-MATHEMATICAL MODELLING OF A ROLLER COASTER

Abstract

Keywords:

Projectile Motion, Roller Coasters, Valley Out

Problem Statement

Your city wants to setup a roller-coaster that is stretched all around the city. Design a MM answering questions like cost of setup and maintenance, ticket price, energy requirements, effect on happiness of the people etc.

Introduction

Roller coaster, elevated railway with steep inclines and descents that carries a train of passengers through sharp curves and sudden changes of speed and direction for a brief thrill ride. Found mostly in amusement parks as a continuous loop, it is a popular leisure activity.

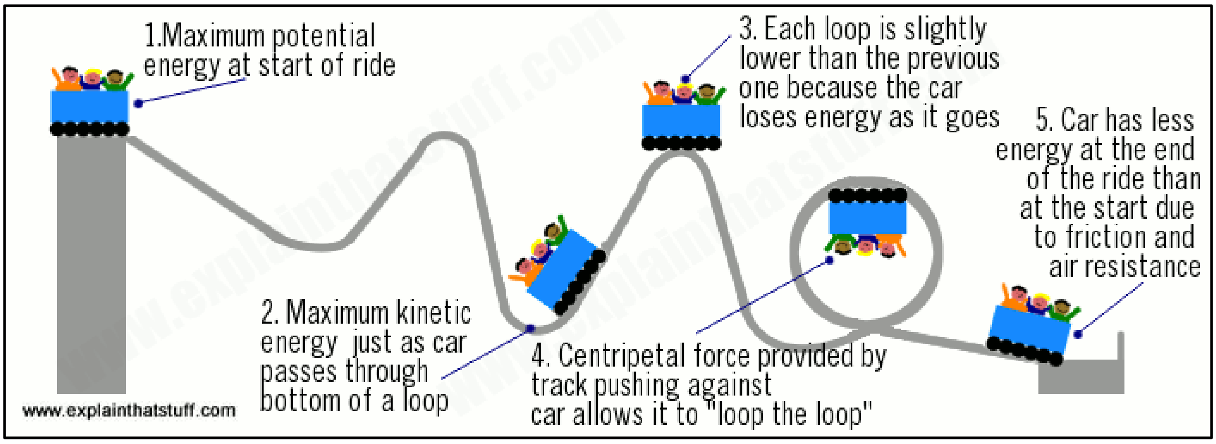
Describing a Roller Coaster

- Mechanism –

Roller coaster trains are not typically powered. Most are pulled up a lift hill by a chain or cable and released downhill.

The potential energy accumulated by the rise in height is transferred to kinetic energy, which is then converted back into potential energy as the train rises up the next hill.

Changes in elevation become smaller throughout the track's course, as some mechanical energy is lost to friction. A properly-designed, outdoor track will result in a train having enough kinetic energy to complete the entire course under a variety of stressful weather conditions.



ELEMENTS / PARTS THAT MAKE UP A ROLLER COASTER –

Roller Coaster is an amusement ride. So, we always want the rider to experience a thrilling ride.
Roller coaster elements are the individual parts of roller coaster design and operation, such as a track, hill, loop, or turn.
Variations in normal track movement that add thrill or excitement to the ride are often called "thrill elements".

Launch Track - A launch track is a section of a launched roller coaster in which the train is accelerated to its full speed in a matter of seconds. A launch track is always straight and is usually banked upward slightly, so that a train would roll backward to the station in the event of a loss of power.

For example, the Ferrari Rossa uses a hydraulic launch system which propels the roller coaster to a speed of 240 km/hr in just 250 m

Lift Hill - A lift hill, or chain lift, is often the initial upward section of track on a typical roller coaster that initially transports the roller coaster train to an elevated point. Upon reaching the top, the train is then disengaged from the lift hill and allowed to coast through the rest of the roller coaster's circuit.

Station - The station is the area where guests waiting in a line queue board a roller coaster. The line often divides into lanes to allow guests to board each row. In addition to boarding, passengers also exit the ride within the station, but this is not always at the same location where boarding passengers are waiting.

Banked turn - A banked turn is when the track twists from the horizontal plane into the vertical plane, tipping the train to the side in the direction of the turn. Banking is used to minimize the lateral G-forces on the riders to make the turn more comfortable.

Braker Run - A brake run on a roller coaster is any section of track meant to slow or stop a roller coaster train. Brake runs may be located anywhere or hidden along the circuit of a coaster and may be designed to bring the train to a complete halt or to simply adjust the train's speed.

Head Chopper - A head chopper is any point on a roller coaster where the support structure of the ride or the track itself comes very close to the passengers' heads, or at least appears to do so.
All head choppers are, of course, designed so that even the tallest rider, with both hands up, would be unable to touch the structure; although if a rider exceeding the maximum height does board the coaster it could be potentially dangerous.

Helix - A helix is a balanced spiral, generally exceeding 360°. Helixes can spiral upward or downward.

Tunnels - Some roller coasters feature tunnels, and they may include special effects such as lighting, fog, and sound. The Iron Rattler at Six Flags Fiesta Texas, for example, features a darkened, above-ground tunnel.

MAIN ASSUMPTIONS MADE

As mentioned in the question. It is mentioned that we need to design a roller coaster “stretching” across the city . It is not mentioned whether the entire track has to loop around the city or whether we plan to build multiple roller coaster tracks stretching around the city.

So for our modelling we assume that we build a roller coaster which covers approx. … area . And to in order to fulfil the criteria of “stretching across the city “ we ensure that we have multiple roller coaster / amusement parks across the city.

Though Roller coaster are of several types and has several features, we assume that our roller coaster is a traditional “sit-down” roller coaster.

More assumptions have been made later on also

<u>Also note that all the calculations are made in terms of S.I units.</u>

QUESTION TO BE ANSWERED

1. **<u>HOW TO BUILD A ROLLER COASTER ?</u>**

   Before even answering the problem, we need a simple mechanical model which can give such useful information such as the speed of roller coaster , the normal reaction on the track surface, change in inclination , lateral forces experienced .

   These useful information helps us the later questions

   We start with a simple model of just hills , planar curves , and circular loops Later we discuss the flaws of the mechanical model and what are the improvements that can be made.

2. **<u>DESIGNING A THRILL FACTOR FUNCTION , PROFIT MODEL , RUNNING COST MODEL</u>**

   Using the insights gained from the mechanical model we discuss about a thrill factor and how we can use our thrill factor function as a metric to make our roller coaster more exciting and thrilling

   To make any business successful it should make a profit , a elaborate profit model is designed such that it includes factors like region of the roller coaster , demography of the region , and also our thrill factor function

To aid the profit model , we have also discussed in brief about the running cost model

3. **HOW TO MAKE A ROLLER COASTER MORE SAFE , MORE THRILLING AND REALISTIC , AND IMPROVING OUR MODEL ?**

4. **HOW DOES A ROLLER COASTER IMPROVE A REGION?**

SIMPLE MECHANICAL MODEL

We assume that our roller coaster is made of five simple components:

1. Lift Hill
2. curves
3. Circular loops
4. Final braking zone

Some notation that will be used throughout the paper :

Symbol	meaning
m	Mass of roller coaster
g	Acceleration due to gravity
v	Velocity of current state
u	Initial velocity with which coaster enters a system(maybe a loop , curve , hill etc.
x,z	Coordinates represent horizontal plane
y	Height of coaster at a certain stage
N	Normal reaction

ESTIMATING A CLIMB /DROP CURVE AS CUBIC POLYNOMIAL

A hill is a 2d curve aligned in the vertical plane and takes the roller coaster from one height to another.

To design the curve let us assume a 2 D coordinate system. So, assume that the roller coaster enters the curve at (0,0,0) and the roller coaster exits at point (R,H,0) where R is defined as Range of the curve and H is defined as Height of the curve.

For sake of simplicity we use the XY plane only there by we use coordinates of the form (x,y) in the following explanation

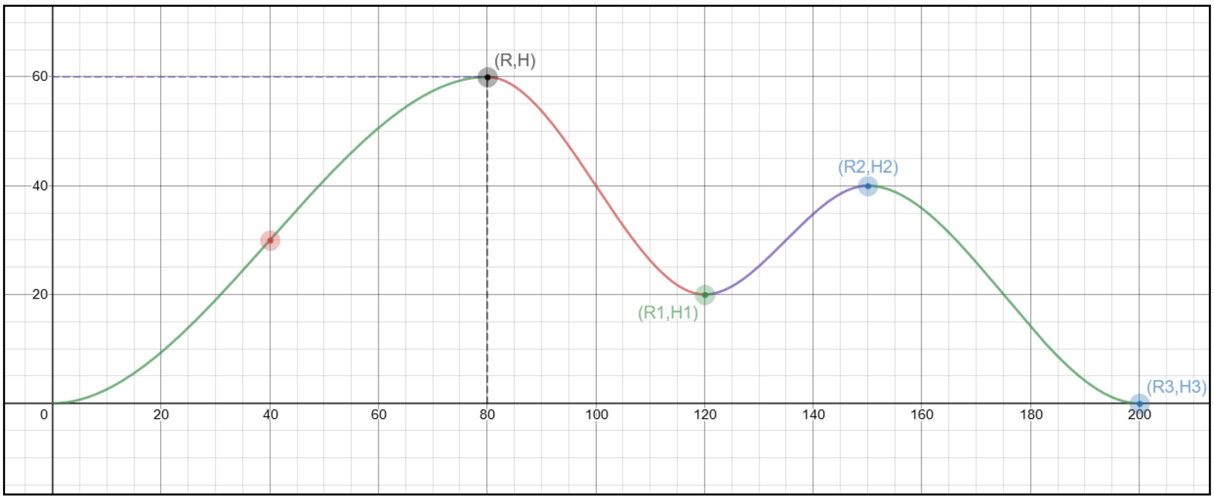
We want a function y = f(x) which is a cubic polynomial which satisfies above parameters . also, the slope of the curve at (0,0) and at (R,H) is 0

By using the above constraints, we get;

$y = f(x) = lx^2 - kx^3 \ldots (1.1)$; Where ; $l = \frac{3H}{R^2}$; $k = \frac{2H}{R^3}$; ... (1.2)

Average slope of the curve = H/R { where H , R are considered with sign }

(Refer appendix (1.1) on how to extend the track by adding more elements)



(plot made in Desmos ; (https://www.desmos.com/calculator/eratejkvna for actual model)

Now we use this model curve for the basis of all the calculations related to the mechanics of the tracks.

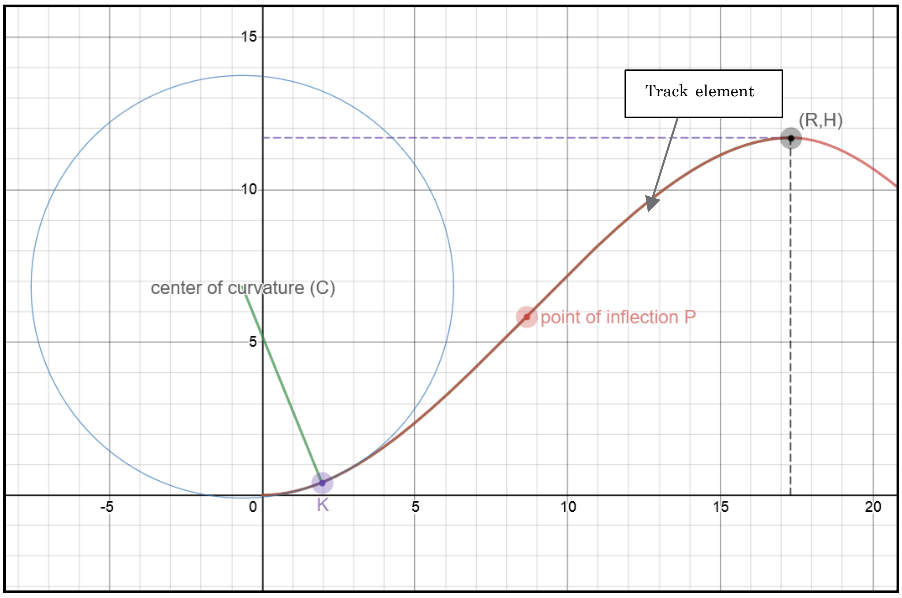
MECHANICS OF THE POLYNOMIAL CURVE

Assumption:

Consider the roller coaster enters the track element with a speed of u m/s, entire roller is assumed to be a point particle of mass m. We neglect friction between the rails and drag produced by the motion of the coaster.

Now any continuous and differentiable curve at a point can be approximated using radius of curvature concept

We use the concept of radius of curvature to estimate the forces experienced by the roller coaster at a particular point (Refer appendix 1.2 for the calculations involved)



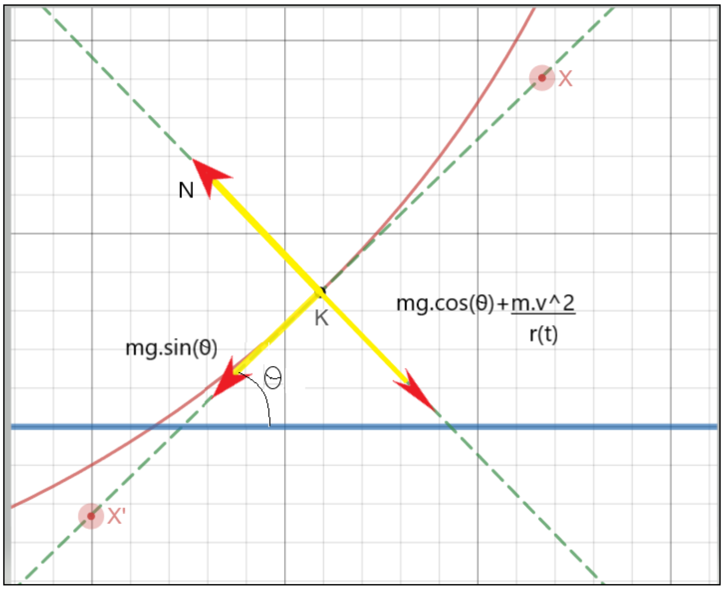
Say at a point K (t,f(t)) radius of the curvature is r(t) where r(x) is a function which gives the radius of curvature at x.

Since energy is conserved as there are no drag forces, we have …

$\frac{1}{2}mv^2 + mg.f(t) = \frac{1}{2}mu^2$;… where f(t) is height gained/lost by the roller coaster.

Using this we can get velocity as function of t so let this function be v(t)

$v(t) = \left(u^2 - 2g.f(t)\right)^{0.5}$ …(2.1)



Now at point (t,f(t)) we can simplify our model curve as following free body diagram ;

By using mechanics, we can see (in the free body diagram given aside) that at only mg.sin(θ) is the only external force and this is cause of deceleration.

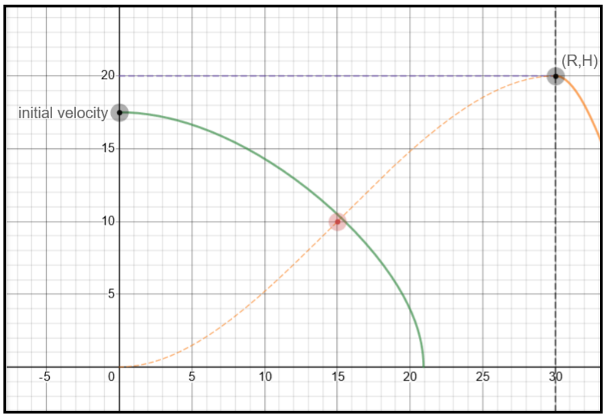
Here slope of XX' is equal to the derivative of the f(x) .

It is very important to note that if the center of the curvature circle is below the XX' line

then N=mg.cos(θ)$-\dfrac{mv^2}{r(x)}$ (more details in appendix 1.2)

Velocity vs position curve

In the following curves green line denotes the velocity function v(t) as in (2.1) , orange(dashed) curve denotes the polynomial curve y= f(x) = lx^2-kx^3 … (1.1) track for the roller coaster.



For this graph u= 17.5 m/s and

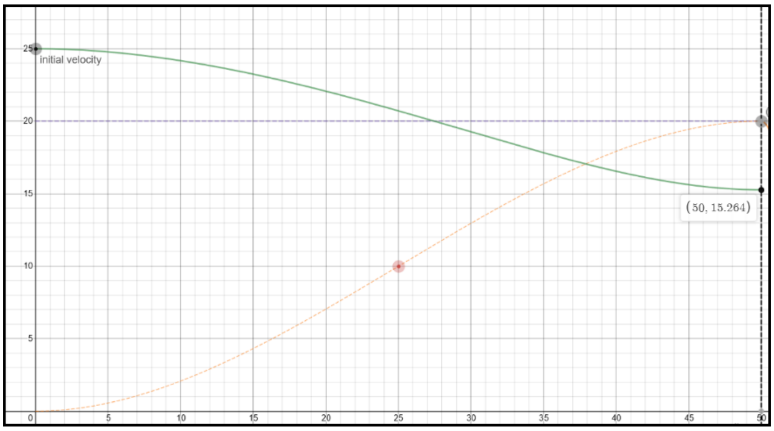
R=30 m and H = 20 m

Clearly, we see that unless u > $\sqrt{(2g.H)}$ we can't climb the hill

However, in real life we need to at least have u = $\sqrt{(2g.H)}$+2.5 m/s (estimate) so that we can overcome the drag forces like friction.

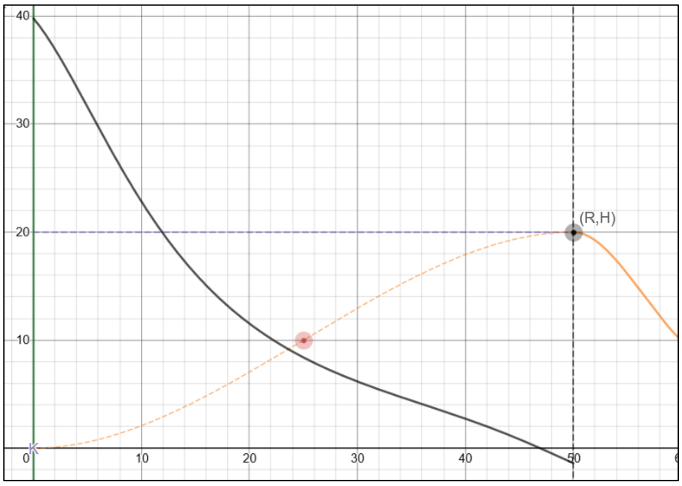
For this graph u=15m/s

R=50m and H= 20m



Normal Reaction vs position curve

Y axis denotes the magnitude of the normal reaction force which is perpendicular to the track (with sign) ...   the N(t) v/s t curve is in black
orange dashed line shows the polynomial track ...



R=20m,H=50m u=25m/s and mass (m) = 1kg

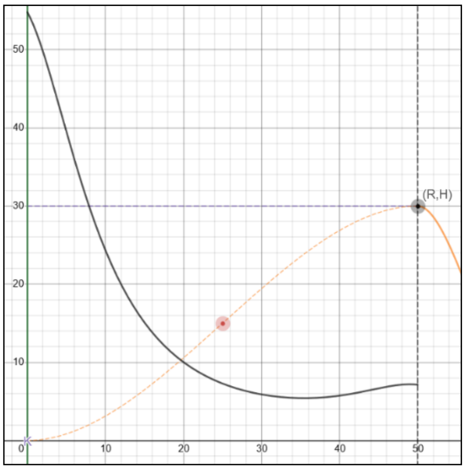
We see that normal reaction is maximum at (0,0) and it is given my the calculation as follows :

$$N(at\ origin) = \ Mg + \frac{Mv^2}{radius\ of\ curvature\ at\ (0{,}0)}$$

We can also see that <u>normal reaction can become negative</u> implying that if we use normal rail tracks then roller coaster can fly off the track hence to prevent the carts from flying of we ensure that the tracks <u>"lock in"</u> the wheels in place and the wheels are always in contact with the track. This also helps us get a sense of the scale of forces we are dealing with.

Clearly we see that N is max either at origin for climbing hills (or at the end point of (R,H)……if we are moving down the hill) , because it is at these two points we have the least radius of curvature.

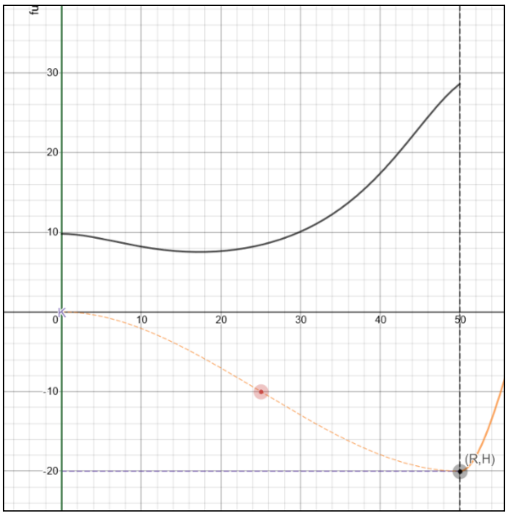
Hence it is also advised to design the roller coaster in such a way that the normal reaction at (R,H) is low because the Normal reaction vs position curve is discontinuous at (R,H)

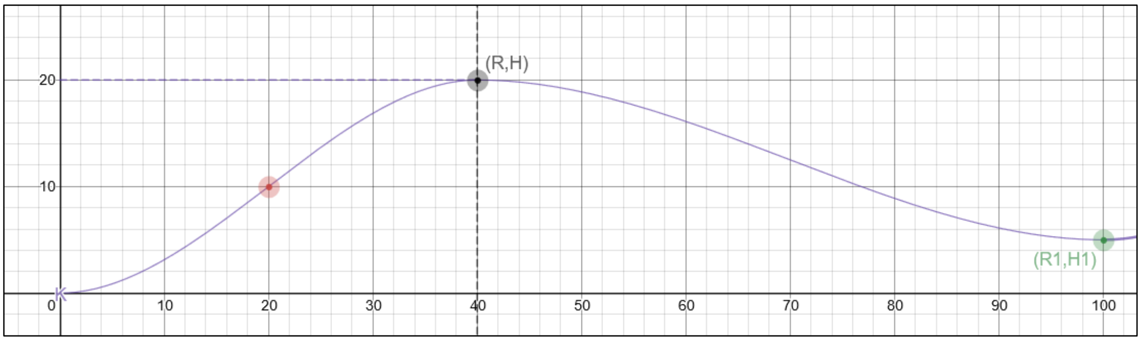


For graph (aside left) the parameters are

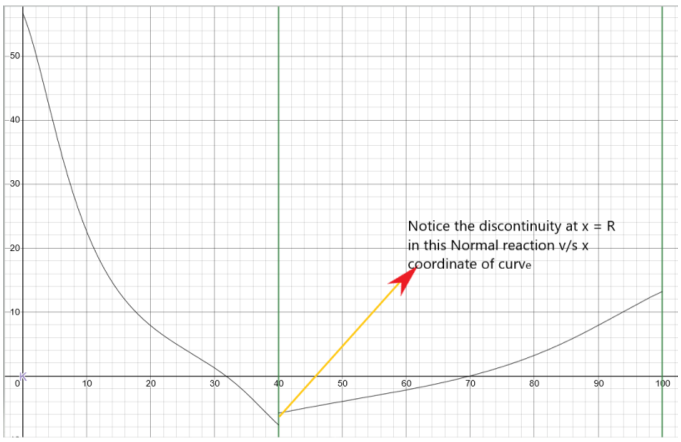
Mass = 1 kg, u = 25m/s ,R=50m H=30m

For graph(aside right) mass = 1 kg u = 0.01 m/s
R=50m and H= -20m (indicating a drop curve)





For given roller coaster curve we have following Normal reaction curve



Hence it is very important that we design tracks such that they don't lead to such discontinuity. Refer the mechanics of loop to understand about easement curves to smoothen out the curves even more

MECHANICS BEHIND A INCLINED CURVE

Since we had already assumed that our roller coaster starts from ground level . and there are no external forces . hence kinetic energy of the body K.E = initial KE – mgh ;

Now the lateral force experienced by a rider in the coaster is same as the centripetal acceleration of the coaster. Which is mv^2/R and since $\frac{1}{2}mv^2 = K.E_{(i)} - mgh$ we have following

Lateral Force $F = \frac{2(initial\ kinetic\ energy - mgh)}{R}$

Lateral acceleration = F/m

Now suppose we have following horizontal roller coaster Now we can calculate radius curvature using the formulas in appendix 1.2

And we also know that initial kinetic energy is fixed and height is also fixed hence we can directly correlate force experienced by rider to the x coordinate of the curve as Radius of curvature can be given as z= f(x)

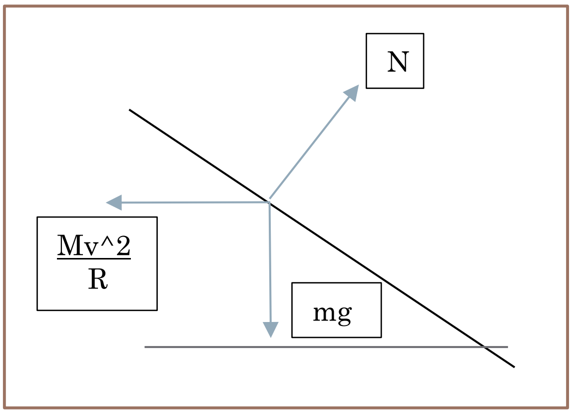
We usually don't always use planar curves in roller coaster as it is let's say too boring.

Now we can actually integrate the two models but it is beyond the scope of our discussion (refer appendix 1.6 for how to integrate the two model… that is how to mix hills with planar curves to make it more realistic)

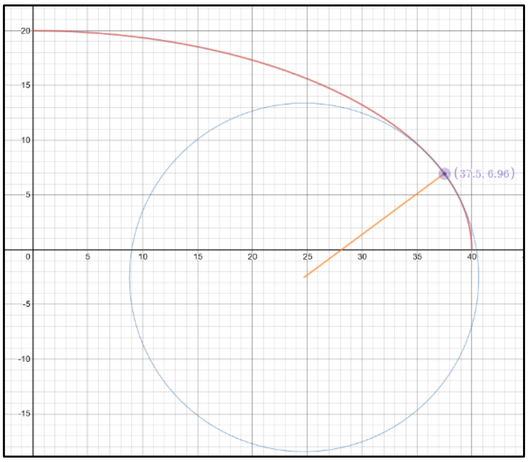
Also by using this model we can tweak our curves such that the change in forces is smooth and safe.

Reason why tracks are banked?

The banking angle of the curve is to prevent too high of lateral forces, injuring the riders. Also if lateral forces are too high the roller coaster might just spill off the track , or can damage the track overtime leading to cracks



Let us assume that our curve in the horizontal plane and is at a height h ; roller coaster is initial given a total energy of U.

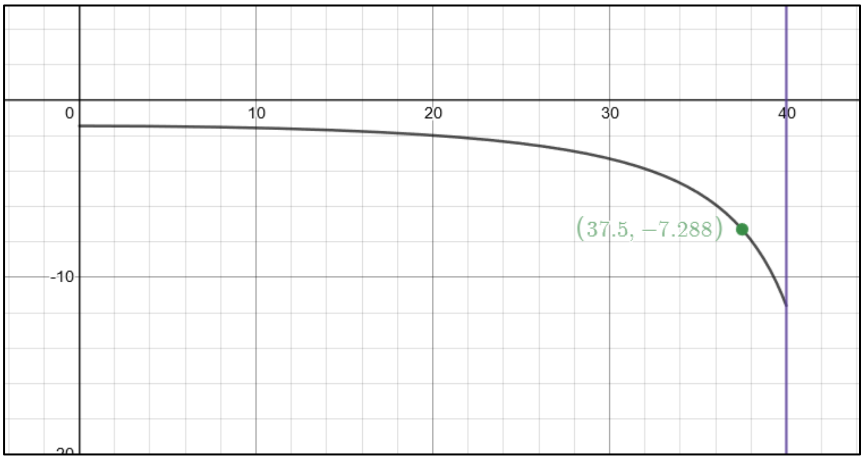


Example 1: z=f(x) = $\sqrt{400 - \frac{x^2}{4}}$ (a elliptical curve)

The graph aside the plot of (x,z) along with its curvature circle at x=37.5

Suppose the roller coaster is at H = 10m and mass is not a factor as we are plotting the lateral acceleration ,

Now the graph below is the lateral force experience perpendicular to elliptical curve at a certain point x (Y axis denotes the g-force



As we can see in graph that at x=37.5 m we lateral acceleration is -7.288 m/s^2

So if we have a mass of 500 kg then net force on he coaster is
500 x 7.288 N

The direction of force can be determined using diagram 1 and magnitude using diagram 2

(**note:** here the negative sign denotes whether the force applied on the body is towards the origin if the graph is having positive value means that it is acting along perpendicular to the curve but away from the curve.)

Using this model, we can gauge whether a planar curve is safe or not …

If the lateral acceleration exceeds 3g it can be fatal (refer appendix 1.4 for limit of acceleration in human body)

Also, this can be used to gauge the thrill factor of the element. By using the range of the function [F_{min} , F_{max}]…(refer the thrill factor section)

MECHANICS BEHIND A CIRCULAR LOOP

A very classic Mechanics problem is to find the height (… or the potential energy) required for a body to complete a loop of radius R freely. In case of frictionless surface, we have that the body must enter the loop with a kinetic energy > 5mgR in order to complete the loop .

Refer appendix (1.3) for detailed mechanics analysis of circular loop

Since we have not considered friction we can't just go with this constraint . Adding Friction force to the mix increases the complexity of the problem, also one may ask what is better curve which better to do a "loop de loop" .

Circular loop mechanics

Refer this article to get a clarity about the mechanics of the loops including ellipses and friction

Kłobus, Waldemar. (2011). Motion on a vertical loop with friction. American Journal of Physics - AMER J PHYS. 79. 913-918. 10.1119/1.3602091.

https://www.researchgate.net/publication/252343740_Motion_on_a_vertical_loop_with_friction

Why can't we use plain circular loops ?

Though circular loops are very easy to visualize and the maths is very easy

, BUT…. the speed needed at the entry of the loop (to allow the roaster to be able to crest over the top), creates too high an acceleration at the bottom! This is a very big disadvantage.

An immediate transition from one radius of curvature to another would give a continuous, smooth track, but with discontinuous second derivatives. Clearly, a function with continuous higher derivatives would be preferrable. From the loop photos, it is obvious that different approaches have been used to achieve the desired transition from a smaller radius of curvature at the top to a larger radius at the bottom. Below, we discuss a number of possible loop shapes with this property.

So remember the two flaws of a circle:

- The g-forces that a body is exposed to at the bottom of the loop exceed what is safe (when travelling at a speed that just allows the car to sail over the top of the loop).
- There will be a very rapid onset of this acceleration as the train starts the loop: The train would be playfully falling down the track, then wham! it will hit a 6g acceleration as it hits the bottom of the loop and starts its way up!

The solution to the first of the two problems (too high a g-load) is to decide on the maximum acceleration you wish to expose your riders to, then modify the track profile to keep this acceleration constant. As the coaster gains height, it loses speed. With this lower speed, the curvature of the track can be decreased to keep the needed centrifugal acceleration.
We know that the centripetal acceleration is proportional to v^2/r, as the velocity reduces then we can decrease the radius to keep the acceleration a constant. Before continuing with the coasters, let's take a short detour and investigate this strategy further by looking at a simpler similar problem. One all in the horizontal plane where we don't need to worry about gravity in the vertical plane.

To solve the second issue of sudden force on the roller coaster

Imagine we have a straight railroad track. A train chugging along this track at constant speed s not experiencing any lateral forces or accelerations.

What happens when this train runs into a corner?

- As soon as the curve is reached the train is subject to a lateral acceleration of v2/r.
- This appears with no warning or run up.
  This might be fine at low speeds, but as the speed increases, the forces can be considerable.
- Never mind the danger of derailment, or damage to the train, it also brutally uncomfortable for the passengers.

- If you ran a model train track at excess speeds you probably found that it was at the entry to curves that the train derailed.
- A similar issue occurs if you enter a corner too fast with your model car race set.
- What we want to do is gradually change the radius of the curve to slowly build up the acceleration (and thus ramp up the force gradually). There are an infinite number of ways for how we might select the profile for how the force is increased, but an obvious suggestion is a linear change.

- Selecting a profile in which the force changes linearly with time (under the assumption that the velocity of the train does not change), can be achieved with a curve whose radius also changes linearly with time. This smooth transition is sometimes called an easement.

This diagram shows how one can use a easement to reduce the forces

A curve whose radius changes linearly with angle (time) is a special kind of spiral called a *Clothoid*

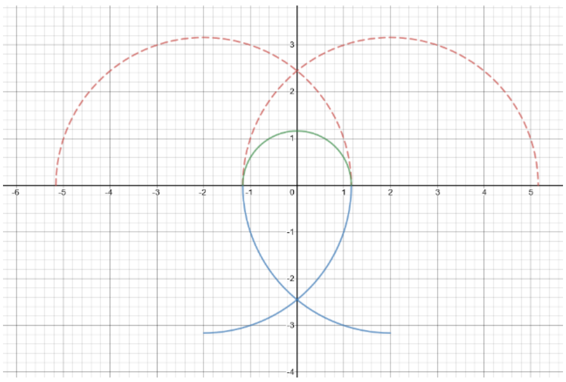
(also known as a *Cornu spiral* and even sometimes as a *Euler spiral*).

Above you can see a Clothoid easement connecting the straight track to the circular track.

Refer this article to know more about clothoid curve and its properties https://mse.redwoods.edu/darnold/math50c/CalcProj/Fall07/MollyRyan/Paper.pdf

How to apply clothoid easement to loops :

- A first order approximate solution to the ideal roller coaster loop shape can be created by simply bolting together circular tracks of the appropriate radii.
- A tight radius at the top where the speed is low, and a large radius at the bottom where the speed is high.
- This is certainly a great improvement over an entirely circular arc, but still has the jerk accelerations at the bottom and at each of the transitions where the radius of the track changes.



- Using what we learned from our Clothoid investigation we can modify the radius of curve smoothly to keep a constant centripetal acceleration all around the loop. The shape of such a track is show to the right.

DESIGNING A THRILL FACTOR

In roller coaster , riders want a exciting and thrilling ride . this is a very subjective question as to which roller coaster is more thrilling ?? The following paragraphs is a assumed model to calculate thrill factor for our given roller coaster

However, we noticed that the following factors somehow affect the thrilling nature of ride

1. Number of features / track elements that constitute the entire ride
2. Types of features
    a. Loops
    b. Banked turns
    c. Special features which won't affect the track itself ; like head chopper , tunnels , and other features
    d. Upside down section
    e. Climbs and drop

    More the variety more combinations we can make and thereby making if more thrilling

3. Lift hill height and maximum velocity
4. Variation of speeds and forces experience by the riders
5. Sense of unpredictability: if we have similar features back to back we make the ride monotonous and boring and thus not much thrilling.
6. Duration of ride
7. Other influences like the theme of the park, location of the park and surrounding scenery.
8. Overall safety : If we assure that our ride is safe and wont cause motion sickness then more people will be interested in trying out the roller coaster.

Now we setup a thrill factor scale by first assigning a thrill coefficient to individual members of the roller coaster

Designing thrill factor for a climb / drop curve

In a climb/drop curve we have following factors that affects the thrill …

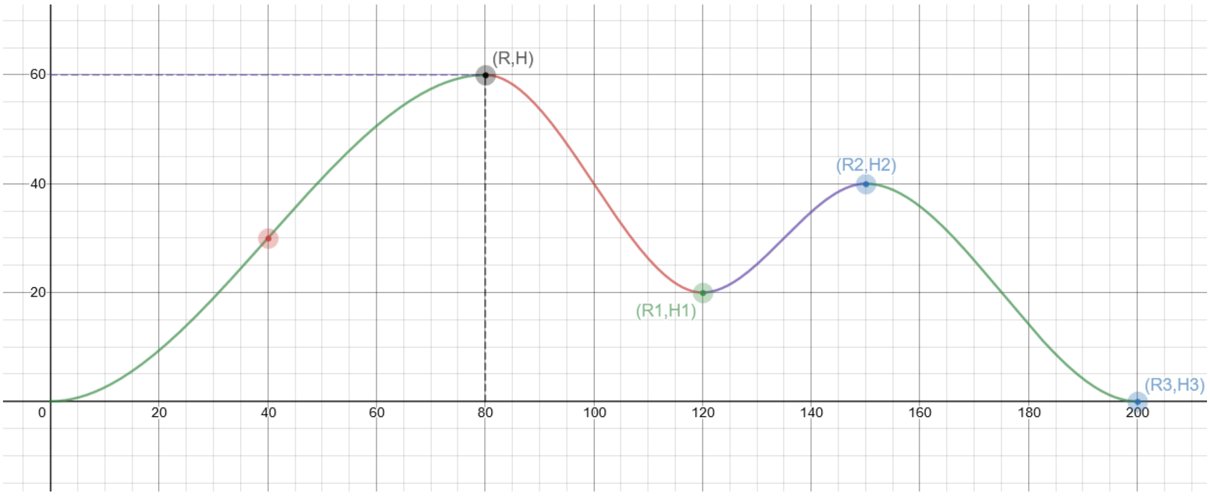
1. Average inclination of fall or rise … (avg(θ) $=\arctan \frac{\Delta H}{\Delta R}$)
2. Height by which roller coaster rises or falls …(ΔH)
3. Initial speed of entry ,… final speed of exit … and acceleration $u, v, a = \frac{vdv}{dx}$

W= arctan(H/R)*H*(v-u)/R*(v+u)/2= inclination*H*average acceleration

W= $\frac{\Delta H}{\Delta R}.tan^{-1}\left(\frac{\Delta H}{\Delta R}\right).\frac{v^2-u^2}{2}$ ...note v^2-u^2=2gH

T.F = $\log_{10}\left\{\frac{g\cdot(\Delta H)^2}{\Delta R}\arctan\left(\frac{\Delta H}{\Delta R}\right)\right\}$

We take logarithm because the inner part sometimes gets too big to make sense ...



(Figure 1)

For above roller coaster track

We have following table :

coordinates	H	R	delta H (h)	delta R (r)	g*h^2/r	avg(θ)	W	abs(W)	log(absW)=T.F
(0,0)	0	0	-	-	-	-	-	-	-
(R,H)	60	80	60	80	441	0.643501	283.784	283.784	2.452987889
(R1,H1)	20	120	-40	40	392	-0.7854	-307.876	307.8761	2.488375948
(R2,H2)	40	150	20	30	130.6667	0.588003	76.83234	76.83234	1.885544061
(R3,H3)	0	200	-40	50	313.6	-0.67474	-211.599	211.5988	2.325513117

Overall thrill factor of this system can be calculated as weighted average as shown below:

$$T.F(for\ polynomial\ hill\ complexes) = \frac{\sum_{i=1}^{i=n} R_i*(T.F)_i}{\sum_{i=1}^{i=n} R_i}$$

for above setup (figure 1) T.F = 2.34308

here R_i is the length of individual curve $T.F_i$ is the Thrill factor for that element.

Designing thrill factor for turn curve

THRILL FACTOR FOR CURVE

Thrill factor of a planar curve is only affected by the following factor:

1. Initial energy given to roller coaster ride U
2. Height of the planar curve H
3. Radius of the curve R

Shorter the radius R the more lateral force hence more the thrill factor

Also the velocity with which the coaster enters the track is dependent on U and H

As K.E = U -mgh

$v=\sqrt{2(U-mgh)}$

we define thrill factor TC.F as follows:

$$TC.F = \frac{v^2}{r}\frac{\cos(\theta)}{g}$$

R ranges from 10 m to 30 m and v ranges from 15 m/s to 22.5 m/S

PLEASE FILL SOME NUMERICAL TABLE LIKE FOR THE ABOVE THRILL FACTOR ONE

THRILL FACTOR FOR LOOP

Thrill factor for loop also only depends on :

1. Initial energy given to roller coaster ride U
2. Height of the planar curve H
3. Radius of the curve R

$$TL.F = \frac{v^2}{Rg} * Loop\ factor$$

Usually v= (10-20m/s) R ranges from 8-12 m

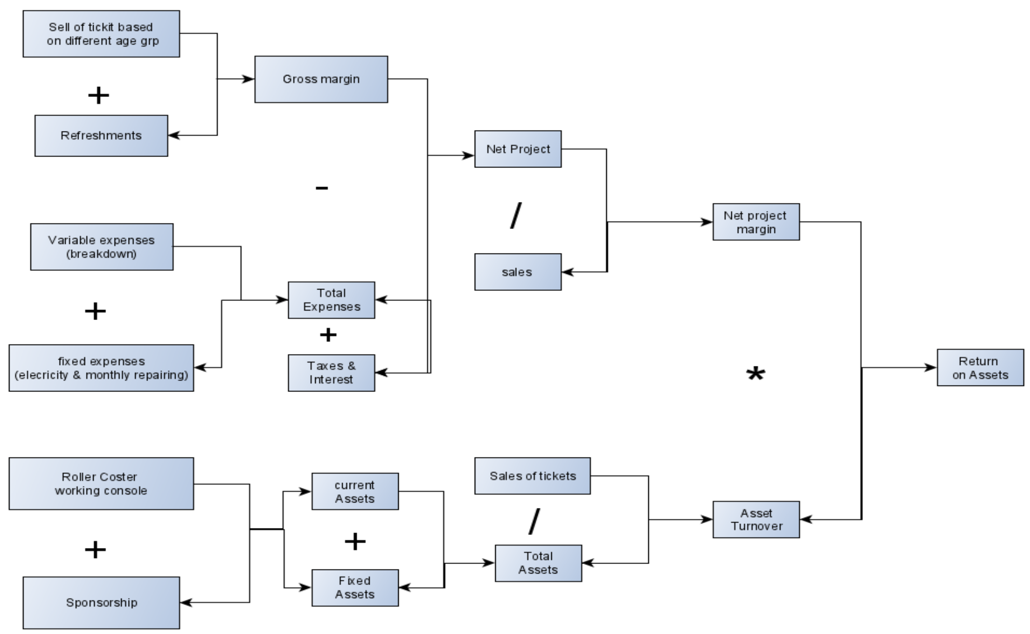
Loop factor is just some arbitrary constant which depends on the geometry of the curve.

PROFIT MODEL OF ROLLER COASTER

- The strategic profit model (SPM) measures the return on net worth (RONW) of a company. RONW is a tool that is used to measure the increase or decrease in the shareholder value of an organization. RONW is made up of three basic components namely:
    a. net profit
    b. asset turnover
    c. financial leverage.
- These components can be controlled by the managers of a company.
- Net profit is defined as the difference between sales and expenses. Related to net profit is a margin of a company which is a percentage of sales. This measures how efficiently a company manufactures the products.
- Asset turnover, which is the sales divided by the total assets of a company, shows how efficiently the company employs its assets in order to achieve a certain level of sales. The return on assets (ROA) of a company is calculated by multiplying the net profit margin with the asset turnover.
- The financial leverage of a company provides a relationship between the total equity (liabilities and shareholder's equity) of the firm and the amount invested by the shareholders. Since total equity is equal to total assets, financial leverage is the total assets under the control of management divided by the net worth or amount of shareholder's investment in the company.

From these financial figures, the RONW is obtained by multiplying the return on assets by the financial leverage. This provides an indication of how well a company is utilizing the investment made by their shareholders.

PROFIT MODEL OF ROLLER COASTER



Assumptions:

There are several assumptions that have been made in using the SPM model to analyze the main players of our roller costar industry. For the purpose of this exercise, and in the spirit of comparison, it is assumed that this industry competes in the same market making the comparisons more valid and meaningful.

Strategic profit model (SPM) assumptions:

The model assumes that the total gross margin is the sum of the tickets sold and refreshment. This is to overcome the difference in definitions of these terms among the companies to enable comparison.

The SPM is designed to allow changes of one variable to be monitored. However, in the case of sales, the model has been modified. Changes in sales cause changes in the total return on assets. For instance, when sales of tickets will increase, the level of turnover of assets and the returning of assets will also increase. The value of these two variables as a percentage of sales is first calculated. It is also assumed that when changing any of the other variables (excluding sales), the remaining variables remain unchanged.

The variables that can be altered for this model are sales of tickets, refreshments and fixed assets. Fixed expenses are not controllable in the short-run since most rentals and insurance policies that constitute this variable, are over a long period of time (30–40 years). In addition, changes in the cash reserves of a company will not lead to significant changes in the total assets since the cash will be reinvested into the business.

Application of SPM in Our Roller Coaster:

Income statement

Sales of Ticket - 1,00,000 /-
+Refreshment - 20,000 /-
= Gross Margin - 1,20,000 /-

- variable expenses - 30,000 /-
- fixed expenses - 10,000 /-
= Total expanses - 40,000/-

- Interest and Taxes - 10,000 /-
= Net Project - 70,000 /-

Balance Sheet

Roller Coster console - 50,000/-
+ Sponcership - 10,000 /-
Total Current assets - 60,000/-
+ fixed assets - 50,000 /-

= Total assets - 1,10,000 /-

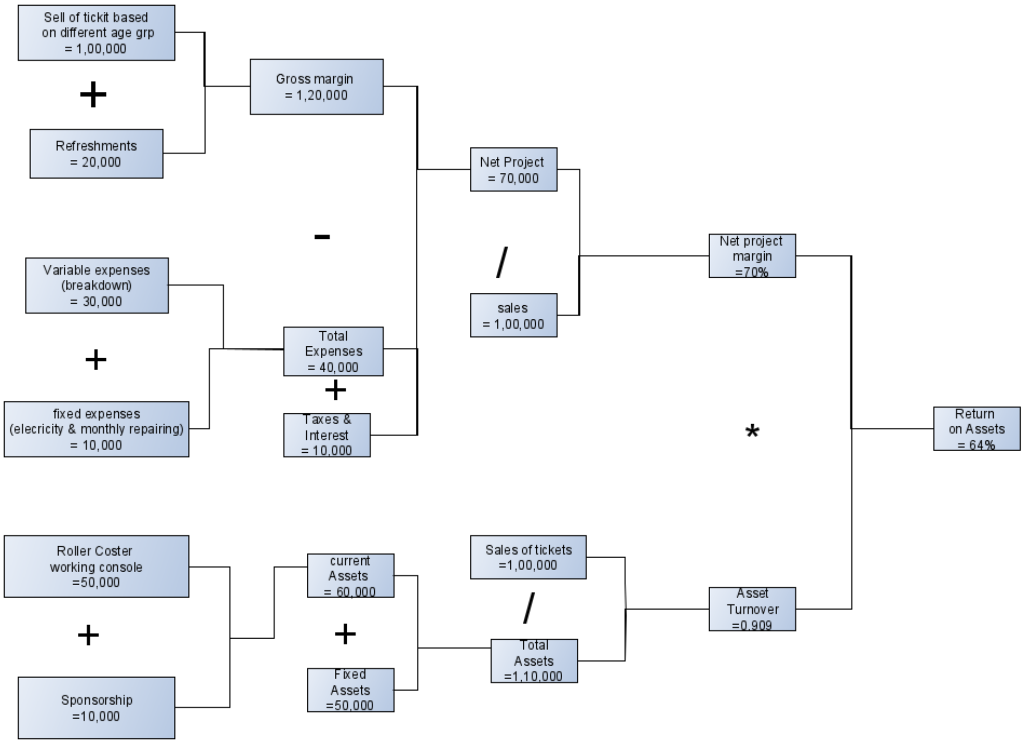
Liabilities

Current Liabilites - 30,000

+ long term liabilites - 50,000

= Total liabilites - 80,000

Simple Financial Statement -



As mentioned in the top financial model, let's assume that our total industry sold total tickets of Rs. 1,00,000 /-. The amount for refreshment is Rs. 20,000 -/. The total amount of gross margin is Rs. 1,20,000 /-. On the margin side of the SPM, only the sale of the ticket will change according to time. That will change the amount of the gross profit and net project income. However, as we assume that the income will grow up, so does the tax bill. The profit model says that the tax will affect the net project margin. Adding these expenses to the net project the costs of Rs.50,000 /- increase your total expanse to Rs. 1,70,000 -/, which has to be subtracted from gross profit. The net income is now Rs. - 70,000 -/. And the net profit margin is 10.5%.

Now, let's look at the assets portion of the model:

The only change will occur in the value of the current assets for sponsorships. So, the current assets are now Rs. 60,000 -/ and total assets are Rs. 1,10,000 -/. For this amount, the asset turnover will improve slightly to 0.909. Now the whole Return on assets is 64%.

So, from this SPM model according to our assumption, we can tell that if we maintain this strategy for our whole business, we can able to get a huge profit in a short time span.

Conclusion:

A methodology for the accurate description of three dimensional track geometries is proposed. Cubic, Akima and shape preserving splines are proposed for track parameterisation. The track description uses Frenet frames that provide the track referential at every point. During dynamic analysis a pre-calculated database is linearly interpolated to form the kinematic constraints that enforce the wheelsets of the vehicle to move along the track with a prescribed angular orientation. This approach allows calculating the reaction forces that develop between the vehicle and the track. The formulation is discussed emphasising the influence of the parameterisation methods to construct the track geometry.

Appendix

1.1 shifting of origin and adding more track elements

Now say we want to append another element to the curve at (R,H)

It is very easy we consider another point (R_1,H_1) and we shift our coordinates to (R,H) by applying following transformation , X=x-R and Y = y-H and we get following curve;

Y=f(X) …. $y - H = g(x - R) \dots \dots \dots \dots \dots where\ g(x) = lx^2 + kx^3$

$$and\ l = \frac{3(H1 - H)}{(R1 - R)^2}\ and\ k = \frac{2(H1 - H)}{(R1 - R)^3}$$

We can use this transformation to add all the track elements in a single coordinate system.

1.2 radius of curvature calculation

$$r(x) = \frac{\left(1 + \left(f'(x)\right)^2\right)^{\frac{3}{2}}}{\mathrm{abs}\left(\left(f''(x)\right)\right)}$$

$$x_0(x) = x - \frac{f'(x)\left(1 + \left(f'(x)\right)^2\right)}{f''(x)}$$

$$y_0(x) = f(x) + \frac{1 + \left(f'(x)\right)^2}{f''(x)}$$

Here x0(x), y0(x) is the center of the curvature circle and it tangent to the curve at the path at (x,f(x)) and the radius of the curvature circle at (x,f(x)) is given by the function r(x)

$$\frac{x - x_0(t)}{y - y_0(t)} = \frac{t - x_0(t)}{f(t) - y_0(t)} \left\{x_0(t) \le x \le t\right\}$$

$t = 3.8$

0

$$\frac{x - x_0(t)}{y - y_0(t)} = \frac{t - x_0(t)}{f(t) - y_0(t)} \left\{x_0(t) \ge x \ge t\right\}$$

$$\left(x - x_0(t)\right)^2 + \left(y - y_0(t)\right)^2 = \left(r(t)\right)^2$$

,

Here t is a parameter and the last equation is the equation of circle

REFERENCES

Links used so far

https://www.worldsciencefestival.com/2015/06/roller-coaster-science-thrills-chills-physics/

https://science.howstuffworks.com/engineering/structural/roller-coaster.htm/printable

http://mathcircle.wustl.edu/uploads/4/9/7/9/49791831/20170409-roller-coasters.pdf

https://www.maa.org/book/export/html/115770#:~:text=The%20track%20must%20start%2075,with%20a%20zero%20degree%20incline
https://web.wpi.edu/Pubs/ETD/Available/etd-042318-205416/unrestricted/Design_Analysis_of_Roller_Coasters_MS_Thesis_KHunt.pdf

http://www.batesville.k12.in.us/physics/phynet/mechanics/circular motion/banked_no_friction.htm

http://mathcircle.wustl.edu/uploads/4/9/7/9/49791831/20170409-roller-coasters.pdf

https://web.wpi.edu/Pubs/ETD/Available/etd-042318-205416/unrestricted/Design_Analysis_of_Roller_Coasters_MS_Thesis_K*Hunt.pdf

http://www.math.wpi.edu/Course_Materials/Calc_Projects/node6.html

https://en.wikipedia.org/wiki/Roller_coaster_elements#cite_note-3

https://www.researchgate.net/publication/252343740_Motion_on_a_vertical_loop_with_friction

http://physics.gu.se/LISEBERG/eng/loop_pe.html