$

$$y_3 = \frac{1}{1 + e^{-[((0\times 1.1)+(1\times 0.5)) - (-0.2)]}} = 0.6682$$

$$y_4 = \text{sigmoid}\left((x_1 w_{14}) + (x_2 w_{24}) - \theta_4\right) \quad \cancel{or} \quad \theta_4 = -1\times 0.6$$

$$y_4 = \frac{1}{1 + e^{-[((0\times 0.8)+(1\times 0.8)) - (-0.6)}} = \cancel{}\ 0.8022.$$

$$y_5 = \text{sigmoid}\left((y_3 w_{35}) + (y_4 w_{45}) - \theta_5\right) \quad \text{or } \theta_5 = -1\times 0.3$$

$$y_5 = \frac{1}{1+e^{-[(0.6682\times 0.9)+(0.8022\times(-1.3)) + 0.3}} = 0.4647.$$

The error :

$$e = y_{d,5} - y_5 = 0 - 0.4647 = -0.4647$$

b) Calculation of the error gradients $\delta_5(P)$, $\delta_3(P)$ and $\delta_4(P)$:

$$\delta_5 = y_5(1-y_5)e = 0.4647(1-0.4647)(-0.4647) = -0.1156.$$

~~The learning rate $\alpha = 0.4$.~~

~~$\Delta_{35} = \alpha \cdot y_3 \cdot \delta_5 = 0.4 \times 0.6682 \times (-0.1156) = -0.0309$~~

~~$\Delta_{45} = \alpha \cdot y_4 \cdot \delta_5 = 0.4 \times 0.8022 \times (-0.1156) = -0.0371$~~

~~$\Delta\theta_5 = \alpha \cdot (-1) \cdot \delta_5 = 0.4(-1)(-0.1156) = 0.0462$~~

~~For δ_3 and δ_4~~ :

$$\delta_3 = y_3(1-y_3)\,\delta_5 \cdot w_{35} = 0.6682(1-0.6682)(-0.1156)(0.9) = -0.0231$$

$$\delta_4 = y_4(1-y_4)\,\delta_5 \cdot w_{45} = 0.8022(1-0.8022)(-0.1156)(-1.3) = 0.0238.$$

c) Determination of the weight Corrections.

The learning rate $\alpha = 0.4$.

$$\Delta_{35} = \alpha \cdot y_3 \cdot \delta_5 = 0.4 \times 0.6682 \times (-0.1156) = -0.0309$$

$$\Delta_{45} = \alpha \cdot y_4 \cdot \delta_5 = 0.4 \times 0.8022 \times (-0.1156) = -0.0371$$

$$\Delta_{13} = \alpha \cdot x_1 \cdot \delta_3 = 0.4 \times 0 \times (-0.0231) = 0$$

$$\Delta_{23} = \alpha \cdot x_2 \cdot \delta_3 = 0.4 \times 1 \times (-0.0231) = -0.0092$$

$$\Delta_{14} = \alpha \cdot x_1 \cdot \delta_4 = 0.4 \times 0 \times (0.0238) = 0$$

$$\Delta_{24} = \alpha \cdot x_2 \cdot \delta_4 = 0.4 \times 1 \times (0.0238) = 0.0095$$

$$\Delta\theta_5 = \alpha \cdot (-1) \cdot \delta_5 = 0.4(-1) \cdot (-0.1156) = +0.0462$$

$$\Delta\theta_3 = \alpha \cdot (1) \cdot \delta_3 = 0.4(1) \cdot (-0.0231) = -0.0092$$

$$\Delta\theta_4 = \alpha \cdot (-1) \cdot \delta_4 = 0.4(-1) \cdot (0.0238) = -0.0095$$

d) update of the weight and threshold accordingly:

$$W_{13} = W_{13} + \Delta W_{13} = 1.1 + 0 = 1.1$$

$$W_{14} = W_{14} + \Delta W_{14} = 0.8 + 0 = 0.8$$

$$W_{23} = W_{23} + \Delta W_{23} = 0.5 + (-0.0092) = 0.4908$$

$$W_{24} = W_{24} + \Delta W_{24} = 0.8 + 0.0095 = 0.8095$$

$$W_{35} = W_{35} + \Delta W_{35} = 0.9 + (-0.0309) = 0.8691$$

$$W_{45} = W_{45} + \Delta W_{45} = -1.3 + (-0.0371) = -1.3371$$

$$\theta_3 = \theta_3 + \Delta\theta_3 = -0.2 + (-0.0092) = -0.2092$$

$$\theta_4 = \theta_4 + \Delta\theta_4 = -0.6 + (0.0095) = \cancel{-0.5905}\ -0.6095$$

$$\theta_5 = \theta_5 + \Delta\theta_5 = -0.3 + (+0.0462) = \cancel{-0.3462}\ -0.2538$$