Assignment 1

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question

Using the factor theorem show that x-2 is a factor of x^3+x^2-4x-4.hence factorise the polynomial completely

solution:

By the factor theorem if $f(a)=0$,then $x-a$ will be factor of f(x)

let the given polynomial be f(x)

$$f(x)=x^3+x^2-4x-4 \tag{1}$$

$$f(2)=2^3+2^2-4\times2-4 \tag{2}$$

$$\implies f(2)=0 \tag{3}$$

so,x-2 is a factor of f(x),now to factorise f(x)

$$\begin{array}{r|l} x-2 & x^2+3x+2 \\ \hline & x^3+x^2-4x-4 \\ & -x^3+2x^2 \\ \hline & 3x^2-4x \\ & -3x^2+6x \\ \hline & 2x-4 \\ & -2x+4 \\ \hline & 0 \end{array}$$

we get x^2+3x+2 which is a quadratic expression so we can factorise it further by finding it's roots

$$x=\frac{-b\pm\sqrt{b^2-4\times a\times c}}{2\times a} \tag{4}$$

here,

$$b=3 \tag{5}$$

$$a=1 \tag{6}$$

$$c=2 \tag{7}$$

so roots would be

$$x=\frac{-3\pm\sqrt{3^2-4\times1\times2}}{2\times1} \tag{8}$$

$$x=-1 \text{ and } -2 \tag{9}$$

so -1 and -2 are the roots

$$\implies \textbf{other two factors are } x+1 \textbf{ and } x+2 \tag{10}$$

$\therefore$ the final factors of f(x) are $x+1$,$x+2$ and $x-2$

$$f(x)=(x+1)(x-2)(x+2) \tag{11}$$