GLUE LECTURES
with
Amy LaViers

Purpose:

To connect what is done in the lectures to the quizzes
To give helpful hints about the quizzes
To clarify and repeat key concepts

There will be one Glue Lecture every week.

GLUE LECTURE 3 – “Systems”

(This will be helpful for Quiz 3!)

Goals of this lecture:

- Show how inputs and outputs define a “system”
- Get used to matrices via two examples
  - Do an example putting a second order system into state space form
  - Do an example of linearization

What is a system?

http://www.lockheedmartin.com/us/products/gps.html

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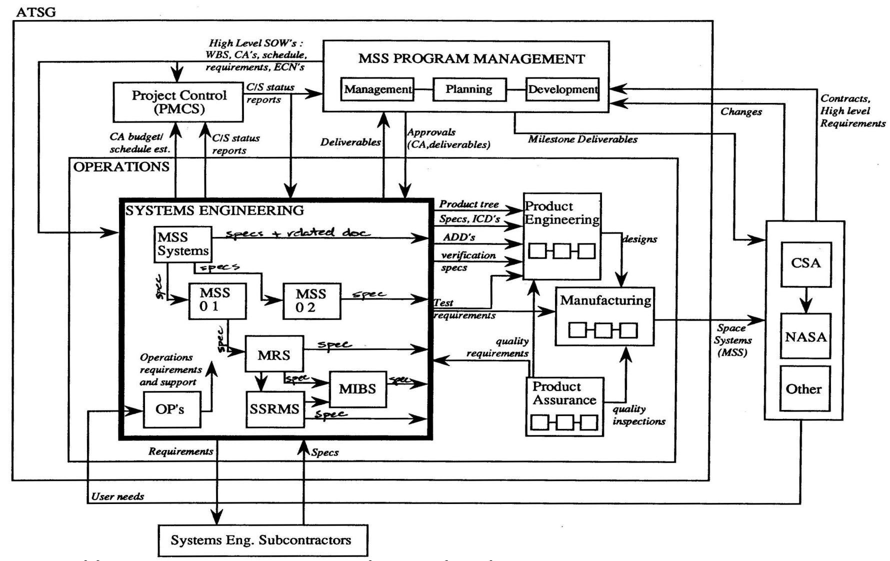

http://gramconsulting.com/2009/05/the-lasting-value-of-the-organization-as-system-map/

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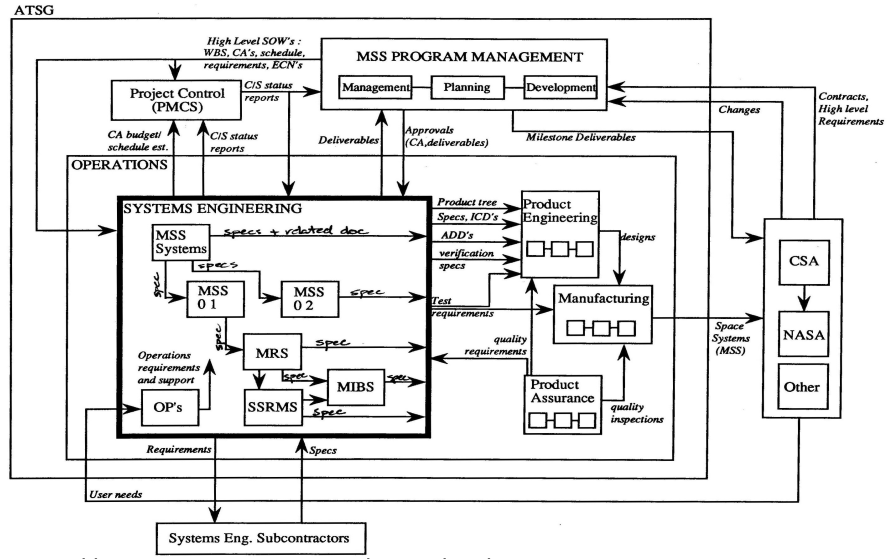

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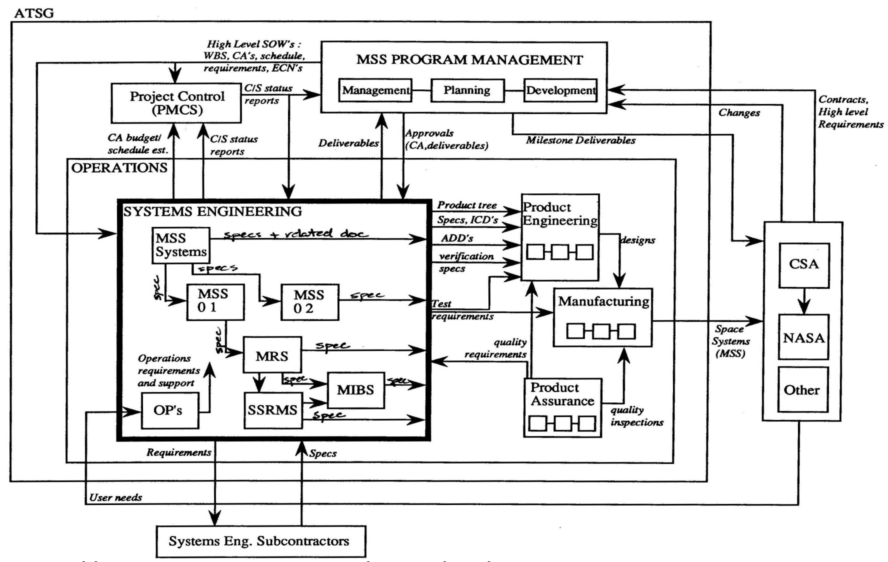

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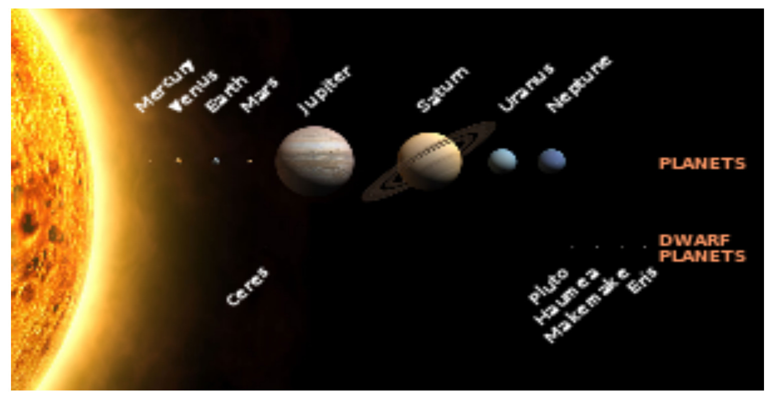

http://en.wikipedia.org/wiki/Solar_System

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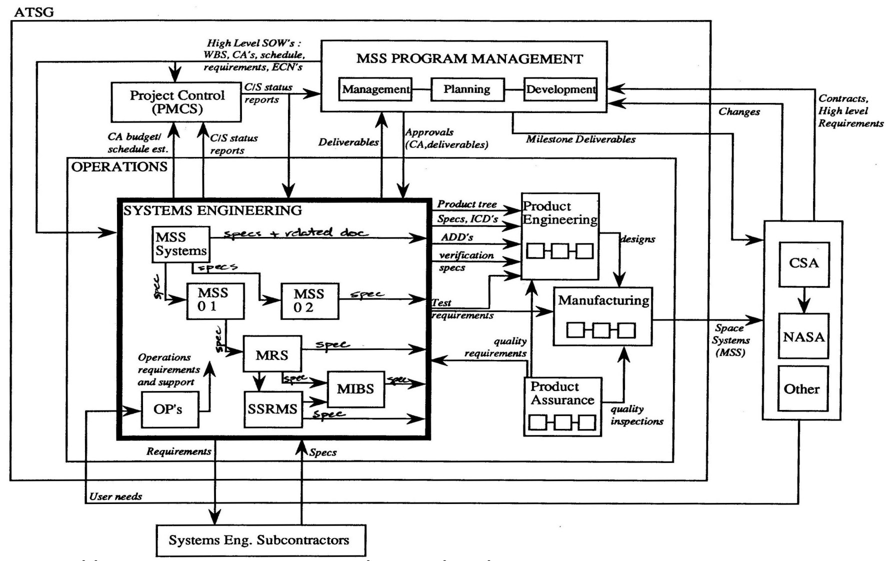

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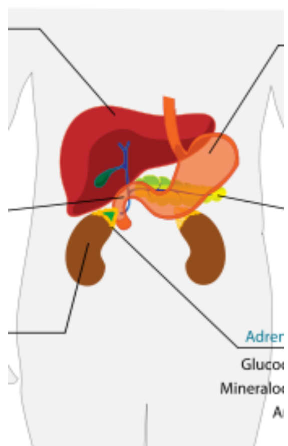
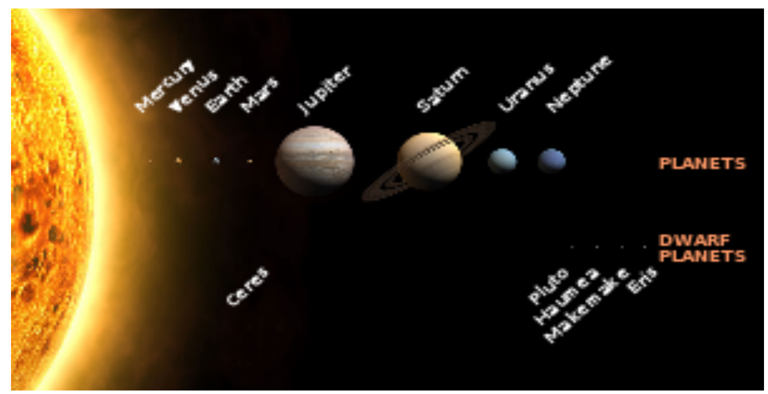


http://en.wikipedia.org/wiki/Endocrine_system



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Example 1:

How do you turn a general dynamical equation into a system in state space form?

$$m\ddot{f} = \alpha\dot{f} + \beta f + cp \qquad \Rightarrow \qquad \left\{ \begin{array}{c} \dot{x} = Ax + Bu \\ y = Cx \end{array} \right.$$

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$$\xrightarrow{\;u\;} \boxed{(A, B, C)} \xrightarrow{\;y\;}$$

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1) Pick the state variables and define inputs and outputs.
2) Write the second order differential equation as a pair of first order ones.
3) Put these in terms of state, input, and outputs.

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1) $$x = \left[\begin{array}{c} f \\ \dot{f} \end{array} \right] \qquad u = p \qquad y = f$$

(These choices define the SIZES of A, B, and C! And, with the dynamical equation, define our system!)

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2) $$\begin{array}{l} \dot{f} = \dot{x}_1 = x_2 \\ \ddot{f} = \dot{x}_2 = \frac{1}{m}(\alpha x_2 + \beta x_1 + cu) \end{array}$$

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$$= \left[\begin{array}{cc} 0 & 1 \\ \frac{\beta}{m} & \frac{\alpha}{m} \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \end{array} \right] + \left[\begin{array}{c} 0 \\ \frac{c}{m} \end{array} \right] u = Ax + Bu$$

$$y = f = \left[\begin{array}{cc} 1 & 0 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \end{array} \right]$$

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$$m\ddot{f} = \alpha \dot{f} + \beta f + cp \qquad \Rightarrow \qquad \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

$$u \longrightarrow \boxed{(A, B, C)} \longrightarrow y$$

$$x = \begin{bmatrix} f \\ \dot{f} \end{bmatrix} \qquad u = p \qquad y = f$$

$$(A, B, C) = \left(\begin{bmatrix} 0 & 1 \\ \frac{\beta}{m} & \frac{\alpha}{m} \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{c}{m} \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix} \right)$$

Example 2:

Linearize the following system around *x*=0.

$$\ddot{z} = \ell z^2 + \gamma \dot{z} + c\tau \qquad x = \begin{bmatrix} z \\ \dot{z} \end{bmatrix} \quad u = \tau$$

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$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{x=(0,0)} :$$

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Linearize the following system around *x*=0.

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$$B = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix}_{x=(0,0)} = \begin{bmatrix} 0 \\ c \end{bmatrix}_{x=(0,0)} = \begin{bmatrix} 0 \\ c \end{bmatrix}$$

Check the forums for more help and good luck with Quiz 3!