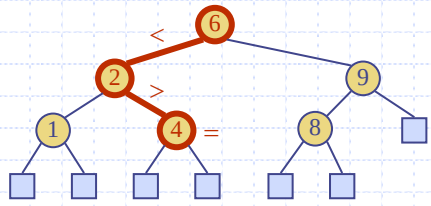
Lecture 12: Binary Search Trees



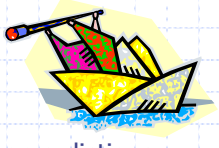
Definitions:

- 2-tree is a tree in which every vertex except the leaves has exactly two children
- Binary tree is an ordered tree in which every node has at most two children
- Binary tree is proper if each internal node has two children

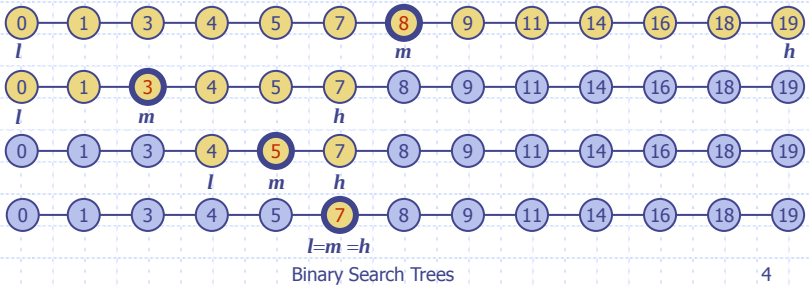
Ordered Dictionaries

- Keys are assumed to come from a total order.

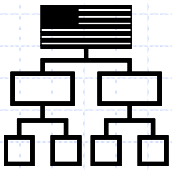
Binary Search (§3.1.1)



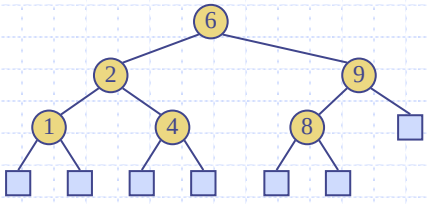
- Binary search performs operation findElement(k) on a dictionary implemented by means of an array-based sequence, sorted by key
  - at each step, the number of candidate items is halved
  - terminates after O(log n) steps
- Example: findElement(7)



Binary Search Tree (§3.1.2)



- A binary search tree is a binary tree storing keys (or key-element pairs) at its internal nodes and satisfying the following property:
  - Let u, v, and w be three nodes such that u is in the left subtree of v and w is in the right subtree of v. We have $key(u) \leq key(v) \leq key(w)$
- External nodes do not store items
- An inorder traversal of a binary search trees visits the keys in increasing order



Search (§3.1.3)

- To search for a key k, we trace a downward path starting at the root
- The next node visited depends on the outcome of the comparison of k with the key of the current node
- If we reach a leaf, the key is not found and we return NO_SUCH_KEY
- Example: findElement(4)

```
Algorithm findElement(k, v)
    if T.isExternal (v)
        return NO_SUCH_KEY
    if k < key(v)
        return findElement(k, T.leftChild(v))
    else if k = key(v)
        return element(v)
    else { k > key(v) }
        return findElement(k, T.rightChild(v))
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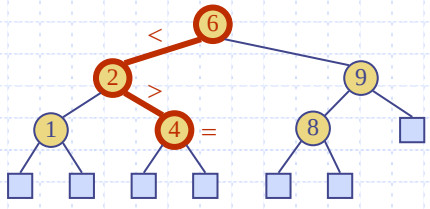
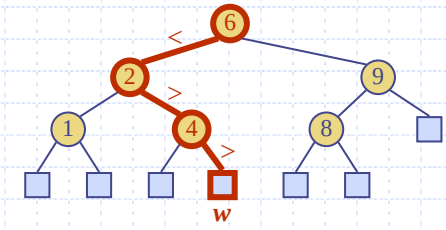
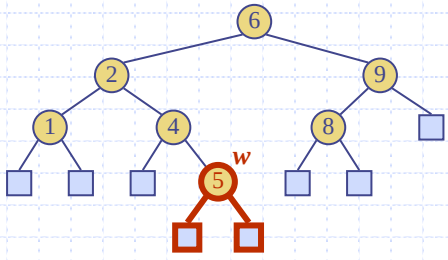


Insertion (§3.1.4)

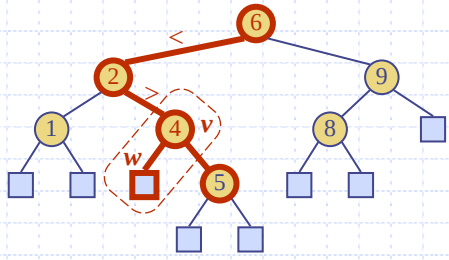
- To perform operation insertItem(k, o), we search for key k
- Assume k is not already in the tree, and let let w be the leaf reached by the search
- We insert k at node w and expand w into an internal node
- Example: insert 5

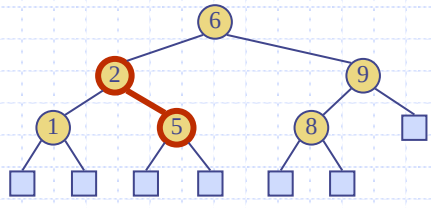




Deletion (§3.1.5)

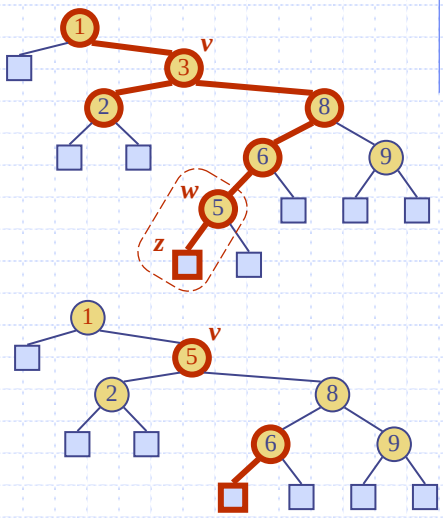
- To perform operation removeElement(*k*), we search for key *k*
- Assume key *k* is in the tree, and let let *v* be the node storing *k*
- If node *v* has a leaf child *w*, we remove *v* and *w* from the tree with operation removeAboveExternal(*w*)
- Example: remove 4





Deletion (cont.)

- We consider the case where the key k to be removed is stored at a node v whose children are both internal
  - we find the internal node w that follows v in an inorder traversal
  - we copy $key(w)$ into node v
  - we remove node w and its left child z (which must be a leaf) by means of operation removeAboveExternal(z)
- Example: remove 3



Performance (§3.1.6)

- Consider a dictionary with n items implemented by means of a binary search tree of height h
  - the space used is $O(n)$
  - methods findElement , insertItem and removeElement take $O(h)$ time
- The height h is $O(n)$ in the worst case and $O(\log n)$ in the best case