# Electric Field Intensity of the TEM01 Mode

Mouhamed Mbengue

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## 1 Hermite-Gaussian Modes

The general expression for the electric field of Gaussian-Hermite modes propagating in the x-direction (ignoring complex exponentials since we care about intensity) is:

$$E_{n,m}(x,y,z) = E_0 \frac{\omega_0}{\omega(x)} H_n\left(\frac{\sqrt{2}z}{\omega(x)}\right) e^{-\frac{z^2}{\omega(x)^2}} H_m\left(\frac{\sqrt{2}y}{\omega(x)}\right) e^{-\frac{z^2}{\omega(x)^2}} \tag{1.1}$$

If we account for the different beam waists in y and z then the expression becomes

$$E_{n,m}(x,y,z) = E_0 \sqrt{\frac{\omega_{0,y}\omega_{0,z}}{\omega_y(x)\omega_z(x)}} H_n\left(\frac{\sqrt{2}z}{\omega_z(x)}\right) e^{-\frac{z^2}{\omega_z^2(x)}} H_m\left(\frac{\sqrt{2}y}{\omega_y(x)}\right) e^{-\frac{y^2}{\omega_y^2(x)}}$$

Where $H_n$, $H_m$ are the physicist's Hermite polynomials. For the TEM01 mode, the field is:

$$E_{01}(x,y,z) = E_0 \sqrt{\frac{\omega_{0,y}\omega_{0,z}}{\omega_y(x)\omega_z(x)}} \left(2\frac{\sqrt{2}z}{\omega_z(x)}\right) e^{-\frac{z^2}{\omega_z^2(x)}} e^{-\frac{y^2}{\omega_y^2(x)}}$$

From this, the intensity will be:

$$I_{01}(x,y,z) = 8\frac{\omega_{0,y}\omega_{0,z}}{\omega_y(x)\omega_z^3(x)} E_0^2 z^2 e^{-2\frac{z^2}{\omega_z(x)^2}} e^{-2\frac{z^2}{\omega_y(x)^2}} \tag{1.2}$$

Where $\omega_y(x)$ and $\omega_z(x)$ are the beam waist's in y and z and are given by

$$\omega_i(x) = \omega_{0,i}\sqrt{1+\left(\frac{x}{x_{R,i}}\right)^2} \tag{1.3}$$

$$x_{R,i} = \frac{\pi\omega_{0,i}^2}{\lambda} \tag{1.4}$$

$x_R$ is the Raleigh length and $\omega_0$ is the minimum beam waist.

# 2 Power

The power delivered by the electric field is just the integral of the intensity over all y and z (not x since we want to know what the power is per unit area and not the power over the entire volume which will be infinity since the beam propagates into infinity)

$$P = \int_{\mathbb{R}^2} I_{01}(x, y, z)\, dA \tag{2.1}$$

$$= 8\frac{\omega_{0,y}\omega_{0,z}}{\omega_y(x)\omega_z^3(x)}E_0^2\left(\int_{-\infty}^{\infty} z^2 e^{-2\frac{z^2}{\omega(x)^2}}\, dz\right)\left(\int_{-\infty}^{\infty} e^{-2\frac{z^2}{\omega(x)^2}}\, dy\right)$$

We can make use of the following gaussian integrals

$$\int_{-\infty}^{\infty} e^{-2\frac{x^2}{a^2}}\, dx = a\sqrt{\frac{\pi}{2}} \tag{2.2}$$

$$\int_{-\infty}^{\infty} x^2 e^{-2\frac{x^2}{a^2}}\, dx = \frac{a^3}{4}\sqrt{\frac{\pi}{2}} \tag{2.3}$$

This means that:

$$P = 8\frac{\omega_{0,y}\omega_{0,z}}{\omega_y(x)\omega_z^3(x)}E_0^2\left(\omega_y(x)\sqrt{\frac{\pi}{2}}\right)\left(\frac{\omega_z^3(x)}{4}\sqrt{\frac{\pi}{2}}\right)$$

$$P = \omega_{0,y}\omega_{0,z}E_0^2\pi \tag{2.4}$$

We can use this to obtain the following expression for the field intensity as a function of power:

$$\boxed{I_{01}(x, y, z) = \frac{8P}{\pi\omega_y(x)\omega_z^3(x)}z^2 e^{-2\frac{z^2}{\omega_z(x)^2}}e^{-2\frac{z^2}{\omega_y(x)^2}}} \tag{2.5}$$