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Problem Statement:

Our model takes in account of factors like the existing population of fungus, its lifespan, the minimum moisture requirement and if the fungus has sufficient food or not. Our model is able to mimic the real-world scenario in terms of population variation of fungus. It also shows how everything falls into a stable equilibrium even after periodic changes in moisture levels. Initially we have developed the model in terms of a system of a DE with the food source and one specie of fungus in play. Once the system started to make sense, we extended it for multiple species. It is very interesting to see how different species with different growth rates, moisture tolerance and lifespans grow alongside one another. Our model is heavily relied on Eulers Method of analyzing DEs using MATLAB. Also, this model neglects one very important part of the fungus that is its hyphae for which we have also tried to develop a different set of DE.

INTRODUCTION

We have tried to mimic the behavior of fungus in a rainforest and for calculation purposes we are only talking about a limited area of 400 m^2 so wherever the word forest is used we mean this Isolated forest. We have assumed that the moisture in a rainforest varies periodically over a cycle of 12 months just like a sine curve from 0 to pi. This paper first analyzes the behavior of one species of fungus in the environment using differential equations and drawing the curve for them using an algorithm for Euler's numerical method of analyzing a system of differential equations (A1). Then we try to extend the idea for multiple species. The MATLAB code for this has been included in the Appendix of this paper.

DESCRIBING FUNGUS

Characteristics:

- They are mostly terrestrial.
- The body is haploid(n) and thalloid i.e., not defeminated into root, steam and leaves. They are multicellular. The fungal body is made of thread like elongated tubular structures, called **hyphae.**
- The cell wall of hyphae is made of **chitin and fungal cellulose**, which is a polysaccharide containing nitrogenous compound and it is basically made up of acetylglucosamine.
- Cells have unicellular Golgi bodies.
- Nutrition is heterotrophic which includes saprophytes, parasites and symbionts.
- [1] In most of the fungi, there are two distinct phases in the life cycle, the vegetative phase and the reproductive phase. In vegetative phase fungi is microscopic hidden in the substratum and hardly visible in the naked eyes. The fungus enters into reproductive phase after having maturity in the vegetative phase.
- Fungi store their food in the form of starch

Nutrition:

Fungi get their nutrition by absorbing organic compounds from the environment. Fungi are **heterotrophic**: they rely solely on carbon obtained from other organisms for their metabolism and nutrition. Fungi have evolved in a way that allows many of them to use a large variety of organic substrates for growth, including simple compounds such as nitrate, ammonia, acetate, or ethanol. Their mode of nutrition defines the role of fungi in their environment.

On the basis of mode of nutrition, fungi are classified into four groups:

(1) **Saprophytes:** These fungi obtain nutrition from dead organic matter. Ex.- *Mucor, Rhizopus*

(2) **Parasites:** Parasites fungi take food from other living plants or animals. The living organisms on which they parasite are called host. Ex.- *Puccinia.*

(3) **Symbionts:** These fungi grow on or with another living organism but both of them are mutually benefited. **Lichens and mycorrhiza are the best example** of symbionts.

- **Lichens** are the symbiotic association between algae and fungi. Here both fungi and algae are mutually benefited. Algae synthesize carbohydrates whereas the fungi provides shelter for algae.
- **Mycorrhiza** are the symbiotic association between fungi and roots of some higher plants. It helps in the absorption of nutrients by the host plant.

(4) **Predacious:** They are animal capturing fungi. They usually inhabit in the soil. Ex: - *Arthrobotrys, Dactylella.*

Idle Environmental Conditions:

They are mostly terrestrial. They prefer to grow in warm and humid places. They may grow in warm and humid places. They may grow on tree bark, dung, wood, burnt wood and keratinous material (e.g., hair, horns).

- **Temperature:** 25- 30 degree Celsius.
- **Humidity:** high moist humid environment.
- **pH:** Moulds differ in their pH requirements. Most will grow well over the pH range 3-7. Some such as *Aspergillus niger* and *Penicillium funiculus* can grow at pH 2 and below.
- **Nutrients:** Nutrient requirements for moulds may vary from mould to mould. Some moulds may thrive well on substrates with high sugar or salt content. Some may prefer simple sugars while others have the ability to utilize complex sugars.
- **Light:** Many moulds species grow well in the dark, but some prefer daylight or alternate light and darkness for them to produce spores.
- **Aeration:** Nearly all moulds require air to grow.

Reproduction:

Fungi reproduce by all the three modes i.e., **vegetation, asexual and sexual.**

Vegetative Reproduction:

It occurs by the following methods:

- **Fragmentation:** The mycelium brakes up into two or more fragment due to mechanical injury, decay or some other reasons. Each fragment grows into independent mycelium.
- **Fission:** Here, simple splitting of vegetative cells into two daughter cells takes place by simple constriction.
- **Budding:** Some fungi like yeast produce small outgrowth, i.e., buds from their vegetative body. Eventually, the buds are cut off from parent cell and mature to from new induvial.

Asexual Reproduction:

It occurs through spores. These are single called specialized structures which separate from the organism, get dispersed and germinate to produce new mycelium after falling on suitable substrate. The spores produced during asexual reproduction in fungi are formed by miotic division and are thus termed, **mitospores**.

The various kind of asexual reproduction are as follows:

- **Zoospore:** Many fungi, especially aquatic members produce these types of spores. They germinate to give rise to new mycelium. Ex. *Saprolegnia, Pythium.*
- **Sporangiospore:** Sporangiospores are thin-walled non-motile spores produced endogenously in a sporangium during favourite conditions, which after liberation give rise to new mycelium. Ex. - *Rhizopus, Mucor.*
- **Conidia:** Conidia are non-motile, thin-walled **exogenous spores** produced at the tips of erect hyphae called conidiophore. They are arranged in chains upon the conidiospore Ex. *Aspergillus and Penicillium*
- **Chlamydospore:** In some fungi the hyphae under unfavourable conditions, forms **thick-walled resting resistant spores** which later get separated from each other. They may remain viable for several years. On return to favourable conditions, they germinate to give rise to new individuals. Thus, chlamydospores are structures for perennation also, Ex.- *Rhizopus.*
- **Oidia:** Non-motile thin-walled spores developing under rich conditions in medium. Their budding condition is called **torula stage.**

Sexual reproduction:

[1] During the process of sexual reproduction, a huge number of spores are produced from the parents' body. The produced spores disperse either floating on the wind or hitching a ride on an animal, as they are lighter and smaller than the seeds. The dispersed spores land in an environment that will support their growth.

In fungi, sexual reproduction frequently takes place under unfavourable environmental conditions. Based on the mating types, they are classified into two types:

1. Homothallic – When both mating types are present in the same mycelium. It is also called self-fertile.
2. Heterothallic – When both mating types are present in two different mycelium.

There are many variations in fungal sexual reproduction, which includes the following three stages.

1. Plasmogamy: The fusion of protoplasm.
2. Karyogamy: The fusion of nucleus.
3. Meiosis: Cell cycle involved with the nuclear division.

This sexual mode of reproduction in fungi is referred to as teleomorph and are of four types:

1. Ascospores

2. Basidiospores
3. Oospores
4. Zygospores

QUESTIONS TO BE ADDRESSED

1. *How fungus interacts with its environment and its role in the ecosystem?*
   Fungus can't produce its own food it generally grows on food items itself, then it releases some digestive enzymes in its surroundings to breakdown the food into simpler proteins and then it absorbs it using its hyphae. This process is very important for the ecosystem of the forest as, if fungus would not do its job the forest would be filled with dead plant materiel and wood chips. Moreover, when the digestive enzymes breakdown food into simpler particles the remaining part which is not absorbed acts as a fertilizer for the plants, without which the forest soil would become dead within years.
   Our model should predict the amount of food (dead plants and wood fibers) consumed by fungus and make an approximate its impact on the ecosystem.

2. *How different species of fungi interact with one another, when on the same territory and while sharing same resources?*
   When different species of funguses coexist, they rarely have a direct impact on each other. They mostly compete for the same resources such as food and space. The species with high growth rate consumes food faster, has a short lifespan, are less moisture tolerant and hence are more sensitive than the species with low growth rate.
   Our model should be able to show the competition between different species of fungi in a forest situation like the way it happens in real world.

3. *Effect of environmental factors such as moisture, temperature, light and airflow on the growth of fungus.*
   The model should take in account of the variation in fungus population due to factors such as moisture, temperature change, effect of natural lighting, amount of food available, airflow of the location, etc.
   But our model only takes in account of the food available, the current fungus population, the lifespan of fungus and the presence of sufficient moisture in the soil.

MATHEMATICAL MODEL OF THE BEHAVIOR OF FUNGUS

Population model:

Firstly, we need to analyze the amount of food available in the forest for the fungus and then we need to find out the trend about how fast dose the population of fungus grow. Which can be done by the Following equations:

F = The total area of the forest covered by fungus in m^2
W = The amount of food (wood and dead plant material) present in the forest in kg
k1 = The rate at which the amount of food increases in the forest in kg per month.
k2 = Amount of food consumed per unit area of fungus per month in kg m^{-2} $month^{-1}$
t = time that has passed in months from the reference point
k3 = this constant varies from fungus to fungus

(1)

(2)

The Equation (1) is very obvious as the rate at which the food changes in equal to the rate at which it is produced minus the rate at which it is consumed by fungus.

The equation (2) states that the growth rate of fungus is directly proportional to the already existing amount of fungus F and ($W-k_2F$) part becomes negative when the amount of food available is less than the requirement and becomes positive when food is sufficient and at last, we have included a proportionality constant k_3 which varies from species to species.

But there is a small problem in equation (2), that fungus generally also has a fixed lifespan which can be accounted for by making the following change in (2).

(3)

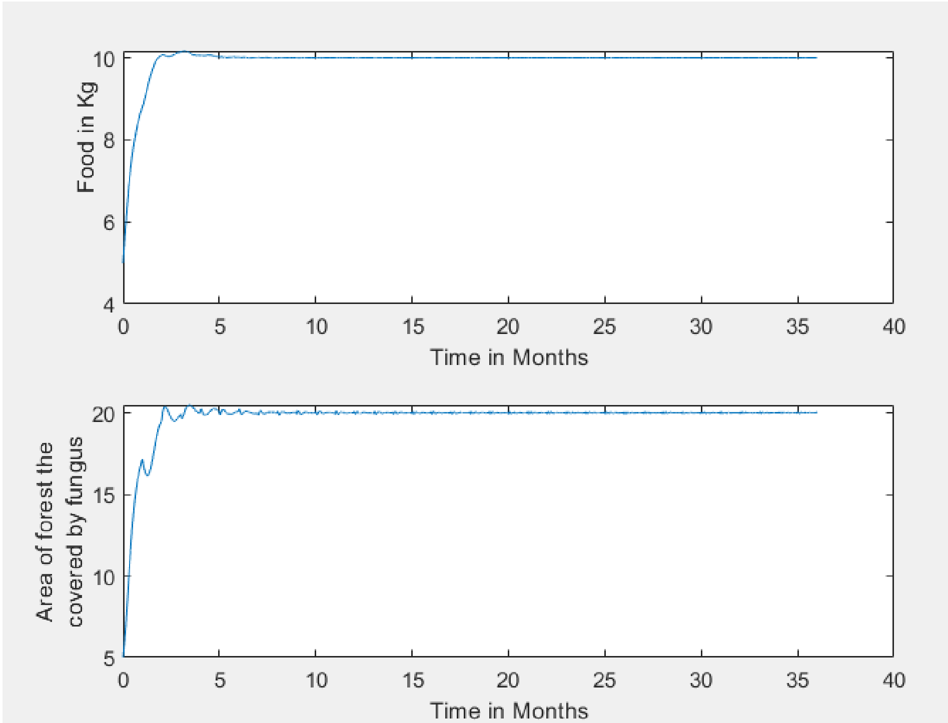
This subtracted part is the amount of fungus that increased exactly t_0 month before the current point. (Assuming the lifespan of fungus is t_0, which also varies from species to species).

This alteration makes it very difficult to for us to find an analytical solution for this system of differential equations. So, we have opted to use Eulers Numerical Analysis of differential equation, the MATLAB code for this is added in (A2).

Let us see what we can observe from the graphs,

Assumptions:

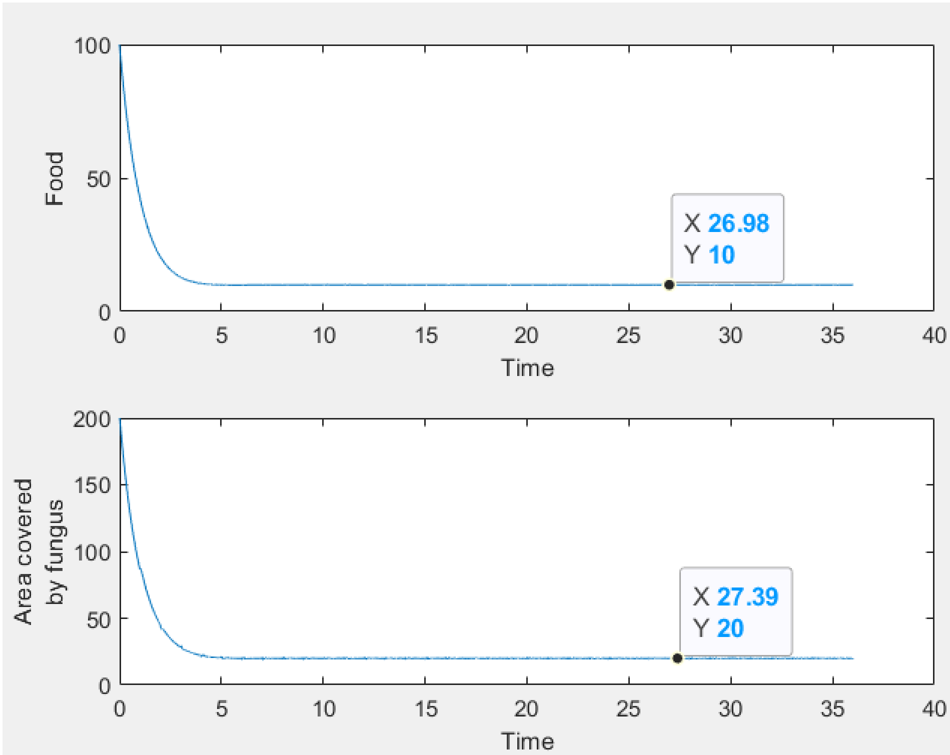
k1 = 10 kg of food produced in the forest every month
k2 = 0.5 kg of food is consumed my one m^2 of fungus in a month
time differential, dt = 0.002 months
Initial conditions (W, F) = (5 kg, 5 m^2) [Taken randomly]



Observations:

- The amount of Fungus keeps on increasing for the first 5 months.
- Both the functions attain an equilibrium state after some time at W=10 and F=20

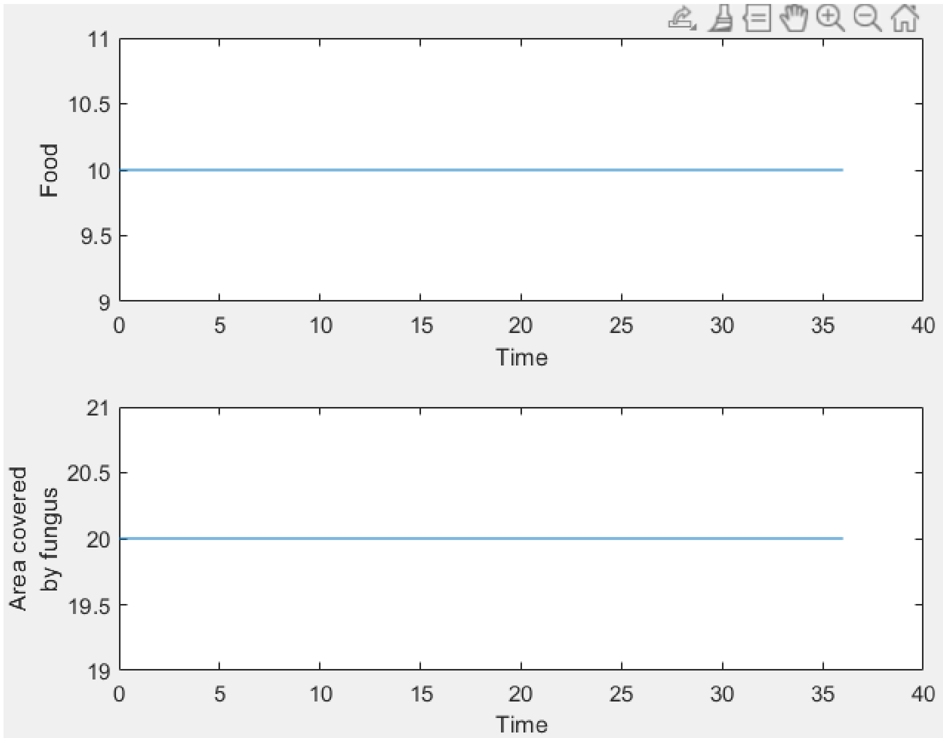
Let's try to give higher initial values to (F, W) say (50, 100) and other assumptions are kept same.



Observation:

- It takes approximately 5 months for them to reach an equilibrium.
- The equilibrium is again seen at W=10 and F=20

Let's try to start the process from the Equilibrium itself, i.e., F = 20 and W = 10



Observation:

- The system maintains its equilibrium throughout.

Let's try to analyze the equations (1) and (3) to figure out more about this equilibrium.

(4)

This verifies with our result. This result is closely consistent with what happens in nature, as we know all things reach an equilibrium in the long run.

But the above equation does not take in account of the different moisture conditions which happens in cycles of 12 months. This trend can be observed in all the forest across the world [3] to account for that (3) can be altered as

(5)

m_0 = the minimum moisture required by a specie of fungus to survive (kg of water present per m^2)

m= the amount of moisture present at any given time

When $(m - m_0)$ becomes negative the then the rate of fungus growth also becomes negative and when more moisture is available the fungus grows faster.

However, since it is multiplied with $(W - k_2F)$ as well, in a condition when sufficient food is not available (i.e., $W - k_2F$ = -ve)and moisture is also below m_0 (i.e., $m - m_0$ = -ve). The rate of change of fungus becomes positive which is not consistent with the real-life situation.

To overcome that we create a function P(x, y) such that,

(6)

Now the final equation looks like this,

(7)

Here the question might arise how dose m change with time, the answer is quite simple we represent in in form of a sine curve which has a time period of 12 months. Like

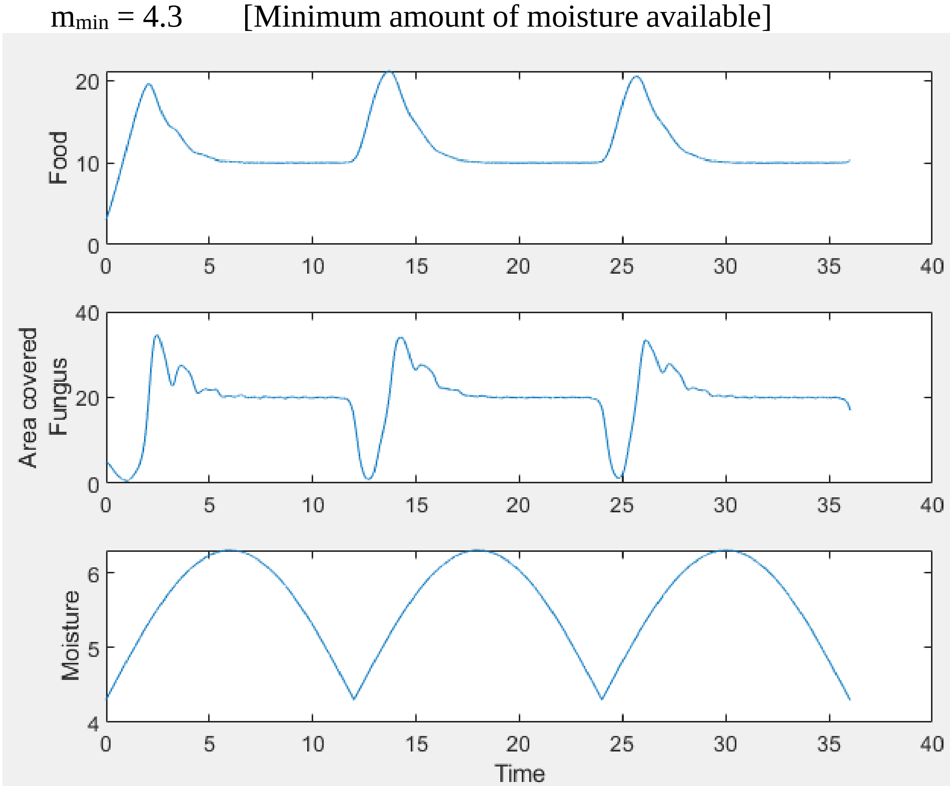
(8)

This is because we have seen in [2] that the moisture is attains a peak at the rainy season, which very closely resembles the absolute value of sine curve. Here m_{min} is the minimum value of moisture in the forest. Also, the amplitude of the sine curve can be changed to show high or low variation in the moisture level. We have assumed it to be 2 for the purpose of simplicity.

MATLAB Code in (A3)
Let us try to plot (1), (7) and (8).

Assumptions:

k1 = 10 kg of food produced in the forest every month
k2 = 0.5 kg of food is consumed my one m^2 of fungus in a month
time differential, dt = 0.002 months
$m_0 = 5$ [Minimum amount of moisture required]
$m_{min} = 4.3$ [Minimum amount of moisture available]



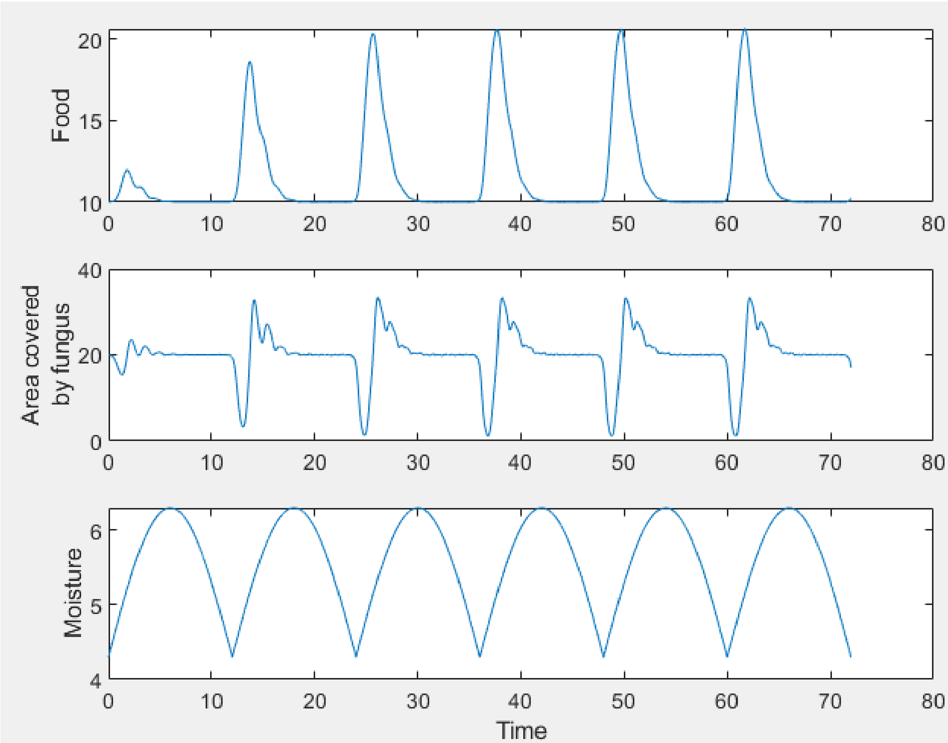
Observations:

- The behavior of fungus and food available is periodic, which happens in reality.
- When the moisture is sufficient then the system again reaches the equilibrium points, which is again consistent with the real time situation.
- When the moisture drops below m_0 the fungus starts to reduce and also the amount of food increases.
- As soon as the moisture curve hits m_0 in an increasing period of m, there is a rapid growth in the amount of fungus as a lot of food is available with sufficient moisture.

When we try to start from the equilibrium position F=20 and W = 10, then we again get a straight-line graph for food and fungus as their slops are 0 at that point.
So, we have tweaked the P(x, y) to also include a condition that if either one of x or y is zero the other value is the answer. The Matlab code for P is

```
function res = P(x, y)
    if(x == 0)
        res = y;
    elseif(y == 0)
        res = x;
    elseif(x < 0 && y < 0)
        res = -1 * x * y;
    else
        res = x * y;
    end
    return;
end
```

Also, it would be more interesting to see it over a long period of time, say 6 years. The graph generated is,



Observations:

- The same as before but here we do not get a straight line due to the minor adjustment we made.
- The graph sill seems to be consistent with the real world, all the features such as attaining an equilibrium, drop in fungus due to low moisture, increase in food due to less fungus and the periodicity as well.

Now we can extend this model for multiple species of fungi, as

(9)

Let's try and visualize what these equations look like,
MATLAB Code in (A4)

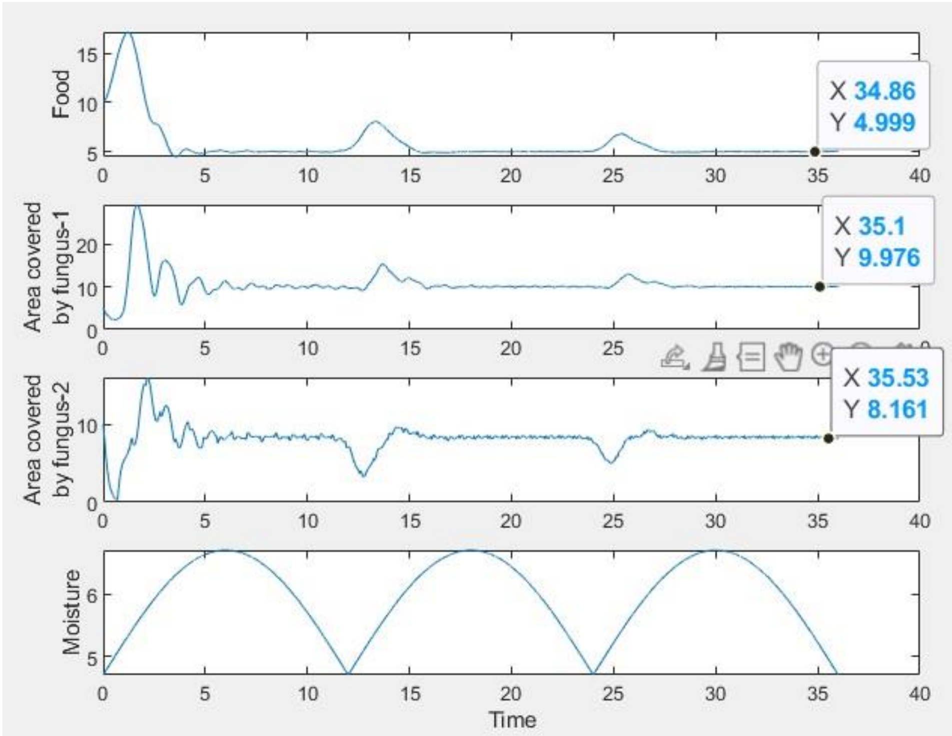
Assumptions:

dt = 0.002

We know that the growth rate of fungus is inversely proportional to its sensitivity so the values of k1, k2, k3, m0 and lifespan have been kept accordingly for the 2 species.

$W_{initial}$	10 kg	-
k_1	0.5 kg / month	-
Species	**Specie-1**	**Specie-2**
k_2	k_{21}= 0.5	k_{22}= 0.6
k_3	k_{31}= 1	k_{32}= 1.3
m_0	m_{01} = 5	m_{02} = 5.3
Life span	1 month	0.7 of a month

$F_{initial}$	$5\ m^2$	$10\ m^2$

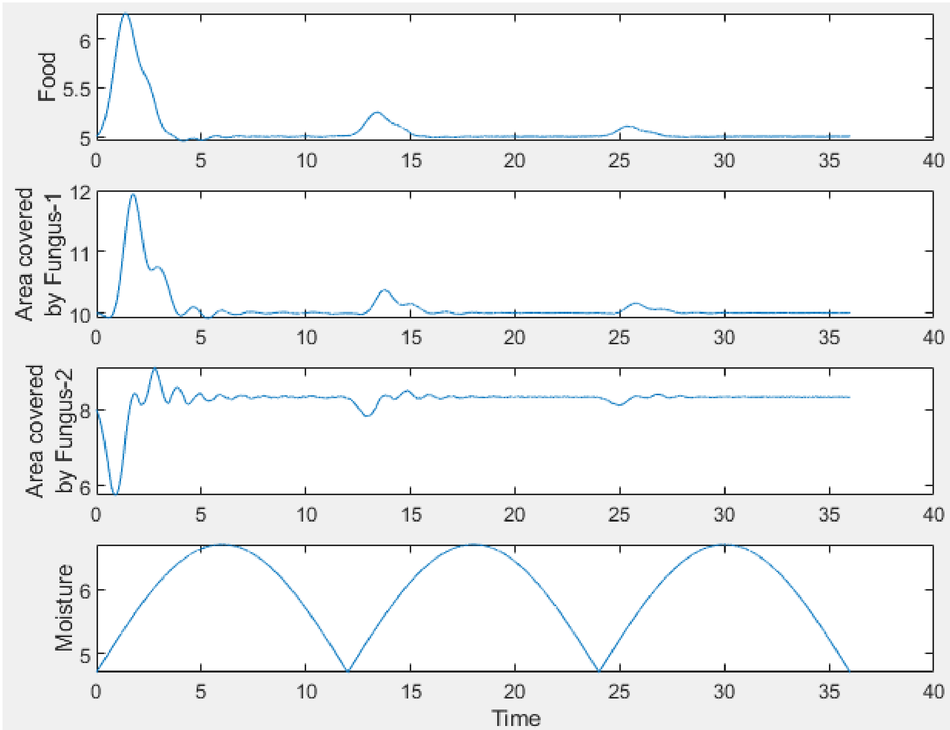


Observations:

- The model again tends to reach an equilibrium.
- The fungus with the shorter lifespan shows more variation in values.
- The population of funguses reduces in the start due to low moisture.
- Since the minimum moisture requirement for Fungus-1 in higher than that of Fungus 2, the decrease in the population of fungus-1 is far less at the minima of moisture.

Let's now try the above model again but with the equilibrium values, just to see if we can figure out something else.

$F_{initial,\,1} = 10$, $F_{initial,\,2} = 8$, $W_{initial} = 5$



Observations:

- After every complete cycle 12 months the equilibrium becomes more and more stable.
- Rest all the observations are consistent with the previous observations.

The final system of equations to determine the behavior of n different species of fungi is:

(11)

The Equilibrium point:

(10)

Effect on Geochemical cycles:

The amount of Food (Wood, dead plant etc.) consumed by fungus can be easily calculated. As the rate at which the food is being added to the system is constant and can be determined by the equation

(12)

We have to now subtract the amount of food currently available in the forest, i.e., W(t) at t=0 which equals to the equilibrium value of k_1/n.

(13)

The value of W_0 is also k_1/n because under real life conditions the system is already at equilibrium.

For calculating it for every year we need to take t = 12 months.

(14)

This is a very obvious result because this is the exact amount of food created each year
The amount of wood in the forest bed varies from forest to forest, let's say it is about p% wood then the amount of wood digested by fungi is,

(15)

Now that we have calculated the amount of wood that rots, we can calculate amount of CO2 and H2O generated by it with the help of following equation.

Photosynthesis by trees and other plants

$$6nCO_2 + 5nH_2O + 677{,}000n \text{ calories} \rightleftarrows (C_6H_{10}O_5)_n + 6n0_2 \quad (1)$$

Respiration by fungi and other destructive organisms

[3]

Hence the impact on Carbon Cycle every year can be calculated as

(16)

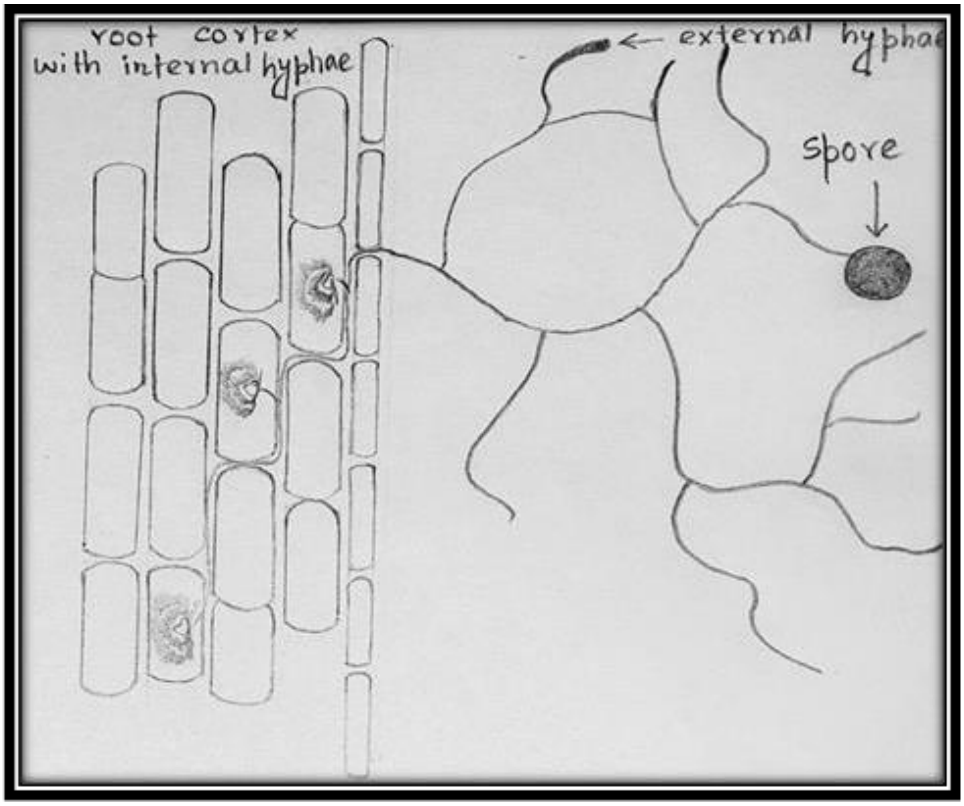
Similarly, the effect on Water cycle can be calculated as

(17)

Hyphae Model:

We haven't discussed much about the most important part of the fungus, i.e., hyphae.

Fungi is dependent on the hyphae for their growth and also for reproduction. Arbuscular mycorrhizal (AM) fungi grow inside plant decay as well as in the surrounding soil. [4] The root-external mycelium spreads several centimetres away from root surfaces into the soil. It can thereby access poorly mobile soil nutrients from outside the nutrient depletion zone formed around actively absorbing roots. Many metres of fungal hyphae may be produced per gram of soil. This results in a greatly increased surface area for plant nutrient absorption. A diagram of an arbuscular mycorrhiza is shown in diagram.



The whole mycelium has two main components: the hyphal length density (r) and the tip density of hyphae(n).

(Hyphal tips are important, because growth occurs due to the elongation of the region just behind the tips. Therefore, we can describe the movement of hyphal tips, with the tips leaving a track for a newly created hypha behind them)

Conservation of tips gives the following equation for the hyphal tip density:

(18)

where ***v*** is the rate of tip movement and *f* is the rate of creation or destruction or both of hyphal tips, i.e. branching, death and anastomosis.

Mycorrhizal fungi are known to branch mainly where one tip splits into two. Let's assuming that branching and tip death are linearly proportional to the existing tip density. So,

(19)

Where bn is the tip branching rate and dn is the tip death rate.

Secondly, we also can assume that branching decreases with increasing tip density and stops at a given maximal point of density, the term f becomes

(20)

From a biological point of view, this behaviour could be due to the competition between hyphal tips for space and growth resources when the tip density is large.

The hyphal length density is dependent on tip density and movement and hyphal death as well. The increase in hyphal density can be written as $n|\mathbf{v}|$, i.e., the length of the track left behind as tips move through the unit volume. We assumed hyphal death to be proportional to the hyphal length density ρ, so that the equation describing the hyphal length density is

(21)

where d is the rate of hyphal death.

In this growth model we need to apply some initial and boundary conditions. For *Arbuscular mycorrhizas* (AM) fungi, fungus can't grow without the hyphae. So, in this point of view we assumed a boundary condition where the hyphal tip density may depend on t, ρ or n. We can use a linear function

(22)

Where $n_{0,b}$ is the tip density initially present at the boundary and a is a boundary parameter. If the soil is fungus free initially, we have as initial condition that ρ=n=0.

We assumed that the growth of fungal hyphae in in the form of cylindrical structure where,

r_0 = root radius

v = is the speed of radial tip movement.

(23)

CONCLUSION

We can see that our model resembles the real-world scenario in terms of its ability to show the fluctuation of food and fungus population over time, also it attains a stable state no matter what the initial conditions. This stable state or equilibrium is very important because in natural habitat, everything stays in an equilibrium, like when the food is in abundance the wildlife population grows and if the food is scarce the population decreases until it reaches a steady state. Our model also accounts for the seasonal change in moisture and its impact on fungus population, even after the periodic fluctuations in moisture the fungus population achieves the steady state which is an important observation. This model gives us an approximate prediction of the amount of Carbon and Water produced by rotting of wood every year by a rainforest of size 400 m^2.

From this Hyphae growth-model, we can say that the growth of fungi individually is also dependent on their hyphae. In this module, we assumed different situations for hyphal growth. From that, we easily understand that during tip growth, cell walls are extended by the external assembly and polymerization of cell wall components and the internal production of a new cell membrane. The direction of hyphal growth can be controlled by environmental stimuli.

APPENDIX

(A1) Eulers Method or Forward Eulers Method

If we are given a DE such as y'(t) = f(t , y) and y(a) = b , then we can numerically analyze the DE by,

First, we assume dt = some very small constant (say 0.001)

Then we rewrite the DE as dy = f(t , y) * dt ….(1)

Next, we pot the values (t = a , y = b , dt = 0.001) in (1) to get a value of dy

Now the next set of points according to DE is (a + dt , b + dy)

Now the entire process is repeated again taking (a + dt , b + dy) as the new initial point

This process is done using a computer program, which generates a very good approximation of the actual graph.

(A2) MATLAB Code for fungus with death rate only

```
k1 = 10; %kg per month

k2 = 0.5; %kg of food consumed per unit fungus per unit time

k3 = 1;

dt = 0.002;

len_of_one_life = 1 / dt;

t = 0; %months

F = 20; %m^2

W = 10; %kg

AREA = 400; %total area available in forest in m^2

while(t(end) < 36)

   dW = (k1 - k2*F(end)) * dt;

   dF = k3 * F(end) * (W(end) - k2*F(end)) * dt;
```

```
    if(length(F)>len_of_one_life)
        dF = dF - (F(end-len_of_one_month+2) - F(end-len_of_one_month+1));
    end
    F(end+1) = F(end) + dF;
    if(F(end) > AREA)
        F(end) = 400;
    end
    W(end+1) = W(end) + dW;
    t(end+1) = t(end) + dt;
end
subplot(2,1,1);
plot(t,W);
subplot(2,1,2);
plot(t,F);
```

(A3) MATLAB Code for fungus with death rate and moisture tolerance

```
k1 = 10; %kg per month
k2 = 0.5; %kg of food consumed per unit fungus per unit time
k3 = 1;
m0 = 5; %kg per m^2
m = 4.3;
dt = 0.002;
len_of_one_life = 1 / dt;
t = 0; %months
F = 20; %m^2
W = 10; %kg
AREA = 400; %total area available in forest in m^2
while(t(end) < 72)
    dW = (k1 - k2*F(end)) * dt;
    dF = k3 * P(m(end) - m0, W(end) - k2*F(end)) * F(end) * dt;
    if(length(F)>len_of_one_life)
        dF = dF - (F(end-len_of_one_month+2) - F(end-len_of_one_month+1));
```

```
    end
    F(end+1) = F(end) + dF;
    if(F(end) > AREA)
        F(end) = 400;
    end
    W(end+1) = W(end) + dW;
    t(end+1) = t(end) + dt;
    m(end+1) = m(1) + 2*abs(sin(pi * t(end) / 12));
end
subplot(3,1,1);
plot(t,W);
subplot(3,1,2);
plot(t,F);
subplot(3,1,3);
plot(t,m);
```

(A4) MATLAB Code for fungus with death rate and moisture tolerance in 2 species

```
k1 = 10; %kg per month
k21 = 0.5; %kg of food consumed per unit fungus per unit time
k22 = 0.6;
k31 = 1;
k32 = 1.3;
m01 = 5; %kg per m^2
m02 = 5.3;
m = 4.7;
dt = 0.002;
len_of_one_life1 = floor(1 / dt); %1 month
len_of_one_life2 = floor(0.7 / dt); %0.7 of a month
t = 0; %months
F1 = 10; %m^2
F2 = 8;
```

```
W = 5; %kg
AREA = 400; %total area available in forest in m^2
while(t(end) < 36)
    dW = (k1 - k21*F1(end) - k22*F2(end)) * dt;
    dF1 = k31 * P(m(end) - m01, W(end) - k21*F1(end)) * F1(end) * dt;
    dF2 = k32 * P(m(end) - m02, W(end) - k22*F2(end)) * F2(end) * dt;
    if(length(F1)>len_of_one_life1)
        dF1 = dF1 - (F1(end-len_of_one_life1+2) - F1(end-len_of_one_life1+1));
    end
    if(length(F2)>len_of_one_life2)
        dF2 = dF2 - (F2(end-len_of_one_life2+2) - F2(end-len_of_one_life2+1));
    end
    F1(end+1) = F1(end) + dF1;
    F2(end+1) = F2(end) + dF2;
    f = F1(end) +F2(end);
    if(f > AREA)
        F1(end) = AREA * F1(end) / f;
        F2(end) = AREA * F2(end) / f;
    end
    W(end+1) = W(end) + dW;
    t(end+1) = t(end) + dt;
    m(end+1) = m(1) + 2*abs(sin(pi * t(end) / 12));
end
subplot(4,1,1);
plot(t,W);
subplot(4,1,2);
plot(t,F1);
subplot(4,1,3);
plot(t,F2);
subplot(4,1,4);
plot(t,m);
```

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