**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

ANS:Time=60mins

Servicing begin after =10mins

Available time =60-10=50

P(s)= 1-stats.norm.cdf(50,45,8)= 0.26598552904870054

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

ANS:X=44

MEAN=38

STANDARD DEVIATION=6

Z score = (Value - Mean)/SD

Z score for 44  = (44 - 38)/6  = 1  =>  84.13 %

=> People above 44 age = 100 - 84.13 =  15.87%  ≈  63    out of 400

Z score for 38  = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44  age = 84.13 - 50 = 34.13 % ≈  137 out of 400

Hence More employees at the processing center are older than 44 than between 38 and 44. is F**ALSE**

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees

ANS: Z score for 30  = (30 - 38)/6 =  -1.33  =  9.15  %   ≈ 36 out of 400

Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS: **According to the Central Limit Theorem, any large sum of independent, identically distributed(iid) random variables is approximately Normal.**

**The Normal distribution is defined by two parameters, the mean, μ, and the variance,**σ2**and written as . X1 ~ N(μ, σ2)**

**Given X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2)   are two independent identically distributed random variables.**

**From the properties of normal random variables,**

**if X~ N(μ, σ2) and Y ~ N(μ, σ2)  and  are two independent identically distributed random variables then**

* **the sum of normal random variables is given by**

**X+Y ~ N(μ1+μ2, σ2 1+ σ22)**

* **and the difference of normal random variables is given by**

**X-Y ~ N(μ1-μ2, σ2 1+ σ22)**

* **When  Z=aX, the product of X is given by**

**Z~N(aμ1,a2σ21)**

* **When  z=aX+bY, the linear combination of X and Y is given by**

**Z~N(aμ1+ bμ2,a2σ21+ b2σ22)**

**Given to find, 2X1**

**Thus, following the property of multiplication, we get**

**2X1~ N(2μ,22σ2)**

**2X1~ N(2μ,4σ2)**

**and following the property of addition,**

**X1+X2 ~ N(μ+μ, σ2+ σ2) ~N(2μ,2σ2)**

**And the difference between the two is given by**

**2X1-(X1+X2) ~ N(2μ-2μ, 2 σ21+ 4 σ22) ~N(0,6σ2)**

**The mean of 2X1 and X1+X2 is same but the var(σ2) of  2X1 is 2 times more than the variance of X1+X2  .**

**The difference between the two says that the two given variables are identically and independently distributed.**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

ANS:stats.norm.cdf(0.99,100,20)

A=48.48341

B=151.5165

Option:D

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

ANS:mean=5+7=12

Mean of two profits=12\*45=540millions

Standard deviation=sqrt(9+16)=5

Sum of two profits of sd=sqrt((5)2\*(45)2=225millions

Stats.norm.interval(0.95,540,225)

P(a) for 95%=(99.00810,980.9918)

1. Specify the 5th percentile of profit (in Rupees) for the company

ANS:Stats.norm.ppf(0.05)=-1.64485

X=540+(-1.64\*225)

= 170 million rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

ANS:stats.norm.cdf(0,5,3)=0.047790

stats.norm cdf(0,7,4)=0.040059

Therefore the profit1 has the larger probability of making a loss in a given year than profit2.