Syntax

Type Variables	X, Y, Z		
Term Variables	x, y, z		
Types	T, S, U	$::=$	X
		$\mid$	I
		$\mid$	$T \oplus T$
		$\mid$	$T \otimes T$
		$\mid$	$T \multimap T$
		$\mid$	$\mu X.T$
Terms	t, u, v	$::=$	x
		$\mid$	$()$
		$\mid$	inl t
		$\mid$	inr t
		$\mid$	$t \times t$
		$\mid$	$t \mapsto t$
		$\mid$	fold t
		$\mid$	t trace$_T$
		$\mid$	$t + t$
		$\mid$	$-\ t$
		$\mid$	$\varnothing$
Type Contexts	Θ	$::=$	
Term Contexts		$\mid$	Θ, X
	Γ	$::=$	
		$\mid$	$\Gamma, t : T$
Type Judgements			$\Theta \vdash T$
Term Judgements			$\Gamma \vdash t : T$
Expressions	e, f, g	$::=$	t
		$\mid$	$f \ @\ t$
		$\mid$	$f^\dagger$

Type Formation rules

$$\frac{}{\Theta, X \vdash X} \qquad \frac{}{\Theta \vdash I} \qquad \frac{\Theta \vdash T_1 \qquad \Theta \vdash T_2}{\Theta \vdash T_1 \oplus T_2} \qquad \frac{\Theta \vdash T_1 \qquad \Theta \vdash T_2}{\Theta \vdash T_1 \otimes T_2} \qquad \frac{\Theta \vdash T_1 \qquad \Theta \vdash T_2}{\Theta \vdash T_1 \multimap T_2} \qquad \frac{\Theta, X \vdash T}{\Theta \vdash \mu X.T}$$

Substitution

$$\begin{aligned}
X[X/S] &= S \\
Y[X/S] &= Y \\
I[X/S] &= I \\
T_1 \oplus T_2[X/S] &= T_1[X/S] \oplus T_2[X/S] \\
T_1 \otimes T_2[X/S] &= T_1[X/S] \otimes T_2[X/S] \\
T_1 \multimap T_2[X/S] &= T_1[X/S] \multimap T_2[X/S] \\
\mu Y.T[X/S] &= \mu Y.(T[X/S])
\end{aligned}$$

Term Typing rules

$$\text{Variable}\ \frac{}{x : T \vdash x : T} \qquad \frac{\Gamma_2, \Gamma_1 \vdash t : T}{\Gamma_1, \Gamma_2 \vdash t : T}\ \text{Exchange}$$

$$I_L\ \frac{\Gamma \vdash t : T}{\Gamma, () : I \vdash t : T} \qquad \frac{}{\vdash () : I}\ I_R$$

$$\oplus_{L_l}\ \frac{\Gamma, t_1 : T_1 \vdash t : T}{\Gamma, \text{inl}\ t_1 : T_1 \oplus T_2 \vdash t : T} \qquad \frac{\Gamma \vdash t_1 : T_1}{\Gamma \vdash \text{inl}\ t_1 : T_1 \oplus T_2}\ \oplus_{R_l}$$

$$\oplus_{L_r}\ \frac{\Gamma, t_2 : T_2 \vdash t : T}{\Gamma, \text{inr}\ t_2 : T_1 \oplus T_2 \vdash t : T} \qquad \frac{\Gamma \vdash t_2 : T_2}{\Gamma \vdash \text{inr}\ t_2 : T_1 \oplus T_2}\ \oplus_{R_r}$$

$$\otimes_L\ \frac{\Gamma, t_1 : T_1, t_2 : T_2 \vdash t : T}{\Gamma, t_1 \times t_2 : T_1 \otimes T_2 \vdash t : T} \qquad \frac{\Gamma_1 \vdash t_1 : T_1 \qquad \Gamma_2 \vdash t_2 : T_2}{\Gamma_1, \Gamma_2 \vdash t_1 \times t_2 : T_1 \otimes T_2}\ \otimes_R$$

$$\multimap_L\ \frac{\Gamma_1 \vdash t_1 : T_1 \qquad \Gamma_2, t_2 : T_2 \vdash t : T}{\Gamma_1, \Gamma_2, t_1 \mapsto t_2 : T_1 \multimap T_2 \vdash t : T} \qquad \frac{\Gamma, t_1 : T_1 \vdash t_2 : T_2}{\Gamma \vdash t_1 \mapsto t_2 : T_1 \multimap T_2}\ \multimap_R$$

$$\mu_L\ \frac{\Gamma, u : U[X / \mu X.U] \vdash t : T}{\Gamma, \text{fold}\ u : \mu X.U \vdash t : T} \qquad \frac{\Gamma \vdash t : T[X / \mu X.T]}{\Gamma \vdash \text{fold}\ t : \mu X.T}\ \mu_R$$

$$\text{Linearity}_L\ \frac{\Gamma, t_1 : U \vdash t : T \qquad \Gamma, t_2 : U \vdash t : T}{\Gamma, t_1 + t_2 : U \vdash t : T} \qquad \frac{\Gamma \vdash t_1 : T \qquad \Gamma \vdash t_2 : T}{\Gamma \vdash t_1 + t_2 : T}\ \text{Linearity}_R$$

$$\text{Opposition}_L\ \frac{\Gamma, u : U \vdash t : T}{\Gamma, -u : U \vdash t : T} \qquad \frac{\Gamma \vdash t : T}{\Gamma \vdash -t : T}\ \text{Opposition}_R$$

$$\text{Trace}_L\ \frac{\Gamma, u : S \oplus U_1 \multimap S \oplus U_2 \vdash t : T}{\Gamma, u\ \text{trace}_S : U_1 \multimap U_2 \vdash t : T} \qquad \frac{\Gamma \vdash t : U \oplus T_1 \multimap U \oplus T_2}{\Gamma \vdash t\ \text{trace}_U : T_1 \multimap T_2}\ \text{Trace}_R$$

Expression Typing rules

$$\text{Application}\ \frac{\vdash f : T_1 \multimap T_2 \qquad \vdash t : T_2}{\vdash f\ t : T_1} \qquad \frac{\vdash f : T_1 \multimap T_2 \qquad \vdash g : T_2 \multimap T_3}{\vdash f \fatsemi g : T_1 \multimap T_3}\ \text{Composition}$$

$$\frac{\vdash f : T \multimap U}{\vdash f^\dagger : U \multimap T}\ \text{Dagger}$$

Operational Semantics

$$\text{Match–var}\ \frac{}{x = t \ \hookrightarrow\ [x \to t]} \qquad \frac{}{[x \to t]\ x \ \rightsquigarrow\ t}\ \text{Subst-var}$$

$$\text{Match–unit}\ \frac{}{() = () \ \hookrightarrow\ [\,]} \qquad \frac{}{[\,]\ () \ \rightsquigarrow\ ()}\ \text{Subst–unit}$$

$$\text{Match–inl}\ \frac{t = u \ \hookrightarrow\ [\Xi]}{\mathrm{inl}\ t = \mathrm{inl}\ u \ \hookrightarrow\ [\Xi]} \qquad \frac{[\Xi]\ t \ \rightsquigarrow\ t'}{[\Xi]\ \mathrm{inl}\ t \ \rightsquigarrow\ \mathrm{inl}\ t'}\ \text{Subst–inl}$$

$$\text{Match–inr}\ \frac{t = u \ \hookrightarrow\ [\Xi]}{\mathrm{inr}\ t = \mathrm{inr}\ u \ \hookrightarrow\ [\Xi]} \qquad \frac{[\Xi]\ t \ \rightsquigarrow\ t'}{[\Xi]\ \mathrm{inr}\ t \ \rightsquigarrow\ \mathrm{inr}\ t'}\ \text{Subst–inr}$$

$$\text{Match–tensor}\ \frac{t_1 = u_1 \ \hookrightarrow\ [\Xi_1] \qquad t_2 = u_2 \ \hookrightarrow\ [\Xi_2]}{t_1 \times t_2 = u_1 \times u_2 \ \hookrightarrow\ [\Xi_1, \Xi_2]} \qquad \frac{[\Xi_1]\ t_1 \ \rightsquigarrow\ t_1' \qquad [\Xi_2]\ t_2 \ \rightsquigarrow\ t_2'}{[\Xi_1, \Xi_2]\ t_1 \times t_2 \ \rightsquigarrow\ t_1' \times t_2'}\ \text{Subst–tensor}$$

$$\text{Match–arrow}\ \frac{t_1 = u_1 \ \hookrightarrow\ [\Xi_1] \qquad t_2 = u_2 \ \hookrightarrow\ [\Xi_2]}{t_1 \mapsto t_2 = u_1 \mapsto u_2 \ \hookrightarrow\ [\Xi_1, \Xi_2]} \qquad \frac{[\Xi_1]\ t_1 \ \rightsquigarrow\ t_1' \qquad [\Xi_2]\ t_2 \ \rightsquigarrow\ t_2'}{[\Xi_1, \Xi_2]\ t_1 \mapsto t_2 \ \rightsquigarrow\ t_1' \mapsto t_2'}\ \text{Subst–arrow}$$

$$\text{Match–fold}\ \frac{t = u \ \hookrightarrow\ [\Xi]}{\mathrm{fold}\ t = \mathrm{fold}\ u \ \hookrightarrow\ [\Xi]} \qquad \frac{[\Xi]\ t \ \rightsquigarrow\ t'}{[\Xi]\ \mathrm{fold}\ t \ \rightsquigarrow\ \mathrm{fold}\ t'}\ \text{Subst–fold}$$

$$\text{Match–lin1}\ \frac{t = u_1 \ \hookrightarrow\ [\Xi] \qquad t = u_2 \ \hookrightarrow\ [\Xi]}{t = u_1 + u_2 \ \hookrightarrow\ [\Xi]} \qquad \frac{[\Xi]\ t_1 \ \rightsquigarrow\ t_1' \qquad [\Xi]\ t_2 \ \rightsquigarrow\ t_2'}{[\Xi]\ t_1 + t_2 \ \rightsquigarrow\ t_1' + t_2'}\ \text{Subst-lin}$$

$$\text{Match–lin2}\ \frac{t_1 = u \ \hookrightarrow\ [\Xi] \qquad t_2 = u \ \hookrightarrow\ [\Xi]}{t_1 + t_2 = u \ \hookrightarrow\ [\Xi]}$$

$$\text{Match–opp–c}\ \frac{v = u \ \hookrightarrow\ [\Xi]}{-v = -u \ \hookrightarrow\ [\Xi]} \qquad \frac{[\Xi]\ v \ \rightsquigarrow\ v'}{[\Xi]\ -v \ \rightsquigarrow\ -v'}\ \text{Subst–opp}$$

$$\text{Match–opp–l}\ \frac{v = -u \ \hookrightarrow\ [\Xi]}{-v = u \ \hookrightarrow\ [\Xi]}$$

$$\text{Match–opp–r}\ \frac{v = u \ \hookrightarrow\ -[\Xi]}{v = -u \ \hookrightarrow\ [\Xi]}$$

$$\text{Match–trace}\ \frac{t = u \ \hookrightarrow\ [\Xi]}{t\ \mathrm{trace}_T = u\ \mathrm{trace}_T \ \hookrightarrow\ [\Xi]} \qquad \frac{[\Xi]\ t \ \rightsquigarrow\ t'}{[\Xi]\ t\ \mathrm{trace}_T \ \rightsquigarrow\ t'\ \mathrm{trace}_T}\ \text{Subst–trace}$$

$$\text{App–arrow}\ \frac{t_1 = t \ \hookrightarrow\ [\Xi] \qquad [\Xi]\ t_2 \ \rightsquigarrow\ t'}{(t_1 \mapsto t_2)\ @\ t \ \Rightarrow\ t'} \qquad \frac{t_1\ @\ t \ \Rightarrow\ t_1' \qquad t_2\ @\ t \ \Rightarrow\ t_2'}{(t_1 + t_2)\ @\ t \ \Rightarrow\ t_1' + t_2'}\ \text{App–lin}$$

$$\frac{f^\dagger \ \looparrowright\ f' \qquad f'\ @\ t \ \Rightarrow\ t'}{f^\dagger\ @\ t \ \Rightarrow\ t'}\ \text{App–dagger}$$

$$\text{App–trace–nix}\ \frac{t\ @\ (\mathrm{inr}\ u) \ \Rightarrow\ \mathrm{inr}\ u'}{(t\ \mathrm{trace}_X)\ @\ u \ \Rightarrow\ u'} \qquad \frac{t\ @\ (\mathrm{inl}\ u) \ \Rightarrow\ \mathrm{inl}\ u' \qquad (t\ \mathrm{trace}_X)\ @\ (\mathrm{inl}\ u') \ \Rightarrow\ u''}{(t\ \mathrm{trace}_X)\ @\ \mathrm{inl}\ u \ \Rightarrow\ u''}\ \text{App–trace–loop}$$

$$\text{App–trace–start}\ \frac{t\ @\ (\mathrm{inr}\ u) \ \Rightarrow\ \mathrm{inl}\ u' \qquad (t\ \mathrm{trace}_X)\ @\ (\mathrm{inl}\ u') \ \Rightarrow\ u''}{(t\ \mathrm{trace}_X)\ @\ u \ \Rightarrow\ u''} \qquad \frac{t\ @\ (\mathrm{inl}\ u) \ \Rightarrow\ \mathrm{inr}\ u'}{(t\ \mathrm{trace}_X)\ @\ \mathrm{inl}\ u \ \Rightarrow\ u'}\ \text{App–trace–end}$$

$$\text{Flip–arrow}\ \frac{}{(t_1 \mapsto t_2)^\dagger \ \looparrowright\ t_2 \mapsto t_1} \qquad \frac{t_1^\dagger \ \looparrowright\ t_1' \qquad t_2^\dagger \ \looparrowright\ t_2'}{(t_1 + t_2)^\dagger \ \looparrowright\ t_1' + t_2'}\ \text{Flip–lin}$$

$$\text{Flip–app}\ \frac{f\ @\ t \ \Rightarrow\ t' \qquad t'^\dagger \ \looparrowright\ t''}{(f\ @\ t)^\dagger \ \looparrowright\ t''} \qquad \frac{t^\dagger \ \looparrowright\ t'}{(t\ \mathrm{trace}_X)^\dagger \ \looparrowright\ t'\ \mathrm{trace}_X}\ \text{Flip–trace}$$

Equivalence relation

$$\frac{}{t_1 + t_2 \;\equiv\; t_2 + t_1} \qquad \frac{}{(t_1 + t_2) + (t_3 + t_4) \;\equiv\; (t_1 + t_3) + (t_2 + t_4)}$$

$$\frac{t_1 \;\equiv\; t_2}{t_1 + t_2 \;\equiv\; t_1} \qquad \frac{t_1 \;\equiv\; t_2}{t_1 + -t_2 \;\equiv\; \varnothing} \qquad \frac{}{t + \varnothing \;\equiv\; t}$$

$$\frac{}{() \;\equiv\; ()} \qquad \frac{}{x \;\equiv\; x}$$

$$\frac{t \;\equiv\; t'}{\text{inl}\ t \;\equiv\; \text{inl}\ t'} \qquad \frac{t \;\equiv\; t'}{\text{inr}\ t \;\equiv\; \text{inr}\ t'}$$

$$\frac{t_1 \;\equiv\; t_1' \qquad t_2 \;\equiv\; t_2'}{t_1 \times t_2 \;\equiv\; t_1' \times t_2'} \qquad \frac{t_1 \;\equiv\; t_1' \qquad t_2 \;\equiv\; t_2'}{t_1 \mapsto t_2 \;\equiv\; t_1' \mapsto t_2'}$$

$$\frac{t \;\equiv\; t'}{\text{fold}\ t \;\equiv\; \text{fold}\ t'} \qquad \frac{t \;\equiv\; t'}{t\ \text{trace}_T \;\equiv\; t'\ \text{trace}_T}$$

$$\frac{t \;\equiv\; t'}{-t \;\equiv\; -t'}$$

$$\frac{}{\text{inl}\ (t_1 + t_2) \;\equiv\; \text{inl}\ t_1 + \text{inl}\ t_2} \qquad \frac{}{\text{inr}\ (t_1 + t_2) \;\equiv\; \text{inr}\ t_1 + \text{inr}\ t_2}$$

$$\frac{}{(t_1 + t_2) \times t_3 \;\equiv\; (t_1 \times t_3) + (t_2 \times t_3)} \qquad \frac{}{t_1 \times (t_2 + t_3) \;\equiv\; (t_1 \times t_2) + (t_1 \times t_3)}$$

$$\frac{}{(t_1 + t_2) \mapsto t_3 \;\equiv\; (t_1 \mapsto t_3) + (t_2 \mapsto t_3)} \qquad \frac{}{t_1 \mapsto (t_2 + t_3) \;\equiv\; (t_1 \mapsto t_2) + (t_1 \mapsto t_3)}$$

$$\frac{}{\text{fold}\ (t_1 + t_2) \;\equiv\; \text{fold}\ t_1 + \text{fold}\ t_2} \qquad \frac{}{(t_1 + t_2)\ \text{trace}_T \;\equiv\; (t_1\ \text{trace}_T) + (t_2\ \text{trace}_T)}$$

$$\frac{}{-(t_1 + t_2) \;\equiv\; -t_1 + -t_2}$$

$$\frac{}{-\text{inl}\ t \;\equiv\; \text{inl}\ (-t)} \qquad \frac{}{-\text{inr}\ t \;\equiv\; \text{inr}\ (-t)}$$

$$\frac{}{-(t_1 \times t_2) \;\equiv\; (-t_1) \times (-t_2)} \qquad \frac{}{-(t_1 \mapsto t_2) \;\equiv\; (-t_1) \mapsto (-t_2)}$$

$$\frac{}{-\text{fold}\ t \;\equiv\; \text{fold}\ (-t)} \qquad \frac{}{-(t\ \text{trace}_T) \;\equiv\; (-t)\ \text{trace}_T}$$

$$\frac{}{-(-t) \;\equiv\; t}$$

partial order

$$x \prec () \prec \text{inl}\ t \prec \text{inr}\ t \prec t_1 \times t_2 \prec t_1 \mapsto t_2 \prec \text{fold}\ t \prec t\ \text{trace}_T \prec t_1 + t_2 \prec -t$$

Type Interpretation

V is Compact Closed Category with Finite Biproduct.

Π_i is projection functor.

K_I is constant I functor.

$[-,-]$ is internal hom functor.

$(-)^*$ is contravariant anafunctor. Using Axiom of Choice, we can define it on strict functor (see `https://ncatlab.org/nlab/show/rigid+monoidal+category#remarks`).

Id is identity functor.

$$\begin{aligned}
[\![\Theta \vdash T]\!] &: \mathbf{V}^{|\Theta|} \to \mathbf{V} \\
[\![\Theta \vdash X_i]\!] &= \Pi_i \\
[\![\Theta \vdash I]\!] &= K_I \\
[\![\Theta \vdash T_1 \oplus T_2]\!] &= \oplus \circ \langle [\![\Theta \vdash T_1]\!], [\![\Theta \vdash T_2]\!] \rangle \\
[\![\Theta \vdash T_1 \otimes T_2]\!] &= \otimes \circ \langle [\![\Theta \vdash T_1]\!], [\![\Theta \vdash T_2]\!] \rangle \\
[\![\Theta \vdash T_1 \multimap T_2]\!] &= \otimes \circ [(-)^*, Id] \circ \langle [\![\Theta \vdash T_1]\!], [\![\Theta \vdash T_2]\!] \rangle \\
[\![\Theta \vdash \mu X.T]\!] &= [\![\Theta, X \vdash T]\!]^\sharp
\end{aligned}$$

Denotational Semantics

$\mathbf{V}(A, B)$ is morphism which Domain and Codomain are $A \in$ Obj($\mathbf{V}$) and $B \in$ Obj($\mathbf{V}$), respectively.

id_T is identity morphism of Object T.

$0_{A,B}$ is zero morphism of $\mathbf{V}(A, B)$.

ι_i is injection morphism.

π_i is projection morphism.

$\pi_i = \iota_i^{-1}$

$\pi_i \circ \iota_j = id$ if $i = j$

$\pi_i \circ \iota_j = 0$ otherwise

$unfold_{\mu X.T} = fold_{\mu X.T}^{-1}$

$f \circ (g \oplus h) = (f \circ g) \oplus (f \circ h)$

$(f \oplus g) \circ h = (f \circ h) \oplus (g \circ h)$

$id_A \otimes id_B = id_{A \otimes B}$

$(f \otimes g) \circ (h \otimes k) = (f \circ h) \otimes (g \circ k)$

$f^{**} = f$

$(f \otimes g)^* = f^* \otimes g^*$

$\Delta_T : T \to T \oplus T$

$\nabla_T : T \oplus T \to T$

$$\begin{aligned}
[\![t : T]\!] &\in \mathbf{V}(I, T) \\
[\![x : T]\!] &:= x_T \\
[\![() : T]\!] &:= id_I \\
[\![\text{inl } t_1 : T_1 \oplus T_2]\!] &:= \iota_1 \circ [\![t_1 : T_1]\!] \\
[\![\text{inr } t_2 : T_1 \oplus T_2]\!] &:= \iota_2 \circ [\![t_2 : T_2]\!] \\
[\![t_1 \times t_2 : T_1 \otimes T_2]\!] &:= [\![t_1 : T_1]\!] \otimes [\![t_2 : T_2]\!] \\
[\![t_1 \mapsto t_2 : T_1 \multimap T_2]\!] &:= [\![t_1 : T_1]\!]^* \otimes [\![t_2 : T_2]\!] \\
[\![\text{fold } t : \mu X.T]\!] &:= fold_{\mu X.T} \circ [\![t : T[X / \mu X.T]]\!] \\
[\![t_1 + t_2 : T]\!] &:= \nabla_T \circ [\![t_1 : T]\!] \oplus [\![t_2 : T]\!] \circ \Delta_T
\end{aligned}$$

Term Typing rules

$$\varnothing \vdash t : T \triangleright \varnothing$$

$$\text{Variable}\ \frac{\Gamma(x) = T}{\Gamma \vdash x : T \triangleright \Gamma \setminus \{x\}}$$

$$I_L\ \frac{\Gamma \vdash t : T \triangleright \Gamma'}{\Gamma, () : I \vdash t : T \triangleright \Gamma'} \qquad \frac{}{\varnothing \vdash () : I \triangleright \varnothing}\ I_R$$

$$\oplus_{L_l}\ \frac{\Gamma, t_1 : T_1 \vdash t : T \triangleright \Gamma'}{\Gamma, \text{inl}\ t_1 : T_1 \oplus T_2 \vdash t : T \triangleright \Gamma'} \qquad \frac{\Gamma \vdash t_1 : T_1 \triangleright \Gamma'}{\Gamma \vdash \text{inl}\ t_1 : T_1 \oplus T_2 \triangleright \Gamma'}\ \oplus_{R_l}$$

$$\oplus_{L_r}\ \frac{\Gamma, t_2 : T_2 \vdash t : T \triangleright \Gamma'}{\Gamma, \text{inr}\ t_2 : T_1 \oplus T_2 \vdash t : T \triangleright \Gamma'} \qquad \frac{\Gamma \vdash t_2 : T_2 \triangleright \Gamma'}{\Gamma \vdash \text{inr}\ t_2 : T_1 \oplus T_2 \triangleright \Gamma'}\ \oplus_{R_r}$$

$$\otimes_L\ \frac{\Gamma, t_1 : T_1, t_2 : T_2 \vdash t : T \triangleright \Gamma'}{\Gamma, t_1 \times t_2 : T_1 \otimes T_2 \vdash t : T \triangleright \Gamma'} \qquad \frac{\Gamma \vdash t_1 : T_1 \triangleright \Gamma' \qquad \Gamma' \vdash t_2 : T_2 \triangleright \Gamma''}{\Gamma \vdash t_1 \times t_2 : T_1 \otimes T_2 \triangleright \Gamma''}\ \otimes_R$$

$$\multimap_L\ \frac{\Gamma \vdash t_1 : T_1 \triangleright \Gamma' \qquad \Gamma', t_2 : T_2 \vdash t : T \triangleright \Gamma''}{\Gamma, t_1 \mapsto t_2 : T_1 \multimap T_2 \vdash t : T \triangleright \Gamma''} \qquad \frac{\Gamma, t_1 : T_1 \vdash t_2 : T_2 \triangleright \Gamma'}{\Gamma \vdash t_1 \mapsto t_2 : T_1 \multimap T_2 \triangleright \Gamma'}\ \multimap_R$$

$$\mu_L\ \frac{\Gamma, u : U[X / \mu X.U] \vdash t : T \triangleright \Gamma'}{\Gamma, \text{fold}\ u : \mu X.U \vdash t : T \triangleright \Gamma'} \qquad \frac{\Gamma \vdash t : T[X / \mu X.T] \triangleright \Gamma'}{\Gamma \vdash \text{fold}\ t : \mu X.T \triangleright \Gamma'}\ \mu_R$$

$$\text{Linearity}_L\ \frac{\Gamma, t_1 : U \vdash t : T \triangleright \Gamma' \qquad \Gamma, t_2 : U \vdash t : T \triangleright \Gamma'}{\Gamma, t_1 + t_2 : U \vdash t : T \triangleright \Gamma'} \qquad \frac{\Gamma \vdash t_1 : T \triangleright \Gamma' \qquad \Gamma \vdash t_2 : T \triangleright \Gamma'}{\Gamma \vdash t_1 + t_2 : T \triangleright \Gamma'}\ \text{Linearity}_R$$

$$\text{Opposition}_L\ \frac{\Gamma, u : U \vdash t : T \triangleright \Gamma'}{\Gamma, -u : U \vdash t : T \triangleright \Gamma'} \qquad \frac{\Gamma \vdash t : T \triangleright \Gamma'}{\Gamma \vdash -t : T \triangleright \Gamma'}\ \text{Opposition}_R$$

$$\text{Trace}_L\ \frac{\Gamma, u : S \oplus U_1 \multimap S \oplus U_2 \vdash t : T \triangleright \Gamma'}{\Gamma, u\ \text{trace}_S : U_1 \multimap U_2 \vdash t : T \triangleright \Gamma'} \qquad \frac{\Gamma \vdash t : U \oplus T_1 \multimap U \oplus T_2 \triangleright \Gamma'}{\Gamma \vdash t\ \text{trace}_U : T_1 \multimap T_2 \triangleright \Gamma'}\ \text{Trace}_R$$