Nonlinear Modeling

Kylie Ariel Bemis

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Nonlinear Models

All models are wrong, but some are useful.

- George Box

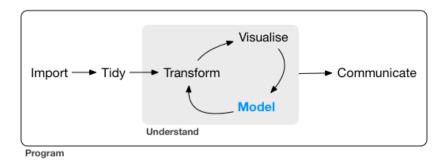


Figure 1: Wickham and Grolemund, R for Data Science

Overview of modeling goals

There are two general classes of analytic methods, designed for different purposes:

- Supervised learning
 - Prediction / classification
- Unsupervised learning
 - Clustering / class discovery

Statistical inference (including testing) can be a component of either supervised or unsupervised learning methods.

So far we have focused on linear regression, which is a type of supervised learning with a continuous response.

Supervised and unsupervised methods are discussed in greater detail in dedicated courses, but we will introduce them here.

Class comparison vs assignment vs discovery

Suppose we are interested in a categorical variable.

Then there are three general approaches we may wish to take:

- Class comparison
 - Use the categorical variable as an explanatory variable in linear regression
 - Use statistical tests to determine whether some continuous response depends on the level of the categorical variable
- Class assignment
 - Predict the level of a categorical response variable
 - ► Many supervised learning models are designed for classification
- Class discovery
 - ▶ The data is unlabeled, and you want to discover the class assignments

Let's focus on class assignment (classification) for now.

Suppose we want to predict delayed flights

```
library(nycflights13)
flights2 <- transmute(flights,</pre>
                      month = factor(month.
                                      levels=1:12).
                      dep_time = factor(dep_time %/% 100,
                                         levels=0:23).
                      arr_delay, dep_delay,
                      origin, dest, distance)
library(modelr)
set.seed(1) # remember to set seeds for reproducibility!
flights3 <- resample(flights2, sample(nrow(flights), 20000))</pre>
flights4 <- as_tibble(flights3) %>%
 mutate(is_delayed = arr_delay > 0,
         status = factor(ifelse(is_delayed, "Delayed", "On_Time"),
                         levels=c("On_Time", "Delayed")))
```

select(flights4, arr_delay:status, -origin, -dest)

```
## # A tibble: 20,000 x 5
##
      arr delay dep delay distance is delayed status
##
          <dbl>
                    <dbl>
                             <dbl> <lgl>
                                              <fct>
##
   1
             30
                        2
                              2565 TRUE
                                              Delayed
                       -5
                               319 FALSE
##
             -1
                                              On_Time
   3
             14
                        9
                              1096 TRUE
##
                                              Delayed
##
             5
                        4
                              2446 TRUE
                                              Delayed
##
             -7
                       -5
                               733 FALSE
                                              On_Time
##
            -31
                              2475 FALSE
                                              On_Time
##
   7
            -22
                       -9
                               479 FALSE
                                              On_Time
##
   8
             -2
                       -5
                               628 FALSE
                                              On_Time
##
             70
                       47
                              1069 TRUE
                                              Delayed
##
  10
             33
                       58
                               944 TRUE
                                              Delayed
## # ... with 19,990 more rows
```

Logistic regression and GLMs

Logistic regression is similar to linear regression, but for a categorical response. It is a type of *generalized linear model* (GLM):

$$g(y) = \beta_0 + \beta_1 x_1 + \dots + \beta_n x_n$$

A generalized linear model is parameterized similarly to a linear model, but the response is related to the linear predictor via a **link function** g(y).

In addition, the response variable may follow a different probability distribution than the normal distribution.

Logistic regression

Logistic regression is defined by a **logit** link function that maps a binary response variable to continuous values:

$$logit(p) = log(\frac{p}{1-p})$$

The response variable is expected to follow either a Bernouli distribution (response is binary), or a Binomial distribution (response is # of "success" occurences out of some total # of binary occurences).

Like linear regression, logistic regression can be used either for prediction or for statistical testing. However, due to the link function, the interpretation of model coefficients is different than for linear regression.

We will focus on prediction.

Logistic regression for flight delays

The glm() function is used to fit generalized linear models.

The model is specified using a formula in the same was as lm().

We specify logistic regression by providing the model family as binomial(link="logit").

Logistic regression for flight delays

summary(fit1)

```
##
## Call:
## glm(formula = status ~ month + dep_time + dep_delay + distance,
      family = binomial(link = "logit"), data = flights4)
##
##
## Deviance Residuals:
##
      Min
               10
                    Median
                                30
                                       Max
## -3.4502 -0.7186 -0.5373
                            0.4173
                                     2.5308
##
## Coefficients:
##
               Estimate Std. Error z value Pr(>|z|)
## (Intercept) 1.623e+00 1.066e+00 1.522 0.127888
## month2
             -4.008e-02 9.105e-02 -0.440 0.659824
## month3
             -2.946e-01 8.958e-02 -3.288 0.001008 **
## month4 5.878e-02 8.817e-02 0.667 0.504975
## month5
             -5.659e-01 9.346e-02 -6.055 1.4e-09 ***
## month6
             -2.200e-01
                         9.200e-02 -2.391 0.016799 *
             -3.111e-01 9.068e-02 -3.430 0.000602 ***
## month7
## month8
             -2.609e-01 8.760e-02 -2.978 0.002902 **
## month9
             -8.414e-01
                         9.712e-02
                                   -8.663 < 2e-16 ***
```

Predicted probabilities

Obtain predictions as a probabilty of status = "Delayed".

```
flights4 %>%
  add_predictions(fit1, type="response") %>%
  select(arr_delay, is_delayed:pred)
```

```
## # A tibble: 20,000 x 4
##
     arr_delay is_delayed status
                                pred
         <dbl> <lgl>
                     <fct> <dbl>
##
            30 TRUE
                         Delayed 0.402
## 1
## 2
            -1 FALSE
                         On_Time 0.243
## 3
            14 TRUE
                         Delayed 0.612
## 4
            5 TRUE
                         Delayed 0.411
## 5
         -7 FALSE
                         On_Time 0.206
## 6
        -31 FALSE
                         On Time 0.420
## 7
          -22 FALSE
                         On_Time 0.0724
## 8
           -2 FALSE
                         On Time 0.174
            70 TRUE
## 9
                         Delayed 0.985
## 10
            33 TRUE
                         Delayed 0.998
## # ... with 19,990 more rows
```

Prediction type

The add_predictions() function from the **modelr** package is actually just a wrapper for the predict() method that most modeling methods implement.

Many models may have predict values beyond just the desired response.

We can use the type argument in either predict() or add_predictions() to specify what kind of prediction is desired.

```
predict(fit1, flights4, type="response") %>% head()
```

```
## 1 2 3 4 5 6
## 0.4015101 0.2431121 0.6120191 0.4105554 0.2060036 0.4204553
```

For GLM's, we need type = "response" because we can obtain either the linear predictions (prior to transformation by the link function) or the response predictions (after transformation by the link function).

Obtaining predictions from models

Because predict() can be specialized for any type of model, the default help page (?predict) is not very useful for determining how to access different types of predictions.

Most modeling functions return an object of the same *class* as the name of the modeling function. The underlying predict() functions will be likewise named.

- ▶ lm() returns an object of type lm
 - predict.lm() is the underlying predict function
- glm() returns an object of type glm
 - predict.glm() is the underlying predict function

Therefore, use ?predict.glm to see how to obtain the correct type of predictions for glm models.

Predicting a binary response

Many classifiers will output a numeric value (such as a probability) that must be converted into a binary value. Obtaining predictions typically requires choosing some kind of cutoff that will decide the class assignments:

```
## # A tibble: 20,000 x 6
##
      arr_delay is_delayed status
                                   pred pred_status correct
                                    <dbl> <chr>
##
          <dbl> <lgl>
                           <fct>
                                                      <1g1>
             30 TRUE
                           Delayed 0.402 On_Time
                                                      FALSE
##
   1
             -1 FALSE
##
                           On_Time 0.243 On_Time
                                                      TRUE
   3
             14 TRUE
                           Delayed 0.612 Delayed
##
                                                      TRUE
##
              5 TRUE
                           Delayed 0.411 On Time
                                                      FALSE
##
   5
            -7 FALSE
                           On_Time 0.206 On_Time
                                                      TRUE
            -31 FALSE
                           On Time 0.420 On Time
                                                      TRUE
##
##
            -22 FALSE
                           On Time 0.0724 On Time
                                                      TRUE
##
             -2 FALSE
                           On_Time 0.174
                                          On_Time
                                                      TRUE
44
             TO TRITE
                           D-1---4 0 00E
                                          D - I - - - - J
                                                      TITITE
```

Predicting a binary response (cont'd)

Note that it is important to know which class is being predicted (i.e., which is level is considered a "success").

When the binary response is a factor, the first level is coded as 0 ("failure") and the second level is coded as 1 ("success"). The predicted probabilities are the probabilities of "success":

```
head(flights4$status)
```

```
## [1] Delayed On_Time Delayed Delayed On_Time On_Time
## Levels: On_Time Delayed
```

"Delayed" is the second level.

Therefore, we are predicting the probability that status = "Delayed".

Predicting a binary response (cont'd)

Varying the cutoff can change the accuracy:

```
flights4 %>%
  add_predictions(fit1, type="response") %>%
  mutate(pred4 = ifelse(pred > 0.4, "Delayed", "On_Time"),
        pred5 = ifelse(pred > 0.5, "Delayed", "On_Time"),
        pred6 = ifelse(pred > 0.6, "Delayed", "On_Time")) %>%
  summarize(acc4 = mean(status == pred4, na.rm=TRUE),
        acc5 = mean(status == pred5, na.rm=TRUE),
        acc6 = mean(status == pred6, na.rm=TRUE))
```

```
## # A tibble: 1 x 3
## acc4 acc5 acc6
## <dbl> <dbl> <dbl> <dbl> ## 1 0.792 0.797 0.795
```

Sensitivity vs specificity

Besides overall accuracy (% classified correctly), there are two types of accuracy that we can balance: **sensitivity** and **specificity**

- Sensitivity is the true positive rate
 - Proportion of correctly-identified positives among actual positives
 - ▶ If a flight will be delayed, how likely are we to classify it as delayed?
- Specificity is the true negative rate
 - Proportion of correctly-identified negatives among actual negatives
 - If a flight will not be delayed, how likely are we to classify it as not delayed?

These definitions depend on which class is considered the "positive" or "success" class.

It can be particularly important to pay attention to sensitivity and specificity when the response class sizes are unbalanced, as the overall accuracy can be misleading in such situations.

Sensitivity vs specificity for delayed flights

[1] 0.9344346

```
flights5 <- flights4 %>%
  add_predictions(fit1, type="response") %>%
  mutate(pred = ifelse(pred > 0.5,
                       "Predicted Delayed",
                       "Predicted On Time"))
table(flights5$status, flights5$pred)[,2:1] # "confusion matrix"
##
##
             Predicted On_Time Predicted Delayed
##
     On_Time
                         10917
                                              766
##
     Delayed
                          3197
                                             4606
4606 / (3197 + 4606) # sensitivity
## [1] 0.5902858
10917 / (10917 + 766) # specificity
```

Calculating sensitivity and specificity

[1] 0.9344346

```
yobs <- flights4$is delayed
ypred <- predict(fit1, flights4, type="response")</pre>
sens <- function(p) {</pre>
  mean((ypred > p)[yobs], na.rm=TRUE)
sens(0.5)
## [1] 0.5902858
spec <- function(p) {</pre>
  mean(!(ypred > p)[!yobs], na.rm=TRUE)
spec(0.5)
```

Receiving operator characteristic (ROC) curve

Changing the cutoff probability for class assignment can affect the sensitivity and specificity.

It can be useful to calculate and plot the tradeoff between sensitivity and specificity for different cutoffs.

This is traditionally visualized as an ROC curve, which plots the *true* positive rate (sensitivity) against the false positive rate (1 - specificity):

roc

```
## # A tibble: 101 x 3
##
         p sensitivity specificity
##
     <dbl>
                 <dbl>
                             <dbl>
##
   1
      0
                          0
##
   2 0.01
                          0
##
   3 0.02
                          0
##
   4 0.03
                          0.000257
##
   5 0.04
                          0.00103
##
   6 0.05
                 1.000
                          0.00282
                          0.00676
##
   7 0.06
                 1.000
##
   8 0.07
                 0.999
                         0.0135
##
   9 0.08
                 0.998
                       0.0254
## 10 0.09
                0.996
                          0.0417
## # ... with 91 more rows
```

Plotting the ROC

A strong classifier will have an area-under-the-curve (AUC) close to 1.

```
ggplot(roc, aes(x=1 - specificity, y=sensitivity)) + geom_line()
   1.00 -
   0.75 -
sensitivity
   0.50 -
   0.25 -
   0.00 -
                         0.25
                                                        0.75
                                        0.50
                                                                        1.00
         0.00
```

- specificity

Logistic regression with many predictors

Suppose we have a classification problem with "many" predictors:

```
N <- 1000
P <- 10
set.seed(2)
x <- matrix(rnorm(N * P), nrow=N, ncol=P)
colnames(x) <- paste0("x", 1:P)
y <- rbinom(N, 1, ifelse(x[,1] > 0, 0.6, 0.4))
data <- bind_cols(as_tibble(x), tibble(y=y))</pre>
```

Which predictors should we include in the model?

```
fit2 <- glm(y ~ ., data=data, family=binomial(link="logit"))
summary(fit2)</pre>
```

##

x2

x3

x4

x6

x8 ## x9

x10

x5

x7

Estimate Std. Error z value Pr(>|z|)

0.02853 0.06561 0.435 0.664

-0.07423 0.06395 -1.161 0.246

0.10222 0.06616 1.545 0.122

0.02463 0.06734 0.366 0.715

-0.03715 0.06700 -0.554 0.579

0.07379 0.06314 1.169

0.06704 0.575

0.06190 0.672

0.06713 -1.467

0.565

0.501

0.243

0.142

(Intercept) 0.10449 0.06557 1.594 0.111 ## x1 0.39305 0.06668 5.894 3.76e-09 ***

0.03855

0.04161

-0.09847

Sparse logistic regression

The glmnet package fits generalized linear models with an L1 and L2 penalty on the coefficients.

The L1 penalty (the "lasso") forces many of the coefficients to be 0, essentially removing them from the model.

```
library(glmnet)
fit3 <- glmnet(x, y, family="binomial")</pre>
```

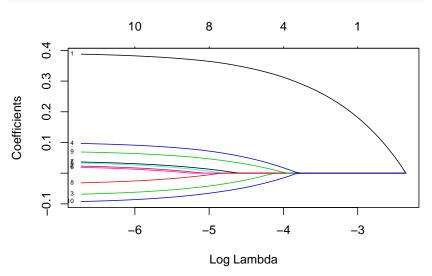
This is a type of "sparse" model.

A sparse model uses only a small subset of the input predictors (because most of the model parameters are forced to 0 by constraints).

Sparse coefficient estimates

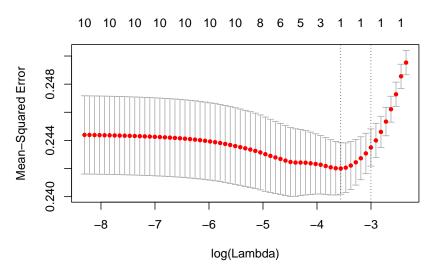
Larger values of the sparsity parameter λ force more coefficients to 0:

plot(fit3, xvar="lambda", label=TRUE)



Cross-validation is used to select lambda

```
fit4 <- cv.glmnet(x, y)
plot(fit4)</pre>
```



Sparse coefficients from selected model

```
coef(fit4, s="lambda.min")
```

```
## 11 x 1 sparse Matrix of class "dgCMatrix"
##
## (Intercept) 0.52291302
## x1
               0.06592032
## x2
## x3
## x4
## x5
## x6
## x7
## x8
## x9
## x10
```

Only x1 has a non-zero coefficient estimate.

Using glmnet for sparse models

Because glmnet implements sparse GLM's, it can also be used for sparse linear regression, and many families of sparse generalized linear models.

- ▶ Good for automated selection of important predictor variables
- ▶ Can be useful for predictive accuracy by removing noisy predictors
- ▶ Statistical testing for sparse models is a subject of research. . .
- ► Model specification is different from lm() or glm()
 - ▶ No model formula, instead use a matrix of predictors
 - Categorical variables (factors) must be manually converted to indicator variables using model.matrix()
- Predictor variables must be on the same units scale for the sparsity constraint to make any sense
 - Predictors are automatically standardized
 - Coefficients are returned to original scale
- ▶ Interpretation of the regression coefficients can be difficult

Modeling packages

One of the advantages of R is almost every conceivable machine learning or statistical model is implemented in some package.

For example:

- ▶ e1071 : support vector machines (SVMs)
- rpart : classification and regression trees
- ▶ igraph : network analysis
- nnet : neural networks and multinomial regression
- randomForest : random forests
- kernlab : kernel-based machine learning

etc.

One of the disadvantages of this is that many of these models are implemented with different function conventions and syntax.

Modeling packages (cont'd)

For example, here are a few different ways to obtain class probabilities from a classifier trained by different packages:

ınction	Package	Code
.da	MASS	<pre>predict(obj)</pre>
lm	stats	<pre>predict(obj, type = "response")</pre>
bm	gbm	<pre>predict(obj, type = "response", n.trees)</pre>
da	mda	<pre>predict(obj, type = "posterior")</pre>
part	rpart	<pre>predict(obj, type = "prob")</pre>
eka	RWeka	<pre>predict(obj, type = "probability")</pre>
ogitboost	LogitBoost	<pre>predict(obj, type = "raw", nIter)</pre>
amr.train	pamr	<pre>pamr.predict(obj, type = "posterior")</pre>

Figure 2: Modeling conventions

Supervised learning with caret

The caret package attempts to provide a consistent interface to 237 machine learning models from 30+ different R packages.

The basic strategy for training machine learning methods with caret is as follows:

Figure 3: https://topepo.github.io/caret

Supervised learning with caret (cont'd)

The caret package provides the following primary functions:

- createDataPartition() : partitions the data into train / test split, using stratified sampling to create balanced partitions
- preProcess() : pre-processes the data (centering, scaling, imputating missing data, etc.)
- trainControl() : controls various computational aspects of how the model is trained (type of cross-validation, etc.)
- train() : trains a model
- predict() : provides a consistent way of accessing predictions from any machine learning model supported by caret

Example: Sonar data

We will use the "Sonar" data from the mlbench package.

```
library(mlbench)
data(Sonar)
```

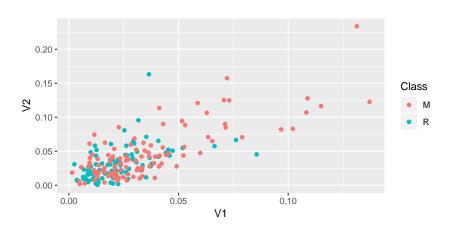
The dataset consists of 208 observations on 60 (continuous) explanatory variables and 1 (categorical) response variable.

The 60 explanatory variables represent energy from different frequencies of sonar signals.

The goal is to predict whether the sonar signals are being bounced off a metal cylinder ("M") or a cylindrical rock ("R").

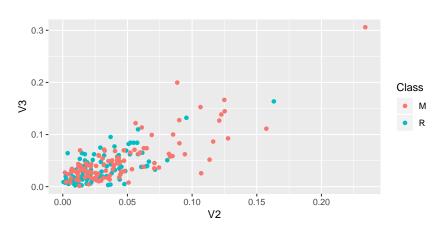
First two frequency channels

ggplot(Sonar, aes(x=V1, y=V2, color=Class)) + geom_point()



Second two frequency channels





Partition the data

First, we use createDataPartition() to partition the data into training and testing sets.

The training set will be used for cross-validation to select tuning parameters for the machine learning models.

Train logistic regression with caret

We use the train() function to train a logistic regression model.

```
## Warning: glm.fit: algorithm did not converge
```

```
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occ
```

Most machine learning models have some kind of tuning parameters, but logistic regression does not.

Therefore, we set the training control to method="none", because no cross-validation is required to select tuning parameters.

glmFit

```
## Generalized Linear Model
##
## 157 samples
## 60 predictor
## 2 classes: 'M', 'R'
##
## No pre-processing
## Resampling: None
```

Confusion matrix for logistic regression

confusionMatrix(predict(glmFit, Sonar_test), Sonar_test\$Class)

```
## Confusion Matrix and Statistics
##
##
             Reference
## Prediction M
##
           M 20 5
           R 7 19
##
##
##
                  Accuracy: 0.7647
                    95% CI: (0.6251, 0.8721)
##
       No Information Rate: 0.5294
##
       P-Value [Acc > NIR] : 0.0004667
##
##
##
                     Kappa : 0.53
##
    Mcnemar's Test P-Value: 0.7728300
##
##
               Sensitivity: 0.7407
               Specificity: 0.7917
##
           Pos Pred Value: 0.8000
##
            Neg Pred Value: 0.7308
##
                Prevalence: 0.5294
##
```

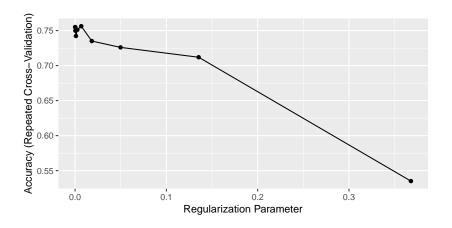
Train sparse logistic regression

We can use the same train() function to train a glmnet sparse logistic regression model.

We will use repeated 5-fold cross-validation to determine the sparsity parameter λ :

```
## glmnet
##
## 157 samples
##
   60 predictor
##
    2 classes: 'M', 'R'
##
## No pre-processing
## Resampling: Cross-Validated (5 fold, repeated 5 times)
## Summary of sample sizes: 125, 126, 125, 126, 126, 125, ...
## Resampling results across tuning parameters:
##
##
    lambda
                  Accuracy
                            Kappa
##
    4.539993e-05 0.7550054 0.5062840
##
    1.234098e-04 0.7549624 0.5064028
    3.354626e-04 0.7498817 0.4960950
##
##
    9.118820e-04 0.7422634 0.4813584
##
    2.478752e-03 0.7511801 0.4996081
##
    6.737947e-03 0.7562634 0.5105089
##
    1.831564e-02 0.7350538 0.4671029
    4.978707e-02 0.7260269 0.4481378
##
##
    1.353353e-01 0.7119516 0.4138395
                             0.0000000
##
    3.678794e-01 0.5350833
```

ggplot(glmnetFit)



Check the coefficients from the best model

coef(glmnetFit\$finalModel, s=glmnetFit\$bestTune\$lambda)

```
## 61 x 1 sparse Matrix of class "dgCMatrix"
##
## (Intercept) 7.0871054
               -25.1603144
## V1
## V2
## V3
                10.6081554
               -11.4336999
## V4
## V5
## V6
                 1.2019394
                 3.8717424
## V7
## V8
                10.0865614
## V9
                -7.3967575
## V10
## V11
                -2.1301003
## V12
                -8.7172197
## V13
                 1.4771280
                 0.4331108
## V14
## V15
## V16
                 0.3511447
## V17
                 1.5755718
```

Confusion matrix for sparse logistic regression

confusionMatrix(predict(glmnetFit, Sonar_test), Sonar_test\$Class)

```
## Confusion Matrix and Statistics
##
##
             Reference
## Prediction M
##
           M 20 5
           R 7 19
##
##
##
                  Accuracy: 0.7647
                    95% CI: (0.6251, 0.8721)
##
       No Information Rate: 0.5294
##
       P-Value [Acc > NIR] : 0.0004667
##
##
##
                     Kappa : 0.53
##
    Mcnemar's Test P-Value: 0.7728300
##
##
               Sensitivity: 0.7407
               Specificity: 0.7917
##
           Pos Pred Value: 0.8000
##
            Neg Pred Value: 0.7308
##
                Prevalence: 0.5294
##
```

Train single-hidden-layer neural network

We can use the same train() function to train a single-hidden layer neural network using the nnet package.

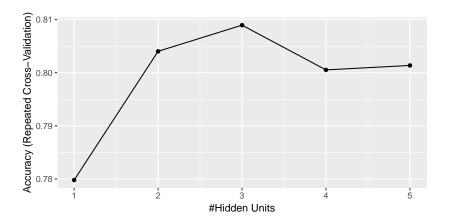
We will use repeated 5-fold cross-validation to determine the optimal number of units in the hidden layer:

nnetFit

Neural Network

```
##
## 157 samples
##
   60 predictor
##
    2 classes: 'M', 'R'
##
## No pre-processing
## Resampling: Cross-Validated (5 fold, repeated 5 times)
## Summary of sample sizes: 126, 127, 125, 125, 125, 125, ...
## Resampling results across tuning parameters:
##
##
    size Accuracy Kappa
##
          0.7798065 0.5516167
    1
##
    2 0.8040349 0.6032537
    3 0.8089543 0.6121657
##
##
    4 0.8005376 0.5965626
##
          0.8013710 0.5983530
##
## Tuning parameter 'decay' was held constant at a value of 0
## Accuracy was used to select the optimal model using the largest valu
## The final values used for the model were size = 3 and decay = 0.
```

ggplot(nnetFit)



Confusion matrix for neural network

confusionMatrix(predict(nnetFit, Sonar_test), Sonar_test\$Class)

```
## Confusion Matrix and Statistics
##
##
             Reference
## Prediction M R.
##
           M 22 6
           R 5 18
##
##
##
                  Accuracy: 0.7843
                    95% CI: (0.6468, 0.8871)
##
       No Information Rate · 0 5294
##
       P-Value [Acc > NIR] : 0.0001502
##
##
##
                     Kappa: 0.5661
##
    Mcnemar's Test P-Value : 1.0000000
##
##
               Sensitivity: 0.8148
               Specificity: 0.7500
##
           Pos Pred Value: 0.7857
##
            Neg Pred Value: 0.7826
##
##
                Prevalence: 0.5294
```

Other supervised learning packages: Keras

Another major machine learning package is the R interface to Keras.

The keras package maintained by R Studio is a high-level neural network API backed by TensorFlow, among other backends.

It provides a flexible interface for building and training many different types of neural networks.

The default installation is CPU-based, but the same models can also be trained on NVIDIA GPUs.

See https://keras.rstudio.com for more information.

Unsupervised learning

Suppose we want to discover classes within unlabeled data.

Consider the following dataset, giving measurements on different flowers.

```
head(as_tibble(iris))
```

```
## # A tibble: 6 x 5
     Sepal.Length Sepal.Width Petal.Length Petal.Width Species
##
##
            <dbl>
                        <dbl>
                                     <dbl>
                                                  <dbl> <fct>
## 1
              5.1
                          3.5
                                        1.4
                                                    0.2 setosa
              4.9
## 2
                          3
                                        1.4
                                                    0.2 setosa
## 3
              4.7
                          3.2
                                       1.3
                                                    0.2 setosa
              4.6
## 4
                          3.1
                                       1.5
                                                    0.2 setosa
                                       1.4
## 5
              5
                          3.6
                                                    0.2 setosa
## 6
              5.4
                          3.9
                                        1.7
                                                    0.4 setosa
```

```
iris2 <- select(iris, Sepal.Length:Petal.Width)</pre>
```

K-means clustering

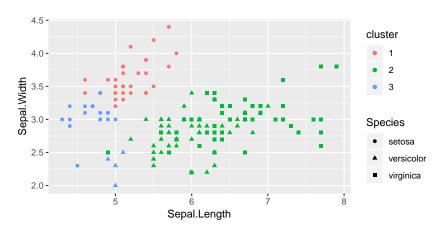
##

##

```
fit_kmeans <- kmeans(iris2, centers=3)</pre>
fit kmeans
## K-means clustering with 3 clusters of sizes 33, 96, 21
##
## Cluster means:
##
   Sepal.Length Sepal.Width Petal.Length Petal.Width
## 1
     5.175758 3.624242 1.472727 0.2727273
## 2 6.314583 2.895833 4.973958 1.7031250
## 3 4.738095 2.904762 1.790476 0.3523810
##
## Clustering vector:
##
   ##
  ## [141] 2 2 2 2 2 2 2 2 2 2 2
##
## Within cluster sum of squares by cluster:
     6.432121 118.651875 17.669524
## [1]
```

(between_SS / total_SS = 79.0 %)

K-means clustering (cont'd)



K-means clustering (cont'd)

