MAT 110 - FINAL

Name: Umme Abira Azmary.

ID: 20101539

SUMMER - 2021.

SET - 19.

Ans. to the ques. no-1.

① . $g(x) = \begin{cases} (1+x)^4 - 1, & x \leq 0. \\ \sin(4x), & x > 0. \end{cases}$

$$L.H.L = \lim_{h \to 0^-} \left[g(0+h) - g(0) \right]$$

$$= \lim_{h \to 0^-} \left[\frac{(1+h)^4 - 1(1-1)}{h} \right]$$

$$= \lim_{h \to 0^-} \left[\frac{h^4 + 4h^3 + 6h^2 + 4h + 1 - 1}{h} \right]$$

$$= \lim_{h \to 0^-} (h^3 + 4h^2 + 6h + 4)$$

$$= 4.$$

Scanned with CamScanner

$$R.H.L = \lim_{h\to 0^+} \left[\frac{g(0+h) - g(0)}{h}\right].$$

$$= \lim_{h\to 0^+} \left[\frac{\sin 4h - \sin(4\times 0)}{h}\right]$$

$$= \lim_{h\to 0^+} \frac{\sin 4h}{4h} \times 4.$$

$$= 1 \times 4$$

$$= 4.$$

So, L.H.L = RHL.

$$\Rightarrow \lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x).$$

So, $g(x)$ is differentiable and continuous at the point $x = 0$.

② . $x(t) = \ln(t^4)$, $y(t) = t.e^{4t}$.

Here,

$$x(t) = \ln(t^4).$$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{t^4} \cdot 4t^3.$$

$$\Rightarrow \frac{dx}{dt} = \frac{4}{t}. \qquad \Rightarrow \frac{dt}{dx} = \frac{t}{4}.$$

$$y(t) = t.\, e^{4t}.$$

$$\Rightarrow \frac{dy}{dt} = e^{4t} + t.e^{4t}.4.$$

$$\Rightarrow \frac{dy}{dt} = e^{4t} + 4t.e^{4t}.$$

So, $$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}.$$

$$= \left(e^{4t} + 4t.e^{4t}\right) \frac{t}{4}.$$

$$= \frac{te^{4t}}{4} + t^2 e^{4t}.$$

Scanned with CamScanner

For 2nd derivative, $\frac{d^2y}{dx^2}$

So, $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}.$$

Now, $$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{t.e^{4t}}{4} + t^2.e^{4t}.$$

$$= \frac{t.e^{4t}.4 + e^{4t}.}{4} + t^2.e^{4t}.4 + 2t.e^{4t}.$$

$$= t.e^{4t} + \frac{1}{4}.e^{4t} + 4t^2.e^{4t} + 2t.e^{4t}.$$

So, $$\frac{d^2y}{dx^2} = \frac{t.e^{4t} + \frac{1}{4}e^{4t} + 4t^2.e^{4t} + 2t.e^{4t}.}{\frac{4}{t}.}$$

$$= \frac{e^{4t}\left(\frac{1}{4} + t + 2t + 4t^2\right).}{4/t.}$$

$$\Rightarrow \frac{t.e^{4t}\left(\frac{1}{4} + t + 2t + 4t^2\right)}{4.}$$

Scanned with CamScanner

③. $g(x,y) = 9x^3 - 81xy + 9y^3$.

$$\Rightarrow \frac{\delta g(x,y)}{\delta x} = 27x^2 - 81y. \quad \text{— (I)}$$

$$\Rightarrow \frac{\delta g(x,y)}{\delta y} = -81x + 27y^2.$$

$$\Rightarrow \frac{\delta g(x,y)}{\delta y} = 27y^2 - 81x \quad \text{— (II)}.$$

From (I) and (II),

$$27x^2 - 81y = 0.$$

$$\Rightarrow 27x^2 = 81y.$$

$$\Rightarrow y = \frac{27}{81}x^2$$

$$\Rightarrow y = \frac{1}{3}x^2.$$

and, $27y^2 - 81x = 0$.

$$\Rightarrow 27\left(\frac{1}{3}x^2\right)^2 - 81x = 0.$$

$$\Rightarrow 27 \cdot \frac{1}{9}x^4 - 81x = 0.$$

$$\Rightarrow 3x^4 - 81x = 0.$$

Scanned with CamScanner

$\Rightarrow x(3x^3 - 81) = 0.$

$\Rightarrow 3x^3 = 81.$

$\Rightarrow x = 3.$

and, $y = \frac{1}{3}.x^2.$

$\Rightarrow \frac{1}{3}.(3)^2.$

$= 3.$

So, critical point $(3, 3)$.

~~Agai~~ $\Rightarrow r = \frac{\delta^2 g(x,y)}{\delta x^2} = 54x - 81.$

at, $(3,3)$ point, $r = 54 \times 3 - 81.$

$= 81.$

$\Rightarrow t = \frac{\delta^2 g(x,y)}{\delta y^2} = 54y - 81.$

at $(3,3)$ point, $t = 54 \times 3 - 84.$

$= 81.$

Scanned with CamScanner

and,

$$s = \frac{\delta^2 g(x,y)}{\delta x \delta y} = -81.$$

at, (3,3) point, $s = -81$.

$$\therefore rt - s^2 = 81 \times 81 - (-81)^2$$
$$= 0.$$

Here,

local minima is (3,3) but there is no global minima or maxima.

④ $f(x,y) = x\ln(xy)$. ; near the point $(e,1)$.

$\Rightarrow f(x,y) = x.\ln(xy)$.

$f_x(x,y) = x.\frac{1}{xy}.y + \ln(xy).1$.

$\Rightarrow f_x(x,y) = 1 + \ln(xy)$.

$f_y(x,y) = x.\frac{1}{xy}.x$.

$\Rightarrow f_y(x,y) = \frac{x}{y}$.

$f_{xx}(x,y) = \frac{1}{xy}.y$.

$\Rightarrow f_{xx}(x,y) = \frac{1}{x}$.

$\Rightarrow f_{yy}(x,y) = -\frac{x}{y^2}$.

$\Rightarrow f_{xy}(x,y) = \frac{1}{xy}.x$.

$\Rightarrow f_{xy}(x,y) = \frac{1}{y}$.

at point $(e,1)$.

$f(e,1) = e.\ln(e)$.

$f_x(e,1) = 1 + \ln e$.

$f_y(e,1) = e$.

$f_{xx}(e,1) = \frac{1}{e}$.

$f_{yy}(e,1) = -e$.

$f_{xy}(e,1) = 1$.

First Taylor Polynomial,

$$L(x,y) = f(x,y) + f_x(x,y)(x-e) + f_y(x,y)(y-1).$$

$$= f(e,1) + f_x(e,1)(x-e) + f_y(e,1)(y-1).$$

$$= e\ln e + (1+\ln e)(x-e) + e.(y-1).$$

Second Taylor Polynomial,

$$Q(x,y) = L(x,y) + \frac{f_{xx}(x,y)}{2!}(x-e)^2 + f_{xy}(x,y)(x-e)(y-1).$$

$$+ \frac{f_{yy}(x,y)}{2!}(y-1)^2.$$

$$= e\ln e + (1+\ln e)(x-e) + e(y-1) + \frac{f_{xx}(e,1)}{2}(x-e)^2$$

$$+ f_{xy}(e,1)(x-e)(y-1) + \frac{f_{yy}(e,1)}{2}(y-1)^2.$$

$$= e\ln e + (1+\ln e)(x-e) + e(y-1) + \frac{1}{2e}(x-e)^2 +$$

$$(x-e)(y-1) - \frac{1}{2}.e(y-1)^2.$$

(5) $\vec{V} = (3x - \sin y)\hat{i} - (z^5 e^{3x})\hat{j} + (z^3 - 5x)\hat{k}$

Now,

$$\vec{\nabla} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ (3x - \sin y) & (-z^5 . e^{3x}) & (z^3 - 5x) \end{vmatrix}.$$

$$= i\left\{\frac{\delta}{\delta y}(z^3 - 5x) - \frac{\delta}{\delta z}(-z^5 e^{3x})\right\} - \hat{j}\left\{\frac{\delta}{\delta x}(z^3 - 5x) - \frac{\delta}{\delta z}(3x - \sin y)\right\} + \hat{k}\left\{\frac{\delta}{\delta x}(z^5 e^{3x}) - \frac{\delta}{\delta y}(3x - \sin y)\right\}.$$

$$= \hat{i}\{0 + 5z^4 . e^{3x}\} - \hat{j}(-5 - 0) + \hat{k}(-z^5 . 3e^{3x} + \cos y).$$

$$= (5z^4 . e^{3x})\hat{i} + 5\hat{j} + (\cos y - 3z^5 . e^{3x})\hat{k}.$$

$$\nabla \times (\nabla \times \vec{V}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ 5z^4 e^{3x} & 5 & (\cos y - 3e^{3x} z^5) \end{vmatrix}$$

$$= i \left\{ \frac{\delta}{\delta y} (\cos y - 3e^{3x} z^5) - \frac{\delta}{\delta z} (5) \right\} - \hat{j} \left\{ \frac{\delta}{\delta x} (\cos y - 3e^{3x} z^5) - \frac{\delta}{\delta z} (5z^4 e^{3x}) \right\} + \hat{k} \left\{ \frac{\delta}{\delta x} (5) - \frac{\delta}{\delta y} (5z^4 . e^{3x}) \right\}.$$

$$= \hat{i} (-\sin y) - \hat{j} (-9e^{3x} z^5 - 20z^3 e^{3x}) + 0.$$

$$= -\sin y \, \hat{i} + \hat{j} (9e^{3x} . z^5 + 20z^3 . e^{3x}).$$

Scanned with CamScanner

(6). $-976 - 216x + 36x^2 - 200y - 25y^2 = 0$.

$\Rightarrow 36x^2 - 216x - (25y^2 + 200y) = 976$

$\Rightarrow 36(x^2 - 6x) - 25(y^2 + 8y) = 976$.

$\Rightarrow 36(x^2 - 2.x.3 + 9) - 25(y^2 + 2.y.4 + 16) = 976 + 324 - 400$

$\Rightarrow 36(x-3)^2 - 25(y+4)^2 = 900$.

$$\Rightarrow \frac{(x-3)^2}{25} - \frac{(y+4)^2}{36} = 1.$$

$$\Rightarrow \frac{(x-3)^2}{(5)^2} - \frac{(y+4)^2}{(6)^2} = 1 \; \ldots\ldots (1).$$

Compairing equ (1) with standard equ whose center is in (α, β) point, $\frac{(x-\alpha)^2}{a^2} - \frac{(y-\beta)^2}{b^2} = 1$. So, $\alpha = 3$, $\beta = -4$.

(i) center, $c(\alpha, \beta)$.

$c(3, -4)$.

(ii) Vertices, $(\pm a + \alpha, \beta)$.

$\Rightarrow (\pm 5 + 3, -4)$.

$\Rightarrow (8, -4), (-2, -4)$.

(iii) Eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}}$.

$$= \sqrt{1 + \frac{36}{25}}$$

$$= \frac{\sqrt{61}}{5}.$$

(iv) Equation of directrices, $\cancel{x = \pm \frac{a}{e}}$.

$$\Rightarrow x - \alpha = \pm \frac{a}{e}.$$

$$\Rightarrow x - 3 = \pm \frac{5}{\frac{\sqrt{61}}{5}}$$

$$\Rightarrow x - 3 = \pm \frac{25\sqrt{61}}{61}.$$

(v) Equation of asymptotes, $y - \beta = \pm\left(\frac{b}{a}\right)(x - \alpha)$.

$$\Rightarrow y + 4 = \pm\left(\frac{b}{a}\right)(x - 3).$$

$$\Rightarrow y + 4 = \pm \frac{6}{5}(x - 3).$$

(7). spherical coordinates $(\rho, \theta, \phi) \equiv (\pi, \frac{\pi}{4}, \frac{\pi}{2})$.

From the diagram we see that.

$$y = r\cos\theta.$$

$$z = r\sin\theta$$

$$x = \rho\cos\phi.$$

and, $r = \rho\sin\phi$.

So, $y = \rho\sin\phi . \cos\theta$.

$$\Rightarrow y = \pi . \sin\frac{\pi}{2} \cos\frac{\pi}{4}.$$

$$= 2.221.$$

$$z = \rho\sin\phi . \sin\theta.$$

$$\Rightarrow z = \pi . \sin\frac{\pi}{2} . \sin\frac{\pi}{4}.$$

$$= 2.221.$$

$$x = \rho\cos\phi.$$

$$= \pi . \cos\frac{\pi}{2}.$$

$$= 0.$$

Cartesian co-ordinates $(x, y, z) \equiv (0, 2.221, 2.221)$.

Now,

$$r = \sqrt{x^2+y^2}.$$

$$= \sqrt{0+(2.221)^2}$$

$$= 2.221.$$

$$\tan\theta = \frac{y}{x}.$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{2.221}{0}\right).$$

$$Z = z.$$

$$\Rightarrow Z = 2.221.$$

~~Cart~~

So, cylindrical co-ordinates are (r, θ, z).