

Assignment 1

ai21btech11007

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question

Using the factor theorem show that $x-2$ is a factor of $x^3 + x^2 - 4x - 4$. hence factorise the polynomial completely

solution:

By the factor theorem if $f(a) = 0$, then $x - a$ will be factor of $f(x)$

let the given polynomial be $f(x)$

$$f(x) = x^3 + x^2 - 4x - 4 \quad (1)$$

$$f(2) = 2^3 + 2^2 - 4 \times 2 - 4 \quad (2)$$

$$\Rightarrow f(2) = 0 \quad (3)$$

so, $x-2$ is a factor of $f(x)$, now to factorise $f(x)$

$$\begin{array}{r} x^2 + 3x + 2 \\ x-2 \overline{) x^3 + x^2 - 4x - 4} \\ \underline{-x^3 + 2x^2} \\ 3x^2 - 4x \\ \underline{-3x^2 + 6x} \\ 2x - 4 \\ \underline{-2x + 4} \\ 0 \end{array}$$

we get $x^2 + 3x + 2$ which is a quadratic expression so we can factorise it further by finding its roots

$$x = \frac{-b \pm \sqrt{b^2 - 4 \times a \times c}}{2 \times a} \quad (4)$$

here,

$$b = 3 \quad (5)$$

$$a = 1 \quad (6)$$

$$c = 2 \quad (7)$$

so roots would be

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times 2}}{2 \times 1} \quad (8)$$

$$x = -1 \text{ and } -2 \quad (9)$$

so -1 and -2 are the roots

other two factors are $x + 1$ and $x + 2$

the final factors of $f(x)$ are $x + 1, x + 2$ and $x - 2$

$$f(x) = (x + 1) \times (x - 2) \times (x + 2) \quad (10)$$