Problem 5 Naive Bayes.

(a) $$P(Y=1 \mid X_1=1, X_2=0, X_3=1) = \frac{P(X_1=1, X_2=0, X_3=1 \mid Y=1)\,P(Y=1)}{P(X_1=1,X_2=0,X_3=1 \mid Y=0)\,P(Y=0) + P(X_1=1,X_2=0,X_3=1 \mid Y=1)\,P(Y=1)}$$

Using Naive Bayes Algorithm,

$$\Rightarrow \frac{P(X_1=1/Y=1)\,P(X_2=0 \mid Y=1)\,P(X_3=1 \mid Y=1)\,P(Y=1)}{P(X_1=1 \mid Y=0)\,P(X_2=0 \mid Y=0)\,P(X_3=1 \mid Y=0)\,P(Y=0) + P(X_1=1 \mid Y=1)\,P(X_2=0 \mid Y=1)\,P(X_3=1 \mid Y=1)\,P(Y=1)}$$

$$\Rightarrow \frac{\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{\cancel{4}}\right)\left(\frac{\cancel{4}}{7}\right)}{\left(\frac{1}{3}\right)\left(\frac{2}{\cancel{3}}\right)(1)\left(\frac{\cancel{3}}{7}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{\cancel{4}}\right)\left(\frac{\cancel{4}}{7}\right)} = \frac{\frac{3}{112}}{\frac{6}{21} + \frac{3}{112}} = \frac{0.027}{\frac{735}{2352}} = \frac{0.0267}{0.3125} = \underline{0.0857}$$

$$P(Y=1 \mid X_1=1, X_2=1, X_3=1) = \frac{P(X_1=1, X_2=1, X_3=1 \mid Y=1)\,P(Y=1)}{P(X_1=1, X_2=1, X_3=1 \mid Y=0)\,P(Y=0) + P(X_1=1, X_2=1, X_3=1 \mid Y=1)\,P(Y=1)}$$

Using Naive Bayes theorem,

$$\Rightarrow \frac{P(X=1 \mid Y=1)\,P(X_2=1 \mid Y=1)\,P(X_3=1 \mid Y=1)\,P(Y=1)}{P(X_1=1 \mid Y=0)\,P(X_2=1 \mid Y=0)\,P(X_3=1 \mid Y=0)\,P(Y=0) + P(X=1 \mid Y=1)\,P(X_2=1 \mid Y=1)\,P(X_3=1 \mid Y=1)\,P(Y=1)}$$

$$\Rightarrow \frac{\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{\cancel{4}}\right)\left(\frac{\cancel{4}}{7}\right)}{\left(\frac{1}{3}\right)\left(\frac{1}{\cancel{3}}\right)\left(\frac{\cancel{3}}{7}\right)\cdot 1 + \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{\cancel{4}}\right)\left(\frac{\cancel{4}}{4}\right)} \Rightarrow \frac{\frac{3}{112}}{\frac{1}{21} + \frac{3}{112}} \Rightarrow \frac{0.0267}{0.075} = \underline{0.357}$$

total parameters = $\underline{7}$.

(b) $$P(Y=1 \mid X_1=1, X_2=0, X_3=1) = \frac{P(X_1=1, X_2=0, X_3=1 \mid Y=1)\,P(Y=1)}{P(X_1=1, X_2=0, X_3=1 \mid Y=0)\,P(Y=0) + P(X_1=1, X_2=0, X_3=1 \mid Y=1)\,P(Y=1)}$$

$$\Rightarrow \frac{0 \times \frac{4}{7}}{\frac{1}{3} \times \frac{3}{7} + 0} = 0.$$

$$P(Y=1 \mid X_1=1, X_2=1, X_3=1) = \frac{P(X_1=1, X_2=1, X_3=1 \mid Y=1)\,P(Y=1)}{P(X_1=1, X_2=1, X_3=1 \mid Y=0)\,P(Y=0) + P(X_1=1, X_2=1, X_3=1 \mid Y=0)\,P(Y=0)}$$

$$\Rightarrow \frac{0 \times \frac{4}{7}}{0 \times \frac{3}{7} + 0 \times \frac{4}{7}} = \infty$$

total parameters = $\underline{15}$.

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(a) parameter estimation:

no. of classes $(k) = 2$.

no. of categorical variables $(p) = 3$.

compute $P(Y=0)$, since $[P(Y=1) = 1 - P(Y=0)]$

compute $P(X_n=0|Y=y)$ $n = \{1,2,3\}$ and $y = \{0,1\}$

$[P(X_n=1|Y=y) = 1 - P(X_n=0|Y=y)]$

$\therefore$ total parameters $= P(X_n=0|Y=y) + P(Y=0)$

$= (k)(p) + (k-1)$

$= (2)(3) + 1$

$= \underline{\underline{7}}$

(b) Parameter Estimation:

no. of classes $= 2$ (k)

no. of categorical variables $(p) = 3$

$\Rightarrow k(2^p - 1) + (k-1)$

take all possible combinations of X_1, X_2 and X_3

$\Rightarrow 2(2^3 - 1) + (1)$

$\Rightarrow 2(8-1) + 1$

$\Rightarrow \underline{\underline{15}}$.

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Problem 6 Logistic Regression.

ⓐ Predicted probability $\Rightarrow \hat{P}(x) = \dfrac{e^{+\beta_0 + \beta_1 x_1 + x_2}}{\left(1+e^{\beta_0+\beta_1 x_1 + x_2}\right)}$

$$h(x) = \frac{e^{-z}}{1+e^{-z}}$$

$$z = (\theta^T x)$$

$$= \frac{e^{-6+0.05x_1 + x_2}}{1+e^{-6+0.05x_1+x_2}}$$

$$1 - P(x) = 1 - \frac{e^{-6+0.05x_1+x_2}}{1+e^{-6+0.05x_1+x_2}}$$

$$= \frac{1}{1+e^{-6+0.05x_1+x_2}}$$

$$P(x) = \frac{e^z}{1+e^z}$$

$$e^z(1-P(x)) = P(x).$$

$$\frac{P(x)}{1-P(x)} = e^z$$

$$\ln\left(\frac{P(x)}{1-P(x)}\right) = z$$

Let $x_1 = 40$hrs $x_2 = 3.5$ GPA.

$$\ln\left(\frac{P(x)}{1-P(x)}\right) = -6 + 0.05(40) + 3.5$$

$$\ln\left(\frac{P(x)}{1-P(x)}\right) = -0.5$$

$$\frac{P(x)}{1-P(x)} = e^{-0.5}$$

The probability that a student gets an A in class $\Big\}$ = $\underline{0.38}$

ⓑ

$$\ln\left(\frac{P(x)}{1-P(x)}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

$$0.5 \Rightarrow -6 + 0.05 * X_1 + 1 * 3.5$$

$$0.05x_1 = 3.5 - 0.5 - \cancel{6}\,6$$

$$X_1 = \frac{-3}{0.05}$$

$$\boxed{X_1 = 60 \text{ hours}}$$

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