MAT 110

Final (Summer 21)

Set 8 (A&D)

E M Rezwan Hossain

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Sec : 05

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Answer to the ~~Q~~ no. 1

Given, $f(x) = x^4 - 12x^3$

$\Rightarrow f'(x) = 4x^3 - 36x^2$

Now, $4x^3 - 36x^2 = 0$

$\Rightarrow x = 9, 0$ [using calculator]

Again, $f'(x) = 4x^3 - 36x^2$

$\Rightarrow f''(x) = 12x^2 - 72x$

$\Rightarrow f''(9) = 12 \times 9^2 - 72 \times 9$

$= 324$; which is greater then 0 & minima exists.

$\Rightarrow f''(0) = 0$

So, minima is $f(x) = x^4 - 12x^3 \Rightarrow$

$\Rightarrow f(9) = 9^4 - 12 \times 9^3$

$= -2187$

Ans.

Answer to the Q no. 2

Given, $f(x) = \frac{1}{x+2}$; $x = 3$

We know,

$$f(x) = f(a) + f'(a)(x-a) + f''(a)(x-a)^2 \cdot \frac{1}{2!} + ----$$

Now,

$$f(x) = \frac{1}{x+2}$$

$$\Rightarrow f(3) = \frac{1}{5}$$

$$f'(x) = -\frac{1}{(x+2)^2}$$

$$\Rightarrow f'(3) = -\frac{1}{5^2}$$

$$f''(x) = \frac{2}{(x+2)^3}$$

$$\Rightarrow f''(3) = \frac{2}{5^3}$$

So, $f(x) = \frac{1}{x+2} \Rightarrow f(3) + f'(3)(x-3) + f''(3)(x-3)^2 \cdot \frac{1}{2!} + ----$

$$= \frac{1}{5} - \frac{1}{25}(x-3) + \frac{2}{125} \cdot \frac{1}{2!}(x-3)^2 + ----$$

Answer to the Q no. 3

Given,

$$T = x^2 y - x y^3 + 2 \quad ; \text{ where, } x = r\cos\theta$$

$$y = r\sin\theta$$

$$\Rightarrow T = (r\cos\theta)^2 (r\sin\theta) - r\cos\theta \cdot (r\sin\theta)^3 + 2$$

$$\Rightarrow T = r^3 \cos^2\theta \sin\theta - r^4 \cos\theta \sin^3\theta + 2$$

Now,

$$\frac{\partial T}{\partial r} = \frac{\partial}{\partial r}\left(r^3 \cos^2\theta \sin\theta - r^4 \cos\theta \sin^3\theta + 2\right)$$

$$= 3r^2 \cos^2\theta \sin\theta - 4r^3 \cos\theta \sin^3\theta$$

$$= 3(r\cos\theta)^2 \sin\theta - 4(r\sin\theta)^3 \cos\theta$$

$$= 3x^2 \sin\theta - 4y^3 \cos\theta$$

$$\frac{\partial T}{\partial \theta} = \frac{\partial}{\partial \theta}\left(r^3 \cos^2\theta \sin\theta - r^4 \cos\theta \sin^3\theta + 2\right)$$

$$= r^3\left[\cos^2\theta \cdot \cos\theta - \sin^2\theta \cdot 2\cos\theta\right] - r^4\left[\cos^2\theta \cdot - \sin^3\theta \cdot \sin\theta\right]$$

II

$$= r^3[\cos^3\theta - 2\sin^2\theta\cos\theta] - r^4[3\sin^2\theta\cos^2\theta - \sin^4\theta]$$

$$= x^3 - 2xy^2 - 3x^2y^2 + y^4$$

Ans.

Answer to the Q no. 4

Given, $f(x,y) = x^2 + xy + y^2 - 3x$

$f_x(x,y) = 2x + y - 3$

$f_{xx}(x,y) = 2$

$f_y(x,y) = 2y + x$

$f_{yy}(x,y) = 2$

$f_{xy}(x,y) = 1$

Now, $f_y = 0$

$\Rightarrow x = -2y$

$\therefore x = 2$

$f_x = 0$

$\Rightarrow 2(-2y) + y = 3$

$\Rightarrow -3y = 3$

$\therefore y = -1$

$\therefore$ Critical point $(2, -1)$

As we know,

$$f_{xx}, f_{yy} > 0$$

$$(f_{xy})^2 < f_{xx} \cdot f_{yy}$$

$\therefore$ It's a minima. $(1 < 4)$

Reletive minima at point $(2, -1)$

Answer to the Q no. 6

Given, $\bar{F} = e^{x}\hat{i} + \ln(xy)\hat{j} + e^{xyz}\hat{k}$

Divergence:

$$\bar{\nabla}\cdot\bar{F} = \frac{\partial}{\partial x}(e^{x}) + \frac{\partial}{\partial y}(\ln(xy)) + \frac{\partial}{\partial z}(e^{xyz})$$

$$= e^{x} + \frac{1}{xy}\cdot x + xy\cdot e^{xyz}$$

$$= e^{x} + \frac{1}{y} + xy\cdot e^{xyz}$$

Curl :

$$\bar{\nabla}\times\bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{x} & \ln(xy) & e^{xyz} \end{vmatrix}$$

$$= \hat{i}\left(xze^{xyz} - 0\right) - \hat{j}\left(yze^{xyz} - 0\right) + \hat{k}\left(\frac{1}{x} - 0\right)$$

$$= xze^{xyz}\hat{i} - yze^{xyz}\hat{j} + \frac{1}{x}\hat{k}$$

Ans.

Answer to the Q no. 7

Given, $-x^2 + 4y^2 - 2x - 16y + 11 = 0$

Now,

$$-x^2 + 4y^2 - 2x - 16y = -11$$

$$\Rightarrow -(x+1)^2 + 4y^2 - 16y = -11 - 1$$

$$\Rightarrow -(x+1)^2 + 4(y^2 - 4y) = -12$$

$$\Rightarrow -(x+1)^2 + 4(y^2 - 2.2y + 4) - 16 = -12$$

$$\Rightarrow -(x+1)^2 + 4(y-2)^2 = 4$$

$$\Rightarrow \frac{(y-2)^2}{1} - \frac{(x+1)^2}{4} = 1$$

here, $a = 1$

$b = 2$

$K = 2$

$h = -1$

$\therefore$ centre $(h, K) = (-1, 2)$

vertices $(h, K \pm a) = (-1, 3), (-1, 1)$

$\therefore$ foci $\left(h, k \pm \sqrt{a^2+b^2}\right)$

$= \left(-1, 2 \pm \sqrt{1+4}\right)$

$= \left(-1, 2+\sqrt{5}\right), \left(-1, 2-\sqrt{5}\right)$