SML2

October 9, 2019

ASSIGNMENT 2
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0.0.1 Problem 1 [Linear regression]

[63]:
```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np

import seaborn as sns

from sklearn.metrics import mean_squared_error
from sklearn import preprocessing

from sklearn.model_selection import train_test_split, cross_val_score
from sklearn.linear_model import LinearRegression
from sklearn.preprocessing import StandardScaler

from numpy.linalg import inv

from statsmodels.regression.linear_model import OLS

from sklearn import warnings

from scipy import stats
```

[64]:
```
# read in data
KChouse_train = pd.read_csv('train.csv')
KChouse_test = pd.read_csv('test.csv')

# ignore the columns id, date, unnamed column as well as the categorical column␣
↪zipcode.
KChouse_train = KChouse_train.drop(columns = ['zipcode','Unnamed: 0'])
KChouse_test = KChouse_test.drop(columns = ['id','zipcode','date','Unnamed: 0'])
```

[65]:
```
KChouse_train.head()
```

[65]:
```
      price  bedrooms  bathrooms  sqft_living  sqft_lot  floors  waterfront  \
0  221900.0         3       1.00         1180      5650     1.0           0
1  538000.0         3       2.25         2570      7242     2.0           0
2  180000.0         2       1.00          770     10000     1.0           0
3  604000.0         4       3.00         1960      5000     1.0           0
4  510000.0         3       2.00         1680      8080     1.0           0

   view  condition  grade  sqft_above  sqft_basement  yr_built  yr_renovated  \
0     0          3      7        1180              0      1955             0
1     0          3      7        2170            400      1951          1991
2     0          3      6         770              0      1933             0
3     0          5      7        1050            910      1965             0
4     0          3      8        1680              0      1987             0

       lat     long  sqft_living15  sqft_lot15
0  47.5112 -122.257           1340        5650
1  47.7210 -122.319           1690        7639
2  47.7379 -122.233           2720        8062
3  47.5208 -122.393           1360        5000
4  47.6168 -122.045           1800        7503
```

[66]:
```
# correlation coefficients of features/variables
KChouse_train.corr()['price'].sort_values(ascending=False)
```

[66]:
```
price            1.000000
sqft_living      0.704776
grade            0.647349
sqft_living15    0.645106
sqft_above       0.582407
bathrooms        0.487157
view             0.445316
sqft_basement    0.367365
lat              0.365770
waterfront       0.317143
bedrooms         0.307058
floors           0.239935
sqft_lot15       0.161746
sqft_lot         0.146645
yr_renovated     0.146348
condition        0.073961
long             0.032846
yr_built         0.016055
Name: price, dtype: float64
```

(a) Use an existing package to train a multiple linear regression model on the training set using all the features (except the ones excluded above). Report the coefficients of the linear regression models and the MSE metric on the training data

[67]:
```
# Linear regression model
X_train = KChouse_train.drop(['price'], axis=1)
y_train = KChouse_train[['price']]

model = LinearRegression()

scores = cross_val_score(model, X_train, y_train, cv=10)
print("Cross-validation scores: {}".format(scores))
print("Average cross-validation score: {:.2f}".format(scores.mean()))
# print("X_train.shape:",X_train.shape)
```

```
Cross-validation scores: [0.67837004 0.70416297 0.6125577  0.72692348 0.55953864
0.79355793
 0.71355718 0.7597062  0.67484746 0.72691946]
Average cross-validation score: 0.70
```

[68]:
```
model_fitted = model.fit(X_train, y_train)
```

[69]:
```
predictions = model_fitted.predict(X_train)
# predictions
```

[70]:
```
print("Mean squared error for train (unscaled) data:
↪",mean_squared_error(y_train,predictions))
```

```
Mean squared error for train data: 31486167775.794888
```

(b) Perform feature standardization so that each feature (including the response) has mean 0 and variance of 1. Train again a linear regression model on the training data and report the MSE on the training data.

Scaling data so that each feature has mean 0 and standard deviation 1.

[71]:
```
scaler = StandardScaler()
data_scaled = scaler.fit_transform(KChouse_train)
data_scaled_test = scaler.fit_transform(KChouse_test)
```

[72]:
```
data_scaled
```

[72]:
```
array([[-0.87974769, -0.40982347, -1.44988843, ..., -0.35519332,
        -0.96563661, -0.3128578 ],
       [ 0.05182493, -0.40982347,  0.28318404, ..., -0.7998304 ,
        -0.44332966, -0.23355563],
       [-1.00323042, -1.58410276, -1.44988843, ..., -0.18307573,
         1.09374507, -0.21669046],
       ...,
       [ 0.0975047 , -1.58410276, -1.44988843, ..., -0.86437449,
        -1.02532883, -0.4185143 ],
       [-0.97390696, -1.58410276, -1.44988843, ...,  1.07911986,
        -0.80148299, -0.40352304],
       [-0.68199849, -0.40982347, -0.06343045, ..., -0.46993837,
```

```
         0.39236146, -0.15736334]])
```

[73]:
```
df = pd.DataFrame(data=data_scaled[:,0:],
                  index=data_scaled[:,0],
                  columns=['price', 'bedrooms', 'bathrooms', 'sqft_living',
        'sqft_lot', 'floors', 'waterfront', 'view', 'condition', 'grade',
        'sqft_above', 'sqft_basement', 'yr_built', 'yr_renovated', 'lat',
        'long', 'sqft_living15', 'sqft_lot15'])
```

[74]:
```
df = df.reset_index(drop=True)
df.shape
```

[74]:
```
(1000, 18)
```

[75]:
```
X_train_scaled = df.drop(['price'], axis=1)
y_train_scaled = df[['price']]

model1 = LinearRegression()

scores = cross_val_score(model1, X_train_scaled, y_train_scaled, cv=10)
print("Cross-validation scores: {}".format(scores))
print("Average cross-validation score: {:.2f}".format(scores.mean()))
# X_train_scaled.columns
```

```
Cross-validation scores: [0.67837004 0.70416297 0.6125577  0.72692348 0.55953864
0.79355793
 0.71355718 0.7597062  0.67484746 0.72691946]
Average cross-validation score: 0.70
```

[76]:
```
model_fitted1 = model1.fit(X_train_scaled, y_train_scaled)
```

[77]:
```
predictions_scaled = model_fitted1.predict(X_train_scaled)
# predictions_scaled
```

[78]:
```
model_fitted1.coef_
```

[78]:
```
array([[-0.03690325,  0.05460245,  0.16724347,  0.03206976,  0.0237055 ,
         0.18785555,  0.14204967,  0.03820676,  0.27181372,  0.14231485,
         0.07997506, -0.19934981,  0.05090017,  0.23097972, -0.00305083,
         0.13432109, -0.03810604]])
```

[79]:
```
print("Mean squared error for scaled train data:
 ↪",mean_squared_error(y_train_scaled,predictions_scaled))
```

```
Mean squared error for scaled train data: 0.2734665681293983
```

(c) Evaluate both models on the testing set. Report the MSE on the testing set

Linear regression model on unscaled test data

[80]:
```
X_test = KChouse_test.drop(['price'], axis=1)
y_test = KChouse_test[['price']]

model2 = LinearRegression()

scores = cross_val_score(model2, X_test, y_test, cv=10)
print("Cross-validation scores: {}".format(scores))
print("Average cross-validation score: {:.2f}".format(scores.mean()))
```

```
Cross-validation scores: [0.572243   0.7302379  0.4195207  0.69421544 0.58056117
0.64406129
 0.58310599 0.67480098 0.69470867 0.5749148 ]
Average cross-validation score: 0.62
```

[81]:
```
model_fitted2 = model2.fit(X_test, y_test)
```

[82]:
```
predictions2 = model_fitted2.predict(X_test)
# predictions
```

[83]:
```
model_fitted2.coef_
```

[83]:
```
array([[-5.09706680e+04,  4.53522145e+04,  1.29557364e+02,
         4.60107448e-01, -4.08290867e+03,  6.67916885e+05,
         6.41825495e+04,  3.74395531e+04,  8.69134817e+04,
         7.58952045e+01,  5.36621579e+01, -2.59297980e+03,
        -5.13466279e+00,  5.67370348e+05, -7.67745545e+04,
         3.41095820e+01, -8.48109171e-01]])
```

[84]:
```
print("Mean squared error for non scaled test data:
 ↪",mean_squared_error(y_test,predictions2))
```

```
Mean squared error for non scaled test data: 54185036855.79511
```

Linear regression on scaled test data

[85]:
```
df_scaled = pd.DataFrame(data=data_scaled_test[1:,0:],
                  index=data_scaled_test[1:,0],
                  columns=['price', 'bedrooms', 'bathrooms', 'sqft_living',
       'sqft_lot', 'floors', 'waterfront', 'view', 'condition', 'grade',
       'sqft_above', 'sqft_basement', 'yr_built', 'yr_renovated', 'lat',
       'long', 'sqft_living15', 'sqft_lot15'])

df_scaled = df_scaled.reset_index(drop=True)
```

[86]:
```
X_test_scaled = df_scaled.drop(['price'], axis=1)
y_test_scaled = df_scaled[['price']]

model3 = LinearRegression()

scores = cross_val_score(model3, X_test_scaled, y_test_scaled, cv=10)
```

```
print("Cross-validation scores: {}".format(scores))
print("Average cross-validation score: {:.2f}".format(scores.mean()))
```

```
Cross-validation scores: [0.61750399 0.72177335 0.41520807 0.69590594 0.58111104
0.63899562
 0.58430498 0.67685041 0.6919166  0.57445912]
Average cross-validation score: 0.62
```

[87]:
```
model_fitted3 = model3.fit(X_test_scaled, y_test_scaled)
```

[88]:
```
predictions_scaled1 = model_fitted3.predict(X_test_scaled)
# predictions_scaled1
```

[89]:
```
model_fitted3.coef_
```

[89]:
```
array([[-0.11656073,  0.08800974,  0.23817802,  0.0661224 , -0.00525078,
         0.1852539 ,  0.12709521,  0.06128675,  0.25616884,  0.21405969,
         0.08813423, -0.17705287, -0.00531226,  0.19009029, -0.02545619,
         0.05815344, -0.0584464 ]])
```

[90]:
```
print("Mean squared error for scaled test data:
↪",mean_squared_error(y_test_scaled,predictions_scaled1))
```

```
Mean squared errorfor scaled test data: 0.3253062062957616
```

(d) Interpret the results in your own words. Which features contribute mostly to the linear regression model? Is the model fitting the data well? How large is the model error?

- Grade feature contributes mostly to the linear regression model. It has the highest coefficient value of 0.25616884.
- We din generalize the model well by taking all the features of the data into modelling.
- The model is fitting the data well and it is evident from the r-sqaured value calculated below as the difference between r squared calculated for training and testing has no much difference.
- The error is not large and it is appropriate for the data used and the model as well.

[91]:
```
from sklearn.metrics import r2_score
r2_test = r2_score(y_test_scaled, predictions_scaled1)
print("Rsquared error on testing data:",r2_test)

r2_train = r2_score(y_train_scaled,predictions_scaled)
print("Rsquared error on training data:",r2_train)
```

```
Rsquared error on testing data: 0.6749701848743102
Rsquared error on training data: 0.7265334318706018
```

0.0.2 Problem 2 [Closed-form solution for linear regression]

(a) Implement simple linear regression using the closed form and train a model for one feature (sqft living) using the training set. Write code to predict a response for a new single-dimensional data point in the testing set.

Simple linear regression with one feature(sqft living)

[92]:
```
# obtain the feature matrix
KChouse_train1 = df.drop(columns = ['price'])
KChouse_test1 = df_scaled.drop(columns = ['price'])

X = KChouse_train1[['sqft_living15']]
X['ones'] = 1
X_test = KChouse_test1[['sqft_living15']]
X_test['ones'] = 1

# obtain the target variable
y = df[['price']]
y_test = df_scaled[['price']]

# calculate coefficients using closed-form solution
coeffs_CF = inv(X.transpose().dot(X)).dot(X.transpose()).dot(y)

y_prediction = X.dot(coeffs_CF)
y_test_prediction = X_test.dot(coeffs_CF)
```

```
C:\Users\mouni\Anaconda3\lib\site-packages\ipykernel_launcher.py:6:
SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas-
docs/stable/indexing.html#indexing-view-versus-copy

C:\Users\mouni\Anaconda3\lib\site-packages\ipykernel_launcher.py:8:
SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas-
docs/stable/indexing.html#indexing-view-versus-copy
```

[93]:
```
# extract the feature names into list

feature_names = list(X.columns)
feature_names

# convert list to array

features = np.asarray(feature_names)
features
```

```
# convert both the array vectors into dataframe
res = pd.DataFrame(coeffs_CF)

res1 = pd.DataFrame(features)

# merge dataframes
results = pd.merge(res1, res, left_index=True, right_index=True)
results = results.rename(columns={'0_x':'Features','0_y':'coeffs_CF'})

results
```

[93]:

```
        Features      coeffs_CF
0  sqft_living15  6.451060e-01
1           ones  6.288373e-17
```

[94]:

```
y_prediction.head()
```

[94]:

```
          0
0 -0.622938
1 -0.285995
2  0.705582
3 -0.603684
4 -0.180098
```

[95]:

```
y_test_prediction.head()
```

[95]:

```
          0
0 -0.355080
1 -0.141438
2 -0.057839
3 -0.292845
4 -0.596589
```

[96]:

```
print("Mean squared error for training data (simple Linear regression):
 ↪",mean_squared_error(y,y_prediction))
print("Mean squared error for testing data (simple Linear regression):
 ↪",mean_squared_error(y_test,y_test_prediction))
```

```
Mean squared error for training data: 0.5838382382385958
Mean squared error for testing data: 0.6764263056279269
```

(b) Implement the closed-from solution for multiple linear regression using matrix operations and train a model on the training set. Write code to predict a response for a new multi-dimensional data point in the testing set.

Multiple linear regression with all features

[97]:

```
X1 = KChouse_train1
X1['ones'] = 1

X_test1 = KChouse_test1
```

```
X_test1['ones'] = 1

# obtain the target variable
y1 = df[['price']]
y_test1 = df_scaled[['price']]

# calculate coefficients using closed-form solution
coeffs_CF_1 = inv(X1.transpose().dot(X1)).dot(X1.transpose()).dot(y1)

y_prediction1 = X1.dot(coeffs_CF_1)
y_test_prediction1 = X_test1.dot(coeffs_CF_1)
```

[98]:
```
# extract the feature names into list

feature_names1 = list(X1.columns)
# print(feature_names1)

# convert list to array

features1 = np.asarray(feature_names1)
# print(features1)

# convert both the array vectors into dataframe
res2 = pd.DataFrame(coeffs_CF_1)

res3 = pd.DataFrame(features1)

# merge dataframes
results1 = pd.merge(res2, res3, left_index=True, right_index=True)
results1 = results1.rename(columns={'0_x':'Features','0_y':'coeffs_CF'})

results1
```

[98]:
```
        Features      coeffs_CF
0   1.819630e-02       bedrooms
1   2.082002e-01      bathrooms
2  -5.465047e-01    sqft_living
3   1.959751e-02       sqft_lot
4   1.108606e-02         floors
5   1.888729e-01     waterfront
6   1.634773e-01           view
7   4.165114e-02      condition
8   2.387074e-01          grade
9   8.340260e-01     sqft_above
10  4.157826e-01  sqft_basement
11 -1.993498e-01       yr_built
12  5.090017e-02   yr_renovated
13  2.309797e-01            lat
```

```
14 -3.050828e-03           long
15  1.343211e-01  sqft_living15
16 -3.810604e-02     sqft_lot15
17  3.165437e-15           ones
```

[99]:
```python
y_prediction1.head()
```

[99]:
```
          0
0 -0.979190
1  0.658167
2 -0.586131
3 -0.085220
4 -0.232246
```

[100]:
```python
y_test_prediction1.head()
```

[100]:
```
          0
0 -0.766049
1 -0.971737
2  0.011118
3 -1.221909
4 -0.026984
```

(c) Compare the models given by your implementation with those trained in Problem 1 by the R or Python packages. Report the MSE metrics for the models you implemented on both training and testing sets

- The package generated linear regression model has appropriate mean squared error value just right for the data used.
- The implemented model's mean squared error values a bit high than package model, but the difference is quite neglegible.

MSE for Python package used models

[101]:
```python
print("Mean squared error for scaled train data:
 ↪",mean_squared_error(y_train_scaled,predictions_scaled))
print("Mean squared errorfor scaled test data:
 ↪",mean_squared_error(y_test_scaled,predictions_scaled1))
```

```
Mean squared error for scaled train data: 0.2734665681293983
Mean squared errorfor scaled test data: 0.3253062062957616
```

MSE for implemented model

[102]:
```python
print("Mean squared error for training data:
 ↪",mean_squared_error(y1,y_prediction1))
print("Mean squared error for testing data:
 ↪",mean_squared_error(y_test1,y_test_prediction1))
```

```
Mean squared error for training data: 0.31194738550839385
Mean squared error for testing data: 0.37329326533579754
```

0.0.3 Problem 3 [Gradient descent]

[103]:
```
X2 = KChouse_train1
X2['ones'] = 1

X_test2 = KChouse_test1
X_test2['ones'] = 1

# obtain the target variable
y2 = df[['price']]
y_test2 = df_scaled[['price']]
```

(a) Write code for gradient descent for training linear regression using the algorithm from class

[104]:
```
def gradient_descent1(X,y,theta,alpha,n):

    m = len(y)

    for i in range(n):
        prediction = np.dot(X,theta)

        theta = theta - (1/m) * alpha * (X.T.dot(prediction - y))

    return theta
```

(b) Vary the value of the learning rate (3 different values) and report the value of after different number of iterations (10, 50, and 100). Include the MSE metric on the training and testing set for all values of and number of iterations.

[105]:
```
# np.random.seed(2)
alpha = 0.01
iter = [10,50,100]
cols = KChouse_train1.columns
theta = np.random.randn(len(cols),1)
theta_df = theta
for i in iter:
    theta_j = gradient_descent1(X2,y2,theta_df,alpha,i)
    y_test_pred_gd = X_test2.dot(theta_j)
    print("mean squared error on scaled test data (GD):
 ↪",mean_squared_error(y_test2,y_test_pred_gd))
```

```
mean squared error on scaled test data: 17.464163451221626
mean squared error on scaled test data: 6.97958920199715
mean squared error on scaled test data: 3.445182931877422
```

[106]:
```
alpha = 0.1
iter = [10,50,100]
```

```
cols = KChouse_train1.columns
theta = np.random.randn(len(cols),1)
theta_df = theta
for i in iter:
    theta_j = gradient_descent1(X2,y2,theta_df,alpha,i)
    y_test_pred_gd = X_test2.dot(theta_j)
    print("mean squared error on scaled test data (GD):
 ↪",mean_squared_error(y_test2,y_test_pred_gd))
```

```
mean squared error on scaled test data: 1.413135824269324
mean squared error on scaled test data: 0.361498220452008
mean squared error on scaled test data: 0.34150739669043473
```

[107]:
```
alpha = 0.2
iter = [10,50,100]
cols = KChouse_train1.columns
theta = np.random.randn(len(cols),1)
theta_df = theta
for i in iter:
    theta_j = gradient_descent1(X2,y2,theta_df,alpha,i)
    y_test_pred_gd = X_test2.dot(theta_j)
    print("mean squared error on scaled test data (GD):
 ↪",mean_squared_error(y_test2,y_test_pred_gd))
```

```
mean squared error on scaled test data: 1.5245356316084024
mean squared error on scaled test data: 0.3824370082195767
mean squared error on scaled test data: 0.3449699649933864
```

[115]:
```
alpha = 0.1
iter = [10,50,100,200,300,400,500,600,700,800,900,1000]
cols = KChouse_train1.columns
theta = np.random.randn(len(cols),1)
theta_df = theta
for i in iter:
    theta_j = gradient_descent1(X2,y2,theta_df,alpha,i)
    y_test_pred_gd = X_test2.dot(theta_j)
    print("mean squared error on scaled test data (GD):
 ↪",mean_squared_error(y_test2,y_test_pred_gd))
```

```
mean squared error on scaled test data: 1.8160006493541812
mean squared error on scaled test data: 0.40963304455213345
mean squared error on scaled test data: 0.3573510297650286
mean squared error on scaled test data: 0.34246820956709867
mean squared error on scaled test data: 0.3409119071387248
mean squared error on scaled test data: 0.3406447292674675
mean squared error on scaled test data: 0.3405909841654776
```

```
mean squared error on scaled test data: 0.34057982865194064
mean squared error on scaled test data: 0.34057750551290006
mean squared error on scaled test data: 0.34057702248141386
mean squared error on scaled test data: 0.34057692224133757
mean squared error on scaled test data: 0.3405769014708617
```

0.0.4 Problem 4

(a) Write the derivation of the closed form solution for parameter that minimizes the loss function J() in ridge regression.

- In paper format at the end of the pdf

(b) Modify your linear regression implementation from Problem 2 to handle ridge regression. Take several values of the regularization parameter and output the MSE metric. Plot the value of MSE as a function of . What is the best value of that you found? Compare the results of linear regression and ridge regression on the dataset.

[116]:
```
I = np.identity(18)
lamda = []
MSE = []

# derivation of the closed form solution for parameter  that minimizes the loss␣
↪function J() in ridge regression

for lamda_i in range(0,10000,20):
    lamda.append(lamda_i)
    coeffs_CF_2 = inv(X1.transpose().dot(X1) + lamda_i * I).dot(X1.transpose()).
↪dot(y1)
    # Y prediction values for training data
    y_prediction1_rg = X1.dot(coeffs_CF_2)
    # y prediction values for testing data
    y_test_prediction1_rg = X_test1.dot(coeffs_CF_2)

    mean_error = mean_squared_error(y1,y_prediction1_rg)
    mean_error_test = mean_squared_error(y_test1,y_test_prediction1_rg)

    print("Mean squared error for training data (ridge):
↪",mean_squared_error(y1,y_prediction1_rg))
    print("Mean squared error for testing data (ridge):
↪",mean_squared_error(y_test1,y_test_prediction1_rg))
    print("\n")
    MSE.append(mean_error_test)
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Mean squared error for training data (ridge): 0.31194738550839385
Mean squared error for testing data (ridge): 0.37329326533579754
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Mean squared error for training data (ridge): 0.27357928858440955
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Mean squared error for testing data (ridge): 0.6982647832568355


Mean squared error for training data (ridge): 0.6801497140067727
Mean squared error for testing data (ridge): 0.6986164886574214
```

[110]:
```python
# convert list to array

Lamda = np.asarray(lamda)
MSE_df = np.asarray(MSE)
# print(features1)

# convert both the array vectors into dataframe
res_1 = pd.DataFrame(Lamda)

res_2 = pd.DataFrame(MSE_df)

# merge dataframes
result_df = pd.merge(res_1, res_2, left_index=True, right_index=True)
result_df = result_df.rename(columns={'0_x':'Lamda','0_y':'MSE'})

result_df.head()
```

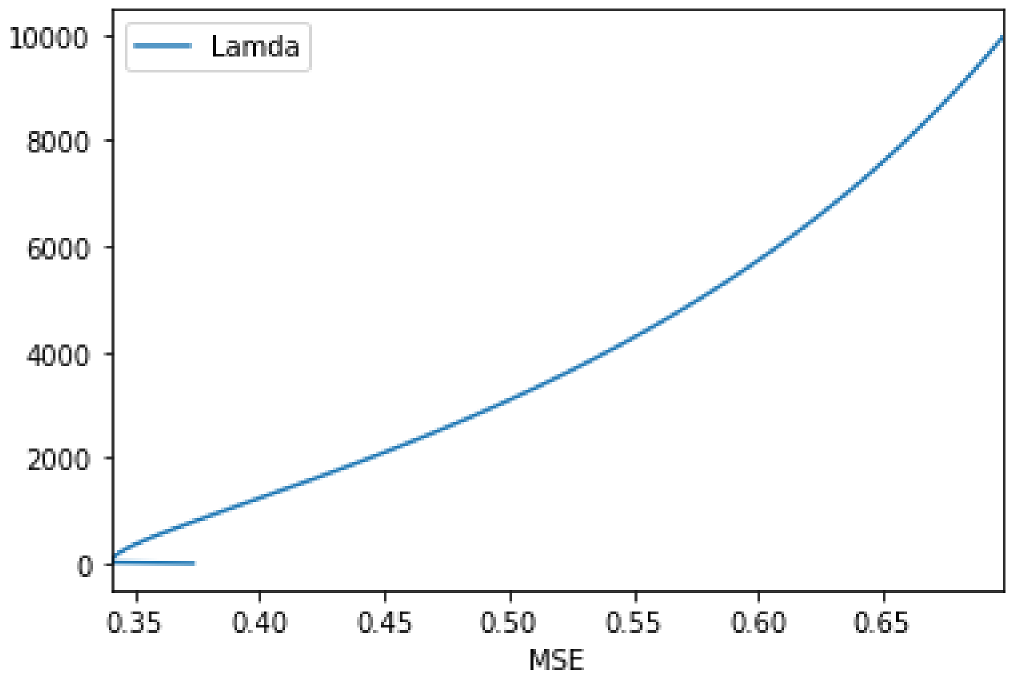
[110]:

	Lamda	MSE
0	0	0.373293

```
1      20   0.340227
2      40   0.340280
3      60   0.340450
4      80   0.340715
```

[111]:
```
result_df.plot.line(x='MSE', y='Lamda')
```

[111]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x291fc58b978>
```



[117]:
```
print("Mean squared error for scaled train data:
 ↪",mean_squared_error(y_train_scaled,predictions_scaled))
print("Mean squared errorfor scaled test data:
 ↪",mean_squared_error(y_test_scaled,predictions_scaled1))
```

```
Mean squared error for scaled train data: 0.2734665681293983
Mean squared errorfor scaled test data: 0.3253062062957616
```

- The best value for lambda is 1, below are the MSE values for lamba = 1.

Ridge regression MSE

- Mean squared error for training data (ridge): 0.27357928858440955
- Mean squared error for testing data (ridge): 0.3402268155372297

package based linear regression MSE

- Mean squared error for scaled train data: 0.2734665681293983
- Mean squared errorfor scaled test data: 0.3253062062957616

There is no much difference in MSE values of rigde and linear regression.

0.0.5 Problem 5 [Dependent features in linear regression]

- In paper format at the end of the pdf

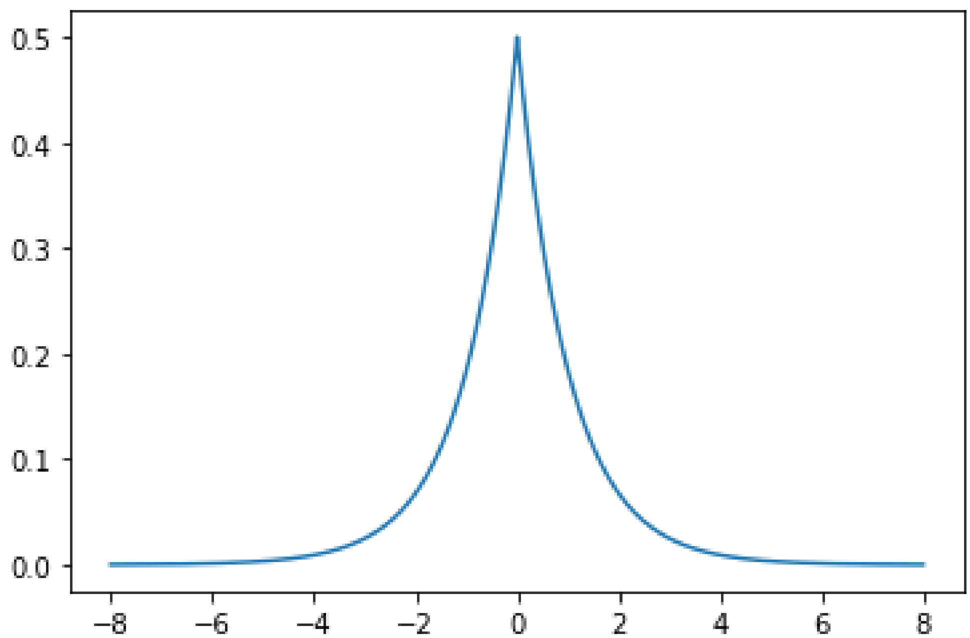
0.0.6 Problem 6 [Laplace noise in MLE estimate]

(a) Plot the Laplace pdf for b {1, 2, 4}. How is the Laplace distribution different from the normal distribution?

[112]:
```
loc, scale = 0., 1.
x_lap = np.random.laplace(loc, scale, 1000)

x = np.arange(-8., 8., .01)
pdf = np.exp(-abs(x-loc)/scale)/(2.*scale)
plt.plot(x, pdf)
```

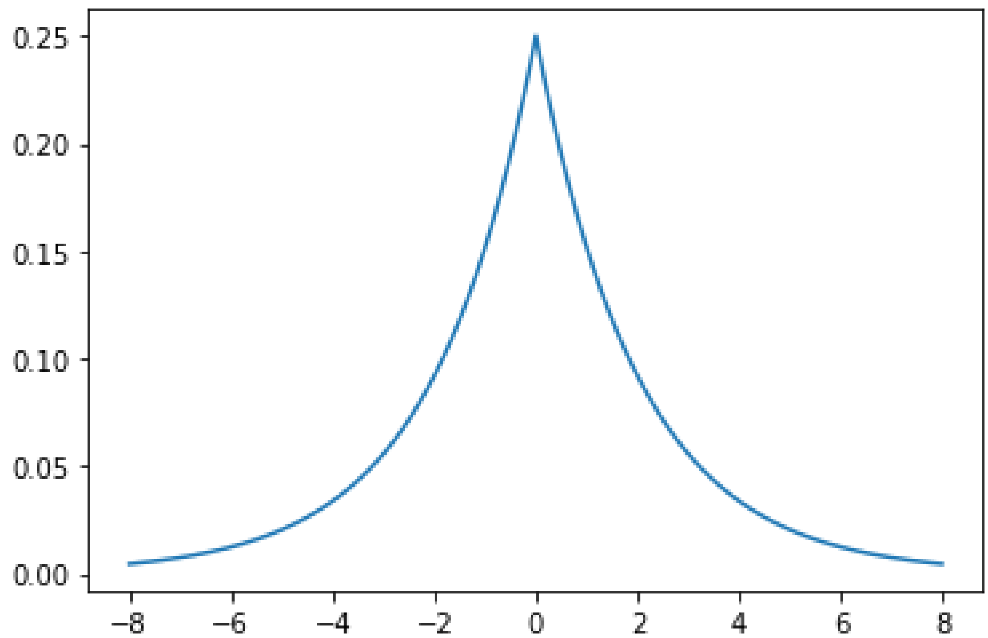
[112]:
```
[<matplotlib.lines.Line2D at 0x291fdad32e8>]
```



[113]:
```
loc, scale = 0., 2.
x_lap = np.random.laplace(loc, scale, 1000)

x = np.arange(-8., 8., .01)
pdf = np.exp(-abs(x-loc)/scale)/(2.*scale)
plt.plot(x, pdf)
```

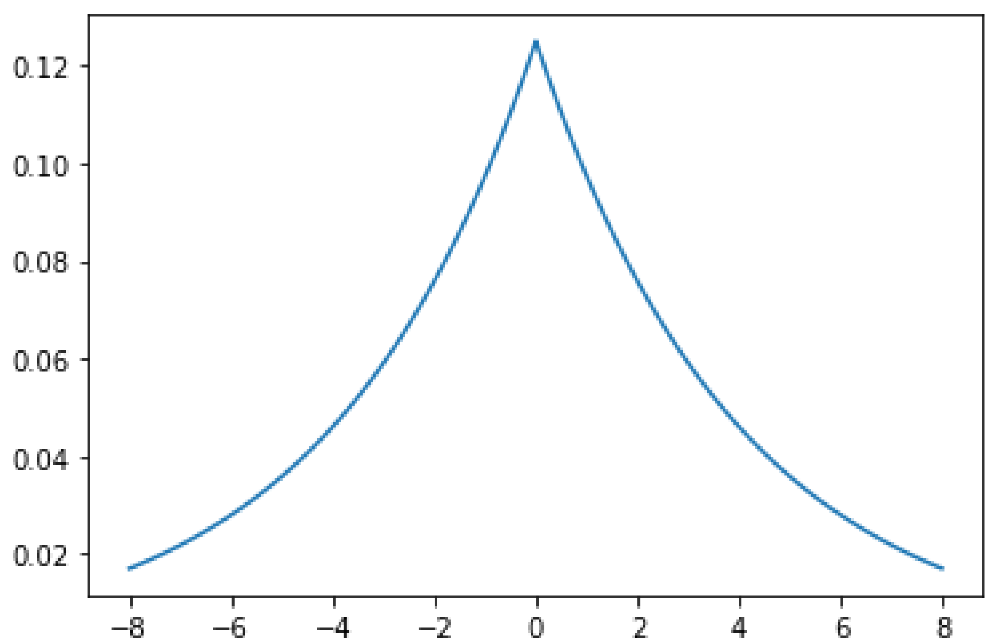
[113]:
```
[<matplotlib.lines.Line2D at 0x291fdb31588>]
```



[114]:
```
loc, scale = 0., 4.
x_lap = np.random.laplace(loc, scale, 1000)

x = np.arange(-8., 8., .01)
pdf = np.exp(-abs(x-loc)/scale)/(2.*scale)
plt.plot(x, pdf)
```

[114]: [<matplotlib.lines.Line2D at 0x291fdb908d0>]



(b) Derive the objective J() that maximizes the MLE for Laplace noise. The objective should depend on , the training dataset (xi, yi), for i {1, . . . , N}, and b.

- In paper format at the end of the pdf

(c) Consider an outlier in the training data, defined as a point of high residual. Which of the two objectives (derived for normal or Laplace noise) are more resilient to the effect of outliers?

- The pdf of laplace distribution is (1/2b)*eˆ(-x/b) and the MLE is 1.
- The pdf of normal distribution is (1/sqrt(2pi))*e$^{(x}$2/b) and the MLE is 2x.
- The normal distribution is more vulnerable to outliers.
- Therefore, laplace noise is more resilient to the effect of outliers.

4a) Derivation of the closed form solution for parameter θ that minimizes the loss function $J(\theta)$ in ridge regression.

$$J(\theta) = \frac{1}{2}\sum_{i=1}^{N}\left(h_\theta(x_i) - y_i\right)^2 + \frac{\lambda}{2}\sum_{j=1}^{d}\theta_j^2$$

The term d in regularization part denotes amount of covariables used in the model.

we can rewrite $J(\theta)$ in matrix-notation and further break it down.

$$J(\theta) = \frac{1}{2}(X\theta - y)^T(X\theta - y) + \frac{\lambda}{2}(\theta^T\theta)$$

the term $\frac{\lambda}{2}\sum_{j=1}^{d}\theta_j^2$ represents regularization we apply on coefficients.

$$\Rightarrow \frac{1}{2}\left[X^T\theta^T X\theta - X^T\theta^T y - X\theta y^T + y^T y + \lambda\theta^T\theta\right]$$

$\left[(X\theta)^T y = (X\theta)y^T\right]$ the a transposed scalar is same scalar.

$$\Rightarrow \frac{1}{2}\left[X^T\theta^T X\theta - 2X^T\theta^T y + y^T y + \theta^T\lambda I\,\theta\right]$$

$[I = \text{Identity matrix}]$

$$\Rightarrow \frac{1}{2}\left[y^T y - 2X^T\theta^T y + \theta^T(X^T X + \lambda I)\theta\right]$$

θ should minimize $J(\theta)$.

By matrix differentiation rule $\frac{\partial x^T A x}{\partial x} = (A + A^T)x = 2Ax$

we can apply it in here as

$$\frac{\partial J}{\partial\theta} \Rightarrow \frac{1}{2}\left[0 - 2X^T y + 2(X^T X + \lambda I)\theta\right] = 0.$$

$$\Rightarrow -X^T y + (X^T X + \lambda I)\theta = 0$$

$$\Rightarrow (X^T X + \lambda I)\theta = X^T y.$$

$\boxed{\theta = (X^T X + \lambda I)^{-1} + X^T y.}$ is a closed form eq.

5) $X = \begin{bmatrix} 1 & x_{i1} & x_{j1} \\ 1 & x_{i2} & x_{j2} \\ 1 & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ 1 & x_{in} & x_{jn} \end{bmatrix}_{n\times 3}$

$X^T = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_{i1} & x_{i2} & \cdots & x_{in} \\ x_{j1} & x_{j2} & \cdots & x_{jn} \end{bmatrix}_{3\times n}.$

$x_{j1} = 2x_{i1}$

So $X = \begin{bmatrix} 1 & x_{i1} & 2x_{i1} \\ 1 & x_{i2} & 2x_{i2} \\ \vdots & \vdots & \vdots \\ 1 & x_{in} & 2x_{in} \end{bmatrix}_{n \times 3}$

$X^T = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_{i1} & x_{i2} & \cdots & x_{in} \\ 2x_{i1} & 2x_{i2} & \cdots & 2x_{in} \end{bmatrix}_{3 \times n}$.

$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_{i1} & x_{i2} & \cdots & x_{in} \\ 2x_{i1} & 2x_{i2} & \cdots & 2x_{in} \end{bmatrix} \begin{bmatrix} 1 & x_{i1} & 2x_{i1} \\ 1 & x_{i2} & 2x_{i2} \\ \vdots & \vdots & \vdots \\ 1 & x_{in} & 2x_{in} \end{bmatrix}$

$X^T X = \begin{bmatrix} 1 & \sum_{i=1}^{n} x_i & 2\sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & 2\sum_{i=1}^{n} x_i^2 \\ 2\sum_{i=1}^{n} x_i & 2\sum_{i=1}^{n} x_i^2 & 4\sum_{i=1}^{n} x_i^2 \end{bmatrix}$ Row 1 and Row 2 are linearly dependent

Rank $(X^T X) = 1$ (using row echelon form).

The columns of $X^T X$ are linearly dependent. col 1 $\times \sum_{i=1}^{n} x_i =$ col 2 and etc.

As per invertible matrix theorem linearly dependent matrix do not have inverses.

So, $X^T X$ is non-invertible.

6 b) Laplace PDF

$$P(x) = \frac{e^{-|x|/b}}{2b}$$

hypothesis be,

$$y_i = h_\theta(x_i) + \epsilon_i$$

ϵ_i - random noise generated from laplace distribution.

$$f(y_i | x_i ; \theta, b) = \frac{1}{2b} e^{\frac{-|y_i - (\theta_0 + \theta_1 x_i)|}{b}}$$

(func of y_i given x_i and parameters for laplace distribution)

For, Assuming that the points are independent.

$f(y_1 \ldots y_n | x_1 \ldots x_n, \theta, b)$. $P[y|x, \theta] = \max_\theta L(\theta)$

MLE $L(\theta) = \prod_{i=1}^{n} f(y_i | x_i, \theta)$

$$\log(L(\theta)) = \sum_{i=1}^{n} \log(f(y_i | x_i, \theta))$$

$$\Rightarrow \sum_{i=1}^{n} \log\left(\frac{1}{2b} \cdot e^{-|y_i - (\theta_0 + \theta_1 x_i)|/b}\right)$$

$$\Rightarrow \sum_{i=1}^{n} \left[\log\left(\frac{1}{2b}\right) - \frac{|y_i - (\theta_0 + \theta_1 x_i)|}{b} \right]$$

Minimize all negative terms to maximize the MLE function.

(ie) $$J(\theta) = \sum_{i=1}^{n} \frac{|y_i - (\theta_0 + \theta_1 x_i)|}{b}$$

$\longrightarrow$ MAE.

$$\therefore J(\theta) = \frac{1}{b} \sum_{i=1}^{n} |y_i - (\theta_0 + \theta_1 x_i)|$$