Assignment 1

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question

Using the factor theorem show that x-2 is a factor of $x^3 + x^2 - 4x - 4$.hence factorise the polynomial completely

solution:

By the factor theorem if $f(a) = 0$,then $x - a$ will be factor of f(x)
let the given polynomial be f(x)

$$f(x) = x^3 + x^2 - 4x - 4 \quad (1)$$
$$f(2) = 2^3 + 2^2 - 4 \times 2 - 4 \quad (2)$$
$$\implies f(2) = 0 \quad (3)$$

so,x-2 is a factor of f(x),now to factorise f(x)

$$\begin{array}{r|l} & x^2 + 3x + 2 \\ \hline x - 2 & x^3 + x^2 - 4x - 4 \\ & -x^3 + 2x^2 \\ \hline & 3x^2 - 4x \\ & -3x^2 + 6x \\ \hline & 2x - 4 \\ & -2x + 4 \\ \hline & 0 \end{array}$$

we get $x^2 + 3x + 2$

which is a quadratic expression so we can factorise it further by finding it's roots
roots are

$$\frac{-b \pm \sqrt{b^2 - 4 \times a \times c}}{2 \times a}$$

here $b = 3$, $a = 1$, $c = 2$
so roots would be

$$\frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times 2}}{2 \times 1}$$

-1 and -2 are roots
so other two factors are
$x + 1$ and $x + 2$
the final factors are
$x + 1$,$x - 2$ and $x + 2$

$$f(x) = (x + 1) \times (x - 2) \times (x + 2) \quad (4)$$