

MAT 110

Final (Summer 21)

Set 8 (A&D)

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See : 05

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Answer to the Q no. 1

Given, $f(x) = x^4 - 12x^3$

$$\Rightarrow f'(x) = 4x^3 - 36x^2$$

Now, $4x^3 - 36x^2 = 0$

$$\Rightarrow x = 9, 0 \quad [\text{using calculator}]$$

Again, $f'(x) = 4x^3 - 36x^2$

$$\Rightarrow f''(x) = 12x^2 - 72x$$

$$\Rightarrow f''(9) = 12 \times 9^2 - 72 \times 9$$

$= 324$; which is greater than 0 & minima

$$\Rightarrow f''(0) = 0$$
 exists.

So, minima is $f(x) = x^4 - 12x^3 \Rightarrow$

$$\Rightarrow f(9) = 9^4 - 12 \times 9^3$$

$$= -2187$$

Ans.

Answer to the Q no. 2

Given, $f(x) = \frac{1}{x+2}$; $x = 3$

We know,

$$f(x) = f(a) + f'(a)(x-a) + f''(a)(x-a)^2 \cdot \frac{1}{2!} + \dots$$

Now,

$$f(x) = \frac{1}{x+2}$$

$$\Rightarrow f(3) = \frac{1}{5}$$

$$f'(x) = -\frac{1}{(x+2)^2}$$

$$\Rightarrow f'(3) = -\frac{1}{5^2}$$

$$f''(x) = \frac{2}{(x+2)^3}$$

$$\Rightarrow f''(3) = \frac{2}{5^3}$$

$$\begin{aligned} \text{So, } f(x) &= \frac{1}{x+2} \Rightarrow f(3) + f'(3)(x-3) + f''(3)(x-3)^2 \cdot \frac{1}{2!} + \\ &= \frac{1}{5} - \frac{1}{25}(x-3) + \frac{2}{125} \cdot \frac{1}{2!}(x-3)^2 + \dots \end{aligned}$$

Answer to the Q no. 3

Given, $T = xy - xy^3 + 2$; where, $x = \pi \cos \theta$

$$y = \pi \sin \theta$$

$$\Rightarrow T = (\pi \cos \theta)(\pi \sin \theta) - \pi \cos \theta \cdot (\pi \sin \theta)^3 + 2$$

$$\Rightarrow T = \pi^3 \cos^2 \theta \sin \theta - \pi^4 \cos \theta \sin^3 \theta + 2$$

Now,

$$\frac{\partial T}{\partial \pi} = \frac{\partial}{\partial \pi} (\pi^3 \cos^2 \theta \sin \theta - \pi^4 \cos \theta \sin^3 \theta + 2)$$

$$= 3\pi^2 \cos^2 \theta \sin \theta - 4\pi^3 \cos \theta \sin^3 \theta$$

$$= 3(\pi \cos \theta)^2 \sin \theta - 4(\pi \sin \theta)^3 \cos \theta$$

$$= 3x^2 \sin \theta - 4y^3 \cos \theta$$

$$\frac{\partial T}{\partial \theta} = \frac{\partial}{\partial \theta} (\pi^3 \cos^2 \theta \sin \theta - \pi^4 \cos \theta \sin^3 \theta + 2)$$

$$= \pi^3 [\cos^2 \theta \cdot \cos \theta - \sin^2 \theta \cdot 2 \cos \theta] - \pi^4 [\cos^2 \theta \cdot$$

$$- \sin^3 \theta \cdot \sin \theta]$$

$$= \pi^3 [\cos^3 \theta - 2\sin^2 \theta \cos \theta] - \pi^4 [3\sin^2 \theta \cos^2 \theta]$$

$$\begin{aligned} & \text{Berechne } x, \text{ dann } y: \quad \underline{x^3 + 3x^2y^2 - 3xy^4 = T} \\ & \text{durch: } x^3 = 3 \\ & = x^3 - 2xy^2 - 3x^2y^2 + y^4 \\ & \underline{(x^3 + 3x^2y^2) - (3xy^2 + y^4) = T} \end{aligned}$$

$$\underline{\underline{x^3 + 3x^2y^2 - 3xy^2 - y^4 = T}} \quad \text{Ans.}$$

$$(x^3 + 3x^2y^2 - 3xy^2 - y^4) \frac{6}{96} = \frac{T_6}{96}$$

$$3x^2y^2(3x^2y^2) - 3xy^2(3x^2y^2) = 0$$

$$3x^2y^2(3x^2y^2) - 3xy^2(3x^2y^2) = 0$$

$$3x^2y^2(3x^2y^2) - 3xy^2(3x^2y^2) = 0$$

$$(x^3 + 3x^2y^2 - 3xy^2 - y^4) \frac{6}{96} = \frac{T_6}{96}$$

Antwort: $T_6 = 3x^2y^2 - 3xy^2 - y^4$

Answer to the Q no. 4

Given,

$$f(x, y) = x^2 + xy + y^2 - 3x$$

$$f_x(x, y) = 2x + y - 3$$

$$f_{xy}(x, y) = 2$$

$$f_y(x, y) = 2y + x$$

$$f_{yy}(x, y) = 2$$

$$f_{xy}(x, y) = 1$$

Now,

$$f_y = 0$$

$$\Rightarrow x = -2y$$

$$\therefore x = \cancel{+} 2$$

$$f_x = 0$$

$$\Rightarrow 2(-2y) + y = 3$$

$$\Rightarrow -3y = 3$$

$$\therefore y = -1$$

\therefore Critical point $(2, -1)$

As we know,

$$f_{xx}, f_{yy} > 0$$

$$\rho \in \mathbb{R}^2$$

$$(f_{xy})^2 < f_{xx} \cdot f_{yy}$$

$$\rho = (1, 4)$$

\therefore It's a minima.

Relative minima at point $(2, -1)$

$$\rho = (1, 0)$$

$$\rho = (0, 1)$$

$$0 = \rho$$

$$0 = \rho$$

$$\rho = \beta + (\beta^2 - 1)^{1/2}$$

$$\beta^2 - 1 \geq 0$$

$$\rho = \beta - 1$$

$$\beta - 1 \geq 0$$

$$1 - \beta \leq 0$$

Answer to the Q no. 6

Given, $\bar{F} = e^x \hat{i} + \ln(xy) \hat{j} + e^{xz} \hat{k}$

Divergence:

$$\begin{aligned}\bar{\nabla} \cdot \bar{F} &= \frac{\partial}{\partial x}(e^x) + \frac{\partial}{\partial y}(\ln(xy)) + \frac{\partial}{\partial z}(e^{xz}) \\ &= e^x + \frac{1}{xy} \cdot x + xy \cdot e^{xz} \\ &= e^x + \frac{1}{y} + xy \cdot e^{xz}.\end{aligned}$$

Curl :

$$\begin{aligned}\bar{\nabla} \times \bar{F} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^x & \ln(xy) & e^{xz} \end{vmatrix} \\ &= \hat{i} (xz e^{xz} - 0) - \hat{j} (yz e^{xz} - 0) + \hat{k} \left(\frac{1}{x} - 0 \right) \\ &= xz e^{xz} \hat{i} - yz e^{xz} \hat{j} + \frac{1}{x} \hat{k}\end{aligned}$$

Ans.

Answer to the Q no. 7

Given,

$$-x^2 + 4y^2 - 2x - 16y + 11 = 0$$

Now,

$$-x^2 + 4y^2 - 2x - 16y = -11$$

$$\Rightarrow -(x+1)^2 + 4(y-4)^2 = -11 - 1$$

$$\Rightarrow -(x+1)^2 + 4(y-4)^2 = -12$$

$$\Rightarrow -(x+1)^2 + 4(y-2)^2 - 16 = -12$$

$$\Rightarrow -(x+1)^2 + 4(y-2)^2 = 4$$

$$\Rightarrow \frac{(y-2)^2}{1} - \frac{(x+1)^2}{4} = 1$$

here,

$$a = 1$$

$$b = 2$$

$$K = 2$$

$$h = -1$$

$$\therefore \text{centre } (h, K) = (-1, 2)$$

$$\text{vertices } (h, K \pm a) = (-1, 3), (-1, 1)$$

\therefore foci $(h, k \pm \sqrt{a^2 + b^2})$

$$= (-1, 2 \pm \sqrt{1+4})$$

$$= (-1, 2+\sqrt{5}), (-1, 2-\sqrt{5})$$

$$H = B^2I - \lambda S = B^2I + \lambda S$$

$$L - H = B^2I - \lambda S + \lambda (I + S) = \lambda$$

$$S = (B^2 - \lambda^2)I + \lambda (I + S) = \lambda$$

$$S = \lambda I + (\lambda^2 - B^2)I + \lambda (I + S) = \lambda$$

$$S = \lambda (I + S) + \lambda (I + S) = \lambda$$

$$I = \frac{\lambda(I + S)}{\lambda} = \frac{\lambda(I + S)}{\lambda}$$

$$I = S$$

$$S = d$$