Control of Mobile Robots: Glue Lectures

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Glue Lecture 2: Robot Models

Pay attention, this lecture will help you all with Quiz 2!

Two models for robots …

(a)

$$\dot{x} = \frac{R}{2}(v_r + v_l)\cos\phi$$

$$\dot{y} = \frac{R}{2}(v_r + v_l)\sin\phi$$

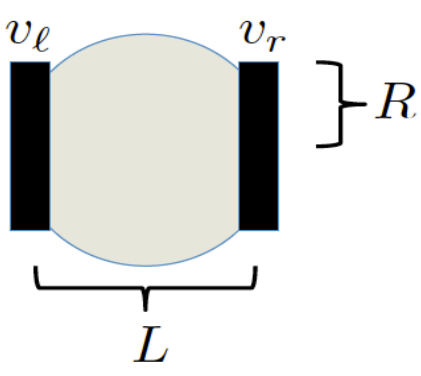
$$\dot{\phi} = \frac{R}{L}(v_r - v_l)$$

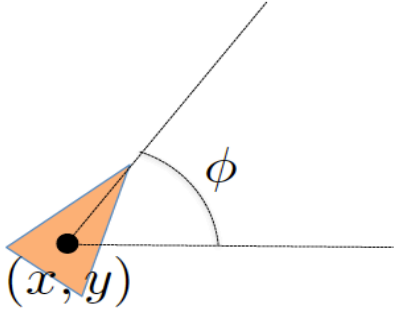
(b)

$$\dot{x} = v\cos\phi$$

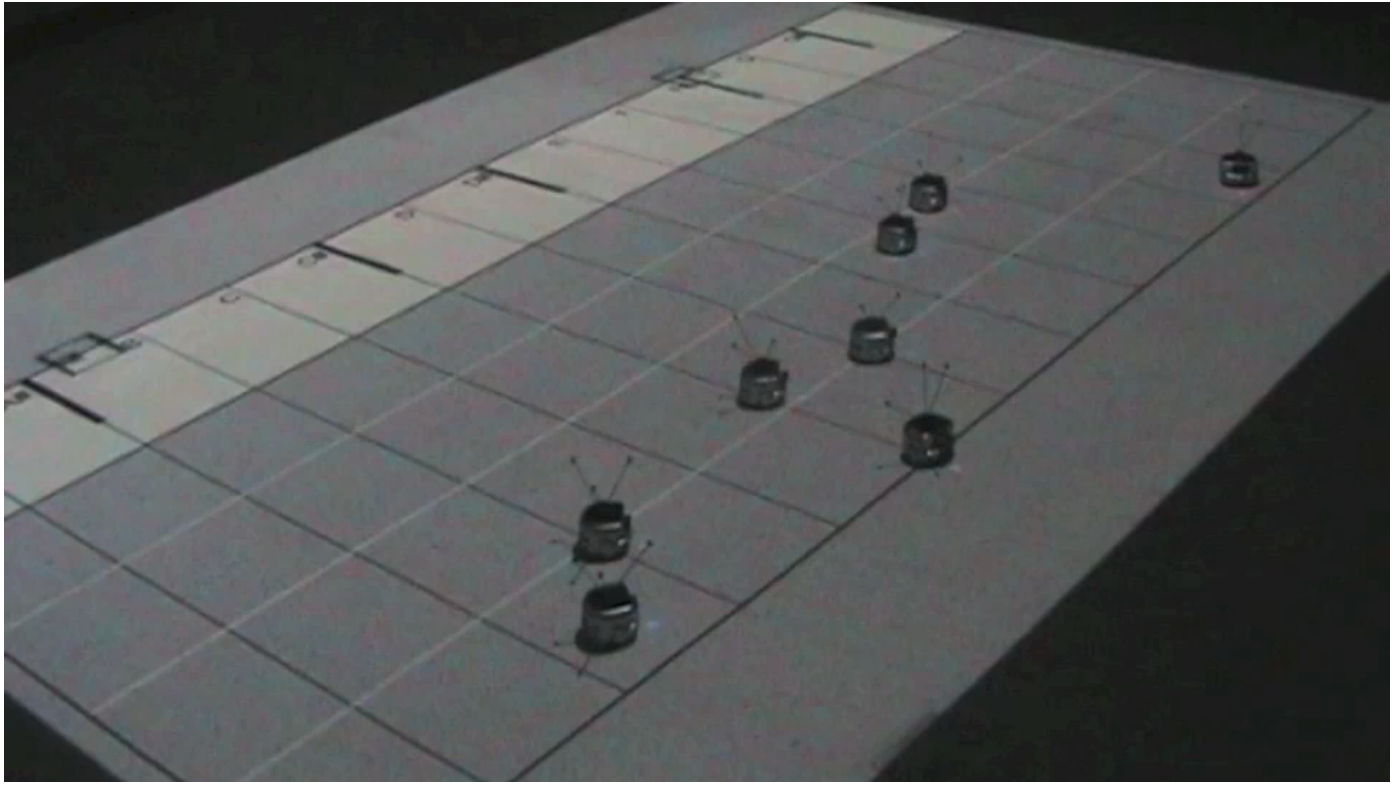
$$\dot{y} = v\sin\phi$$

$$\dot{\phi} = \omega$$





In Action …



From one robot model to another …

$t = 10s$

B

A

$$\dot{x} = v\cos\phi$$
$$\dot{y} = v\sin\phi$$
$$\dot{\phi} = \omega$$

High level task !

From one robot model to another …

$t = 10s$

B

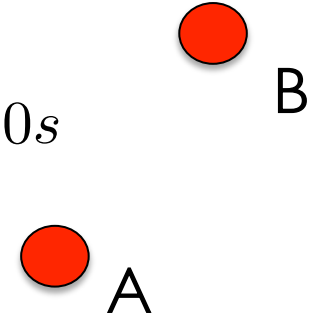
A

$$\dot{x} = v\cos\phi$$
$$\dot{y} = v\sin\phi$$
$$\dot{\phi} = \omega$$

High level task !

Control design - $v,\ \omega$

From one robot model to another …



$$\dot{x} = v\cos\phi \qquad \dot{x} = \frac{R}{2}(v_r + v_l)\cos\phi$$
$$\dot{y} = v\sin\phi \qquad \dot{y} = \frac{R}{2}(v_r + v_l)\sin\phi$$
$$\dot{\phi} = \omega \qquad \dot{\phi} = \frac{R}{L}(v_r - v_l)$$

High level task !

Control design - $v,\ \omega$

Commands sent to the robots - $v_r,\ v_l$

$$v_r = \frac{2v + \omega L}{2R}$$
$$v_l = \frac{2v - \omega L}{2R}$$

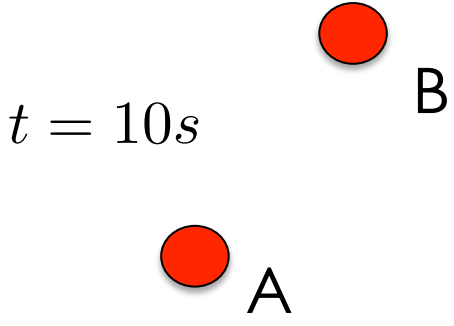
An intuitive example …

For inputs $v = 0,\ \omega = constant$, find the corresponding angular wheel velocities $v_r,\ v_l$

$$v_r = \frac{2v + \omega L}{2R}$$
$$v_l = \frac{2v - \omega L}{2R}$$

$$v_r = \frac{CL}{2R}$$
$$v_l = -\frac{CL}{2R}$$

Back to our musical bot …

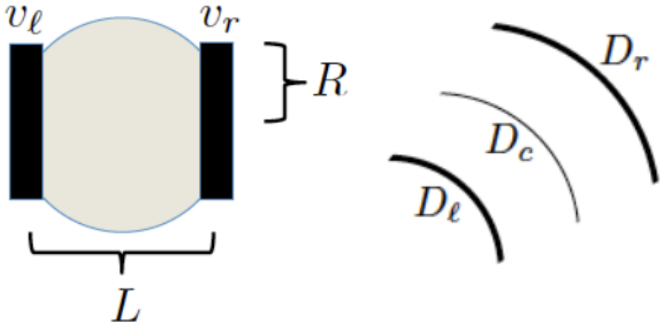


$$\dot{x} = v\cos\phi$$
$$\dot{y} = v\sin\phi$$
$$\dot{\phi} = \omega$$

Finding $v,\ \omega$ (and consequently $v_r,\ v_l$) – simple? No Feedback !!!

Important – state information (where is the bot currently)

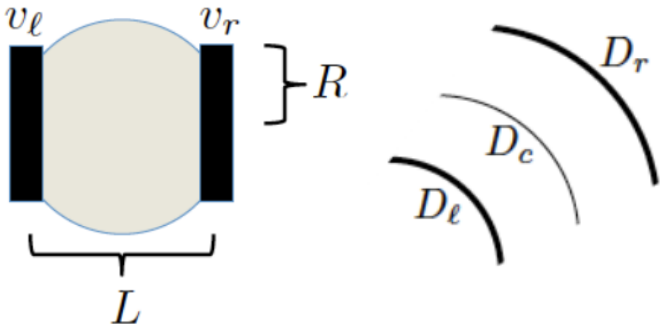
Wheel encoders …



$$x' = x + D_c \cos\phi$$
$$y' = y + D_c \sin\phi$$
$$\phi' = \phi + \frac{D_r - D_l}{L}$$

- Total N ticks per revolution
- $D_l = 2\pi R \frac{\Delta tick_l}{N}$
- $D_r = 2\pi R \frac{\Delta tick_r}{N}$ } dt
- $D_c = \frac{D_r + D_l}{2}$

For your general curiosity …



$$\dot{x} = \frac{R}{2}(v_r + v_l)cos\theta$$
$$\dot{y} = \frac{R}{2}(v_r + v_l)sin\theta$$
$$\dot{\theta} = \frac{R}{L}(v_r - v_l)$$

$$x' = x + D_c \cos\phi$$
$$y' = y + D_c \sin\phi$$
$$\phi' = \phi + \frac{D_r - D_l}{L}$$

$$D_r = Rv_r dt$$
$$D_l = Rv_l dt$$

$$x(t + dt) = x(t) + \dot{x}dt$$

An Odometry example …

If my robot starts at the origin (pos and orientation is 0), where is it located after 0.1s, given that 10 ticks were recorded for the right wheel and 6 ticks for the left wheel. Wheel radius is 2m, total ticks is 100, distance between wheels is 4m.

$$D_l = 2\pi R \frac{\Delta tick_l}{N}$$

$$D_r = 2\pi R \frac{\Delta tick_r}{N}$$

$$D_c = \frac{D_r + D_l}{2}$$

$$x' = x + D_c \cos\phi$$
$$y' = y + D_c \sin\phi$$
$$\phi' = \phi + \frac{D_r - D_l}{L}$$

$$x' = 1.0053\, m$$
$$y' = 0\, m$$
$$\phi' = 0.1257\, rad$$

Check the forums, and good luck with Quiz 2!