CSE340: Computer Architecture

Assignment 3 Solution

Chapter 3 (Arithmetic for Computers)

Question 01

Multiplicand = 13 (Decimal) and multiplier = 17 (Decimal)
Complete the multiplication following the Optimized multiplication algorithm.

Solution:
17 = 10001
13 = 01101 (converting it into 5 bits as well)

Iteration	Multiplicand 01101	Product 00000 10001
1	01101	01101 10001
		00110 11000
2	01101	00011 01100
3	01101	00001 10110
4	01101	00000 11011
5	01101	01101 11011
		00110 11101

Solve of Question - 02

A B B 9 6 0 9

= 1010 1011 1011 1001 0110 0000 1001

$$\underbrace{1}_{\text{Sign bit}}\ \underbrace{0101011}_{\text{exponent (biased)}}\ \underbrace{1011\ 1001\ 0110\ 0000\ 1001}_{\text{fraction}}$$

$\therefore$ biased exponent = 0101011 = 43

$$\text{bias} = 2^{n-1} - 1 = 2^{7-1} - 1 = 63$$

$\therefore$ exponent = $43 - 63 = -20$

$$\text{the decimal value} = (-1)^{\text{sign}} (1 + \text{fraction}) \times 2^{\text{exponent}}$$

$$= (-1)^{1} (1.1011\ 1001\ 0110\ 0000\ 1001) \times 2^{-20}$$

$$= -1.1011\ 1001\ 0110\ 0000\ 1001 \times 2^{-20}$$

$$= -1.050100336 \times 10^{-6}$$

(Ans.)

Solve of Question- 03:

$70 \cdot 78955 = 1000110 \cdot 1100\ 1010\ 000\ 111111111$

$\hookrightarrow 1 \cdot 000110\ 1100\ 1010\ 000\ 111111111 \times 2^6$

$\text{bias} = 2^{n-1} - 1 = 2^{9-1} - 1 = 255$

$\therefore$ biased exponent $= 6 + 255 = 261$

$261 = 100000101$ —— biased exponent

Sign bit $= 0$

floating point pepresentation:

$\underbrace{0}_{\text{sign bit}} \quad \underbrace{100000101}_{\text{biased exponent}} \quad \underbrace{000110\ 1100\ 1010\ 0000\ 11111111}_{\text{fraction}}$

(Ans.)

Solve of Question 4-(a):

$50 \cdot 7869 = 110010 \cdot 1100\,1001\,0111$

~~79~~ $= 1 \cdot 10010\,1100\,1001\,0111 \times 2^5$

$79 \cdot 83 = 1001111 \cdot 1101\ 0100\ 0111$

$= 1 \cdot 001111\ 1101\ 0100\ 0111 \times 2^6$

$29 \cdot 58 = 11101 \cdot 1001\ 0100\ 0111$

$= 1 \cdot 1101\ 1001\ 0100\ 0111 \times 2^4$

Match the lower exponent with higher ones

$50 \cdot 7869 = \cancel{0 \cdot 0}\ 0 \cdot 110010\ 1100\ 1001\ 0111 \times 2^6$

$79 \cdot 83 = 1 \cdot 001111\ 1101\ 0100\ 0111 \times 2^6$

$29 \cdot 58 = 0 \cdot 011101\ 1001\ 0100\ 0111 \times 2^6$

$10 \cdot 1\ 00000\ 001100\ 1001101 \times 2^6$

(Ans.)

Solve of Question 4-(b)

$a = 64.2486 = 1000000.0011111110$

$= 1.000000\,00\,11111110 \times 2^{6}$

$b = 49.1832 = 110001.001011101110$

$= 1.10001\,00\,10\,1110\,1110 \times 2^{5}$

$a \times b = (1.00000000\,11\,111110 \times 1.10001\,0010111\,01110)$

$\times 2^{6+5}$

$= 1.1\,000\,1010111\,1111\,001110 \times 2^{11}$

Answer to the question no:5

we have to subtract -4.0210 from 28.4810

$\therefore$ $28.4810 - (-4.0210)$

$= 28.4840 + 4.0210$

$28.4840 = 11100.01111\,0111\ldots$ Normalize, $\boxed{1.110001111\,011 \times 2^4}$

11100

$0.4840 \times 2 = \underline{0}.968$

$0.968 \times 2 = \underline{1}.936$

$0.936 \times 2 = \underline{1}.872$

$.872 \times 2 = \underline{1}.744$

$.744 \times 2 = \underline{1}.488$

$.488 \times 2 = \underline{0}.976$

$.976 \times 2 = \underline{1}.952$

$.952 \times 2 = \underline{1}.904$

$.904 \times 2 = \underline{1}.808$

$4.0210 = 100.000001010\ldots$ (DO)

Normalize, $\boxed{1.00000001010 \times 2^2}$

$\therefore$ add $1.110001111011 \times 2^4 + 1.00000001010 \times 2^2$

$\Rightarrow 1.110001111011 \times 2^4 + 0.0100000001010 \times 2^4$

$\Rightarrow 2^4 (1.110001111011 + 0.010000000001010)$

$\Rightarrow 10.00001 \times 2^4 \Rightarrow 1000001$

normalize $\Rightarrow \approx 32.5$ (Ans)

$= 1.000001 \times 2^5$

value for addition = 1.000001×2^5

Actual exponent 5

Bias = $2^{8-1} - 1 = 2^7 - 1 = 128 - 1 = 127$

$\therefore$ Biased exponent = 5+127 = 132

size of exponent field = 8 bits

$\therefore$ range 0 to $2^8 - 1$ = 0 to 255

0 & 255 reserved.

$\therefore$ range = 1 to 254

as $1 < 132 < 254$

$\therefore$ No overflow or underflow

Answer to the question no: 6

$A = 0.000101 \times 2^{-85}$

$= 1.01 \times 2^{-89}$

$B = 10.1 \times 2^{-90}$

$= 1.01 \times 2^{-89}$

$A \times B = 1.01 \times 2^{-89} \times 1.01 \times 2^{-89}$

$= 1.01 \times 1.01 \times 2^{-89-89}$

$= 1.1001 \times 2^{-178}$

```
   1.01
   1.01
 -------
   1.01
  000x
 1.01xx
 -------
 1.1001
```

Actual exponent = -178

12 fraction bits, 1 sign bit

$\therefore$ exponent bit $(18-12-1)$

= 5 bits

$\therefore$ bias $2^{5-1}-1 = 2^4-1 = 16-1 = 15$

$\therefore$ Biased exponent $= -178+15$

$= -163 < 1$

$\therefore$ underflow.

Answer to the question no: 7

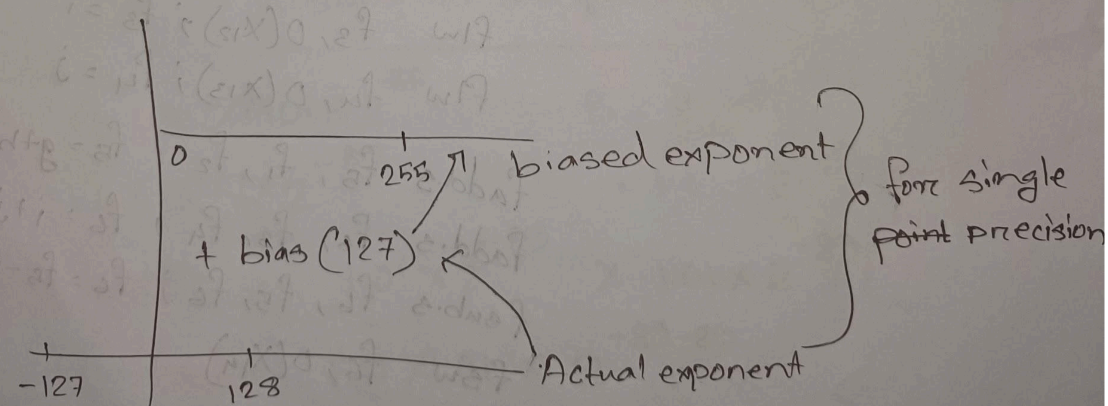
$(g+h)-(i+j) \longrightarrow$

```
flw    f1, 0(x10)  ; f1 = g
flw    f2, 0(x11)  ; f2 = h
flw    f3, 0(x12)  ; f3 = i
flw    f4, 0(x13)  ; f4 = j
fadd.s f5, f1, f2  ; f5 = g+h
fadd.s f6, f3, f4  ; f6 = i+j
fsub.s f6, f5, f6  ; f6 = f5 - f6
fsw    f6, 0(x14)
```

Answer to the question no: 8

ⓐ

In our exponent field there is no sign bit. For single precision, all 8 exponent bits are used to represent the value of the exponent. In that case we can only show the positive values ranging from 0 to 255. We can represent a huge number but no small numbers where the exponent is below 0. To include those numbers we add a bias vale to the actual exponent which simply places the number from the negative ranges into the positive range as well.



(b)

	long multipication	Optimized multiplication
product	2 registers needed	2 registers needed
multiplier	1 register needed	No registers needed directly placed into the lower half of the product
multiplicand	2 registers needed	1 register needed
shifting	multiplicand left shift multiplier right shift. 2 shift instructions to be executed	product right shift 1 shift instruction to be executed

Based on the discussion above we can clearly identify that optimized multiplier uses lesser amount of physical devices and fewer instructions as well making it easier to compute