MODULE 2 LESSON 4

AN IMPROVED EKF: THE ERROR-STATE EXTENDED KALMAN

The Error-State EKF (ES-EKF)

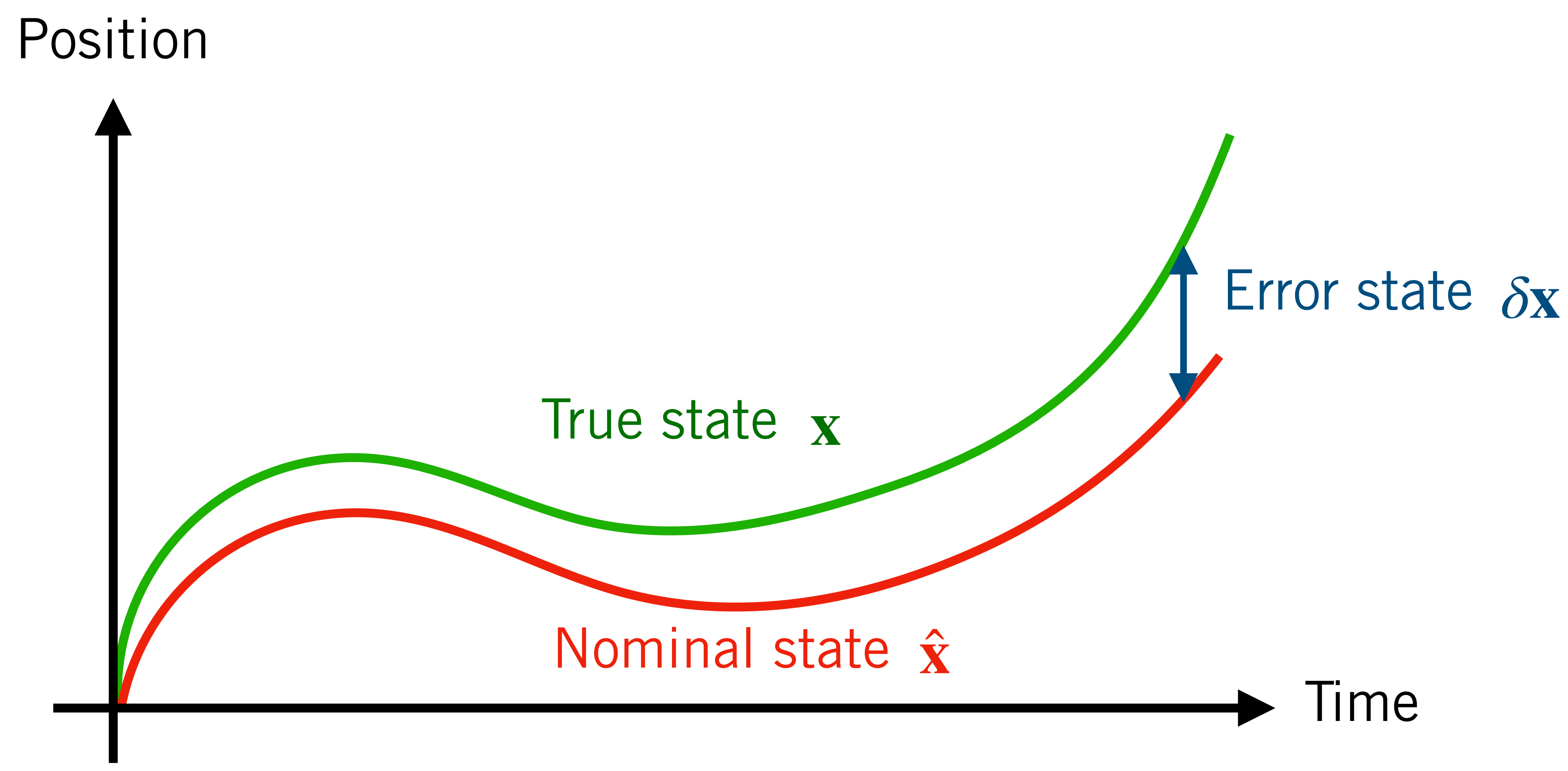
By the end of this video, you will be able to

- Describe the error-state formulation of the Extended Kalman Filter
- Describe the advantages of the Error-state EKF over the vanilla EKF

What's in a State?

We can think of the vehicle state as composed of two parts:

$$\underbrace{\mathbf{x}}_{\text{True State}} = \underbrace{\hat{\mathbf{x}}}_{\substack{\text{Nominal State} \\ \text{("Large")}}} + \underbrace{\delta\mathbf{x}}_{\substack{\text{Error State} \\ \text{("Small")}}}$$



- We can continuously update the *nominal state* by integrating the motion model
- Modelling errors and process noise accumulate into the *error state*

The Error-State Extended Kalman Filter

The Error-State Extended Kalman Filter estimates the error state directly and uses it as a correction to the nominal state:

Linearized motion model

$$\mathbf{x}_k = \mathbf{f}_{k-1}(\hat{\mathbf{x}}_{k-1}, \mathbf{u}_{k-1}, \mathbf{0}) + \mathbf{F}_{k-1}\left(\mathbf{x}_{k-1} - \hat{\mathbf{x}}_{k-1}\right) + \mathbf{L}_{k-1}\mathbf{w}_{k-1}$$

$$\underbrace{\mathbf{x}_k - \mathbf{f}_{k-1}(\hat{\mathbf{x}}_{k-1}, \mathbf{u}_{k-1}, \mathbf{0})}_{\delta\mathbf{x}_k} = \mathbf{F}_{k-1}\underbrace{\left(\mathbf{x}_{k-1} - \hat{\mathbf{x}}_{k-1}\right)}_{\delta\mathbf{x}_{k-1}} + \mathbf{L}_{k-1}\mathbf{w}_{k-1}$$

Error state

Linearized measurement model

$$\mathbf{y}_k = \mathbf{h}_k(\check{\mathbf{x}}_k, \mathbf{0}) + \mathbf{H}_k\left(\mathbf{x}_k - \check{\mathbf{x}}_k\right) + \mathbf{M}_k\mathbf{v}_k$$

$$\mathbf{y}_k = \mathbf{h}_k(\check{\mathbf{x}}_k, \mathbf{0}) + \mathbf{H}_k\underbrace{\left(\mathbf{x}_k - \check{\mathbf{x}}_k\right)}_{\delta\mathbf{x}_k} + \mathbf{M}_k\mathbf{v}_k$$

Error state

The Error-State Extended Kalman Filter

Loop:

1. Update nominal state with motion model

$$\check{\mathbf{x}}_k = \mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{0})$$

This could be
$\check{\mathbf{x}}_{k-1}$ or $\hat{\mathbf{x}}_{k-1}$

The Error-State Extended Kalman Filter

Loop:

1. Update nominal state with motion model
2. Propagate uncertainty

$$\check{\mathbf{P}}_k = \mathbf{F}_{k-1}\mathbf{P}_{k-1}\mathbf{F}_{k-1}^T + \mathbf{L}_{k-1}\mathbf{Q}_{k-1}\mathbf{L}_{k-1}^T$$

This could be

$\check{\mathbf{P}}_{k-1}$ or $\hat{\mathbf{P}}_{k-1}$

The Error-State Extended Kalman Filter

Loop:

1. Update nominal state with motion model
2. Propagate uncertainty
3. If a measurement is available:
    1. Compute Kalman Gain

$$\mathbf{K}_k = \check{\mathbf{P}}_k \mathbf{H}_k^T (\mathbf{H}_k \check{\mathbf{P}}_k \mathbf{H}_k^T + \mathbf{R})^{-1}$$

The Error-State Extended Kalman Filter

Loop:

1. Update nominal state with motion model
2. Propagate uncertainty
3. If a measurement is available:
   1. Compute Kalman Gain
   2. Compute error state

      $$\delta\hat{\mathbf{x}}_k = \mathbf{K}_k(\mathbf{y}_k - \mathbf{h}_k(\check{\mathbf{x}}_k, \mathbf{0}))$$

The Error-State Extended Kalman Filter

Loop:

1. Update nominal state with motion model
2. Propagate uncertainty
3. If a measurement is available:
    1. Compute Kalman Gain
    2. Compute error state
    3. Correct nominal state

$$\hat{\mathbf{x}}_k = \check{\mathbf{x}}_k + \delta\hat{\mathbf{x}}_k$$

The Error-State Extended Kalman Filter

Loop:

1. Update nominal state with motion model
2. Propagate uncertainty
3. If a measurement is available:
    1. Compute Kalman Gain
    2. Compute error state
    3. Correct nominal state
    4. Correct state covariance

$$\hat{\mathbf{P}}_k = (\mathbf{1} - \mathbf{K}_k \mathbf{H}_k)\check{\mathbf{P}}_k$$

Why Use the ES-EKF?

1. **Better performance compared to the vanilla EKF**
   The “small” error state is more amenable to linear filtering than the “large” nominal state, which can be integrated nonlinearly

2. **Easy to work with constrained quantities (e.g., rotations in 3D)**
   We can also break down the state using a generalized composition operator

$$\mathbf{x} = \hat{\mathbf{x}} \oplus \delta\mathbf{x}$$

True State

Nominal State
(Overparametrized, constrained)

Error State
(Minimal parametrization, unconstrained)

Summary | The Error-State EKF (ES-EKF)

- The error-state formulation separates the state into a “large” nominal state and a “small” error state.
- The ES-EKF uses local linearization to estimate the error state and uses it to correct the nominal state.
- The ES-EKF can perform better than the vanilla EKF, and provides a natural way to handle constrained quantities like rotations in 3D.