Assignment 4

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CBSE Probability Grade 12

Exercise 13.3.13 Probability that A speaks truth is $\frac{4}{5}$. A coin is tossed.A reports that a head appears.The probability that actually there was a head is

1) $\frac{4}{5}$
2) $\frac{1}{2}$
3) $\frac{1}{5}$
4) $\frac{2}{5}$

Solution. Let random variables $X, Y \in \{0, 1\}$ denote the following events in Table (1)

Event	Description
X=0	A tells truth
X=1	A tells false
Y=0	head appears on coin
Y=1	tails appears on coin

TABLE 1: Description of events

Probability	Value
Pr(X = 0)	4/5
Pr(X = 1)	1/5
Pr(Y = 0) | $Pr(X = 0)$	1/2
Pr(Y = 0) | $Pr(X = 1)$	1/2
Pr(X = 0) | $Pr(Y = 0)$	?

TABLE 2: Input probabilities

The desired probability is given by:

$$\Pr(X = 0|Y = 0) \tag{1}$$
$$= \frac{\Pr(X = 0, Y = 0)}{\Pr(Y = 0)} \tag{2}$$
$$= \frac{\Pr(Y = 0|X = 0)\Pr(X = 0)}{\sum_{i=0}^{1}\Pr(Y = 0, X = i)} \tag{3}$$
$$= \frac{\Pr(Y = 0|X = 0)\Pr(X = 0)}{\sum_{i=0}^{1}\Pr(Y = 0|X = i)\Pr(X = i)} \tag{4}$$

On substituting the values from Table we get:

$$\Pr(X = 0|Y = 0) = \frac{\frac{4}{5} \times \frac{1}{2}}{\frac{4}{5} \times \frac{1}{2} + \frac{1}{5} \times \frac{1}{2}} \tag{5}$$
$$= \frac{4}{5} \tag{6}$$