Flat Trap Potential Calculations

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1 Total Potential

The total potential experienced by the atoms is of the form

$$U_{tot} = U_{grav} + U_{mag} + U_{dip} \tag{1.1}$$

Which is just the sum of the gravitational, magnetic and dipole potentials respectively. The Cs chemical potential will be relatively small so we really need to focus on making sure that the Li Atoms are sufficiently well trapped. For good trapping, the trap depth needs to be at least 2-3 times the Fermi energy.

1.1 Gravitational Potential

The gravitational potential is simply

$$U_{grav} = mgz \tag{1.2}$$

1.2 Magnetic Potential

The magnetic potential comes from the bitter coils, we'll assume that the atoms are at the maximum of the magnetic potential and that the magnetic field is cylindrically symmetric about the z-axis due to the orientation of the coils:

$$U_B = -\frac{\mu_B g_J}{2} B \tag{1.3}$$

Where μ_B is the Bohr magneton (J/T) and g_J is the fine structure Landé g-factor. The magnetic field is given by:

$$B(x, y, z) = -\omega_B(x^2 + y^2) + 2\omega_B z^2 \tag{1.4}$$

$\omega_B = 20 \frac{J}{A \cdot m^4}$ is the approximate frequency of the magnetic trap used in the experiment

1.3 Dipole Potential

The dipole potential is taken from page 6 of (1):

$$U_{dip}(x, y, z) = \frac{\pi c^2 \Gamma}{2\omega_0^2} \left(\frac{2 + P g_F m_F}{\Delta_{2,F}} + \frac{1 - P g_F m_F}{\Delta_{1,F}} \right) I(x, y, z)$$

Where:

- Γ is the decay rate of the excited state in rad/s

- ω_0 is the resonant frequency of the transition in Hz

- P is a coefficient which characterizes the polarization

  - $P = 0$ for linear polarization
  - $P = \pm 1$ for circular polarization

- g_F is the Landé factor

- m_F is the magnetic quantum number

- $\Delta_{i_f} = \omega - \omega_0$ is the frequency detuning for the D1 or D2 transition

- $I(x, y, z)$ is the intensity as a function of position

Since we're using linearly polarized light, the dipole potential reduces to

$$U_{dip}(x, y, z) = \frac{\pi c^2 \Gamma}{2\omega_0^2} \left(\frac{2}{\Delta_{2,F}} + \frac{1}{\Delta_{1,F}} \right) I(x, y, z) \tag{1.5}$$

1.4 Electric field intensity

The general expression for the electric field of a Hermite-Gaussian mode propagating in the x-direction is given by (2):

$$E_{n,m}(x, y, z) = E_0 \frac{\omega_0}{\omega(x)} H_n \left(\frac{\sqrt{2}z}{\omega(x)} \right) e^{-\frac{z^2}{\omega(x)^2}} H_m \left(\frac{\sqrt{2}y}{\omega(x)} \right) e^{-\frac{z^2}{\omega(x)^2}} \tag{1.6}$$

Where H_n, H_m are the physicist's Hermite polynomials (4). The intensity of the TEM01 mode, assuming that we're at the focal point of the laser in x, is:

$$I_{01}(x, y, z) = \frac{8P}{\pi \omega_y(x) \omega_z^3(x)} z^2 e^{-2\frac{z^2}{\omega_z(x)^2}} e^{-2\frac{z^2}{\omega_y(x)^2}} \tag{1.7}$$

Where P is the beam power in W, $\omega_y(x)$ and $\omega_z(x)$ are the beam waists in y and z and are given by

$$\omega(x) = \omega_0 \sqrt{1 + \left(\frac{x}{x_R} \right)^2}$$

$$x_R = \frac{\pi \omega_0^2}{\lambda}$$

x_R is the Raleigh length and ω_0 is the minimum beam waist. For these calculations, there will be a minimum beam waist in y and a minimum beam waist in z which we will determine based on the system constraints. The maximum allowed laser power will also be determined by the constraints.

2 System Constraints and Beam Parameters

2.1 Constraints

Due to constraints on the size of the aperture that the beam is passing through, we can't actually achieve an arbitrarily small beam waist. The minimum possible beam waist is given by the expression:

$$\omega_{0,min} = \frac{\lambda}{\pi\theta}$$

Where λ is the wavelength of light we are going to use and θ is the divergence angle that determined by the diameter of the aperture and the distance to the center of the vacuum chamber. For 635 nm light, $\theta = 0.025$, this means that $\omega_{0,min} = 8.085 \times 10^{-6}$ m. To get to the 2d regime, the beam waist in y needs to be significantly larger than the beam waist in z so that the Li atoms spread out in the x-y plane. Additionally, the maximum amount of power available to us is 100 mW.
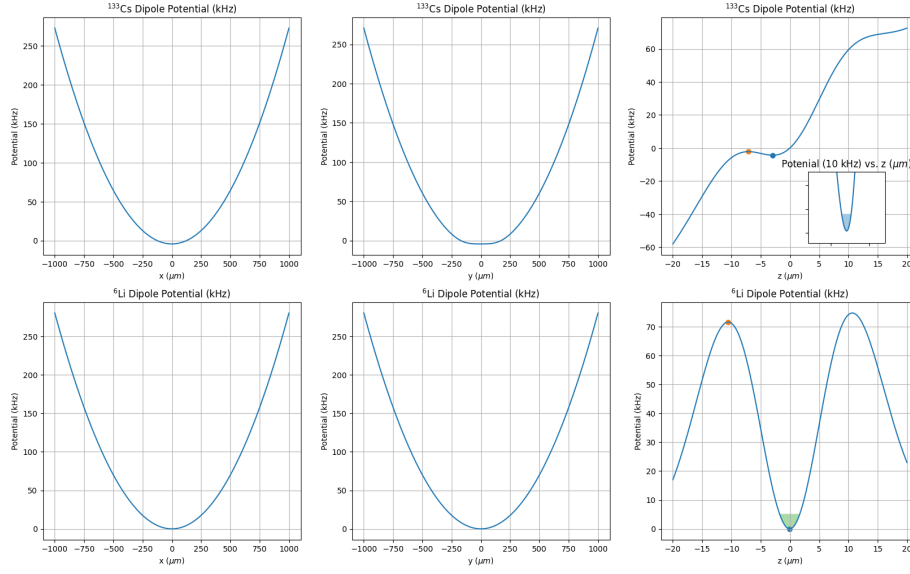
2.2 635 Laser Parameters

The power of the 635 nm laser was set to 70 mW and the minimum beam waists in y and z were set to 200 μm and 15 μm respectively.

3   Results

Potential Optimization   Below are the results of the trap calculations:

^{133}Cs and ^{6}Li Dipole Potentials (kHz) about the minimum for each species

Relevant Results   The relevant results are tabulated below:

	parameter	Values
0	Cs min (μm)	[-0.0, -0.0, -2.92687]
1	Cs trap depth (kHz)	2.177467
2	Cs trap frequency (kHz)	[0.00646, 0.0018, 0.24067]
3	Cs chemical potential (kHz)	0.144155
4	Cs atom density (atoms / μm)	3.37665
5	Li min (μm ^2)	[-0.0, -0.0, -0.04179]
6	Li trap depth (kHz)	71.599528
7	Li trap frequency (kHz)	[0.03069, 0.03068, 2.43774]
8	Li Fermi Energy (kHz)	5.164121
9	Li atom density (atoms / μm ^2)	0.17269

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4 References

1. Optical Dipole Traps for Neutral Atoms

2. General Hermite-Gaussian Modes

3. Gaussian Modes

4. Physicist's Hermite Polynomial

5. BEC and Fermi Gas Calculations