EE3900 Assignment1

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3.2 : Determine the z-transform of the sequence

$$x[n] = \begin{cases} n, & 0 \le n \le N-1 \\ N, & N \le n \end{cases} \tag{1}$$

Solution : Z-transform of $x[n]$ is

$$\sum_{n=-\infty}^{\infty} x[n]z^{-n} \tag{2}$$

$$= \sum_{n=0}^{\infty} x[n]z^{-n} \tag{3}$$

$$= \sum_{n=0}^{N-1} x[n]z^{-n} + \sum_{n=N}^{\infty} x[n]z^{-n} \tag{4}$$

$$= \sum_{n=0}^{N-1} nz^{-n} + \sum_{n=N}^{\infty} Nz^{-n} \tag{5}$$

$$\tag{6}$$

First term is in AGP.

$$\sum_{n=0}^{N-1} nz^{-n} = z^{-1} + 2z^{-2} + ... + (N-1)z^{-N+1} \tag{7}$$

$$= \frac{z^{-1} - z^{-N}}{(z^{-1}-1)^2} - \frac{(N-1)z^{-N}}{1-z^{-1}} \tag{8}$$

$$\tag{9}$$

Second term is in GP.

$$\sum_{n=N}^{\infty} Nz^{-n} = N(z^{-N} + z^{-N-1} + ... \tag{10}$$

$$= N\frac{z^{-N}}{1-z^{-1}} \tag{11}$$

On adding

$$= \frac{z^{-1} - z^{-N}}{(z^{-1}-1)^2} - \frac{(N-1)z^{-N}}{1-z^{-1}} + N\frac{z^{-N}}{1-z^{-1}} \tag{12}$$

$$= \frac{z^{-1} - z^{-N-1}}{(1-z^{-1})^2} \tag{13}$$

Hence Z transform of $x[n]$ is

$$\frac{z^{-1} - z^{-N-1}}{(1-z^{-1})^2} \tag{14}$$