Assignment 6: Papoullis Text Book

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Outline

1. Question
2. solution

Question

Example 7.13

In this example, we shall find the two-dimensional density f($x_2, x_3|x_1$). This involves the evaluation of five parameters ; two conditional means, two conditional variances, and the conditional covariance of the random variables x_2 and x_3 assuming x_1

solution

The first four parameters are determined as

$$E\{x_2|x_1\} = \frac{R_{12}}{R_{11}}x_1 \tag{2.0.1}$$

$$E\{x_3|x_1\} = \frac{R_{13}}{R_{11}}x_1 \tag{2.0.2}$$

$$\sigma^2_{x_2|x_1} = R_{22} - \frac{R^2_{12}}{R_{11}} \tag{2.0.3}$$

$$\sigma^2_{x_3|x_1} = R_{33} - \frac{R^2_{13}}{R_{11}} \tag{2.0.4}$$

The conditional covariance

$$C_{x_2x_3|x_1} = E\left\{\left(x_2 - \frac{R_{12}}{R_{11}}x_1\right)\left(x_3 - \frac{R_{13}}{R_{11}}x_1\right)\middle| x_1 = x_1\right\} \tag{2.0.5}$$

is found as follows: We know that the errors $x_2 - \frac{R_{12}}{R_{11}}x_1$ and $x_3 - \frac{R_{13}}{R_{11}}x_1$ are independent of x_1. Hence the condition $x_1 = x_1$ can be removed. Expanding the product, we obtain

$$C_{x_2x_3|x_1} = R_{23} - \frac{R_{12}R_{13}}{R_{11}} \tag{2.0.6}$$