Assignment 7: Papoullis Text Book

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Outline

Question

Example 8.9

Suppose that $x = \theta + \mathbf{v}$ is an $N(0, \sigma)$ random variable and θ is the value of an $N(\theta_0, \sigma_0)$ random variable θ . Find the bayesian estimate $\hat{\theta}$ of θ.

solution

The density f(x — θ) of x is $N(\theta, \sigma)$. We conclude that the function $f_\theta(\theta|x)$ is $N(\theta_1, \sigma_1)$ where

$$\sigma_1^2 = \frac{\sigma^2}{n} \times \frac{\sigma_0^2}{\sigma_0^2 + \frac{\sigma^2}{n}} \tag{2.0.1}$$

$$\theta_1 = \frac{\sigma_1^2}{\sigma_0^2}\theta_0 + \frac{n\sigma_1^2}{\sigma_0^2}\vec{x} \tag{2.0.2}$$

From this it follows that $E\{\theta|x\} = \theta_1$; in other words $\hat{\theta} = \theta_1$.

The classical estimate of θ is the average $\bar{x}$ of x_i. Furthermore, its prior estimate is the constant θ_0. Hence $\hat{\theta}$ is the weighted average of the prior estimate θ_0 and the classical estimate $\bar{x}$. As n tends to ∞, $\sigma_1 \to 0$ and $\frac{n\sigma_1^2}{\sigma_0^2} \to 1$; hence $\hat{\theta}$ tends to $\bar{x}$. Thus, as the number of measurements increases, the bayesian estimate $\hat{\theta}$ approaches the classical estimate $\bar{x}$; the effect of the prior becomes negligible.