1 構文

```
Type Variable
                       \ni X, Y, Z
Term Variable
                       \Rightarrow x, y, z
Type
                       \ni S, T, U
                                       ::= X \mid \mathbf{I} \mid T \oplus T \mid T \otimes T \mid T \multimap T \mid \mu X.T
Term
                       \ni s, t, u
                                       Expression
                       \ni e, f, g
                                       ::= t \mid e @ e
Type Context
                                            |\Theta, X|
                       \ni \Theta
                                       ::=
                                            |\Gamma, t:T
Term Context
                       \ni \Gamma
                                       ::=
Type Judgement
                       \ni \Theta \vdash T
Term Judgement
                       \ni \Gamma \vdash t : T
Expression Judgement \ni \vdash e:T
Reduction
                       \ni t \Rightarrow t
```

図 1 Syntax

2 型の構成

図 2 Formation rules

$$[\](X) = X$$

$$[\sigma, X \to T](X) = T$$

$$[\sigma, X' \to S](X) = X$$

$$[\sigma](I) = I$$

$$[\sigma](T_1 \oplus T_2) = [\sigma](T_1) \oplus [\sigma](T_2)$$

$$[\sigma](T_1 \otimes T_2) = [\sigma](T_1) \otimes [\sigma](T_2)$$

$$[\sigma](T_1 \to T_2) = [\sigma](T_1) \to [\sigma](T_2)$$

$$[\sigma](\mu X.T) = \mu X.[\sigma](T)$$

図 3 Type Substitution

図 4 Typing rules

図 5 Syntax-Directed Typing rules

$$\begin{aligned} & \textit{unify}(\{X = T\} \cup C) = \textit{unify}(\{X \to T\} \cap (s \mid X \to T] \\ & \textit{unify}(\{T = X\} \cup C) = \textit{unify}(\{X \to T\} \cap (s \mid X \to T] \\ & \textit{unify}(\{S \mid s S = T\} \mid e T_2) \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_2\} \cup C) = \textit{unify}(C \cup \{S \mid T_1, S_2 = T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_1, S_2 \ni C) = \textit{unify}(C \cup \{S \mid T_1, S_2 \ni T_1, S_2 \ni T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_1, S_2 \ni C) = \textit{unify}(C \cup \{S \mid T_1, S_2 \ni T_1, S_2 \ni T_2\}) \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_1, S_2 \ni C) = \textit{unify}(C \cup \{S \mid T_1, S_2 \ni C) = \textit{unify}(C \cup \{S \mid T_1, S_2 \ni C)] \\ & \textit{unify}(\{S \mid s S_2 = T_1 \ni T_1, S_2 \ni C) = \textit{unify}(C \cup \{S \mid T_1, S_2 \ni C) = \textit{unify}(C \cup \{S \mid T_1, S_2 \ni C) = \textit{unify}(C \cup \{S \mid T_1, S_2 \ni C)] \\ & \textit{unify}(\{S \mid s S \mid s S \mid T_1, S_1 \ni C) = \textit{unify}(C \cup \{S \mid s S_1, S_1 \ni C) = \textit{unify}(C \cup \{S \mid s S_1, S_1 \ni C) = \textit{unify}(C \cup$$

 $unify := Set of Constraint \rightarrow Substitution$

 $unify(\{\}) = []$

☑ 7 Type Inference rules

 $\frac{ \vdash f: X_1 \multimap X_2 \triangleright \mid C_1 \qquad \vdash e: X_1 \triangleright \mid C_2}{\vdash f @ e: V \triangleright \mid C_1 \cup C_2 \cup \{V = X_2\}} \operatorname{App}$

$$\begin{split} \llbracket\Theta \vdash T\rrbracket : \mathcal{V}^{|\Theta|} \rightarrow \mathcal{V} \\ \llbracket\Theta \vdash X_i\rrbracket := \Pi_i \\ \llbracket\Theta \vdash \Pi\rrbracket := K_I \\ \llbracket\Theta \vdash T_1 \oplus T_2\rrbracket := \oplus \circ \langle \llbracket\Theta \vdash T_1\rrbracket, \llbracket\Theta \vdash T_2\rrbracket \rangle \\ \llbracket\Theta \vdash T_1 \otimes T_2\rrbracket := \otimes \circ \langle \llbracket\Theta \vdash T_1\rrbracket, \llbracket\Theta \vdash T_2\rrbracket \rangle \\ \llbracket\Theta \vdash T_1 - T_2\rrbracket := \otimes \circ \langle \llbracket\Theta \vdash T_1\rrbracket, \llbracket\Theta \vdash T_2\rrbracket \rangle \\ \llbracket\Theta \vdash \mu X.T\rrbracket := \llbracket\Theta, X \vdash T\rrbracket^\sharp \\ \llbracket\Theta \vdash T[X \rightarrow U]\rrbracket := \llbracket\Theta \vdash T\rrbracket \circ \langle Id, \llbracket\Theta \vdash U\rrbracket \rangle \\ \llbracketT\rrbracket := \llbracket\vdash T\rrbracket (*) \in Obj(\mathcal{V}) \end{split}$$

図 8 Type Interpretation

図 9 Context Interpretation

$$\|T + t : T\| : \|T\| \to \|T\|$$

$$\|x : T + x : T\| : \|T\| \to \|T\|$$

$$\|T_1, T_2 + t : T\| : \|T\| \to \|T\| \to \|T\|$$

$$\|T_1, T_2 + t : T\| : \|T\| \to \|T\| \to \|T\|$$

$$\|T_1, T_2 + t : T\| : \|T\| \to \|T\| \to \|T\|$$

$$\|T_1 \to T_1 + T_2 \to T_2 + t : T\| : \|T\| \to \|T\| \to \|T\|$$

$$\|T_1 \to T_1 + T_2 \to T_2 + t : T\| : \|T\| \to \|T\| \to \|T\| \to \|T\|$$

$$\|T_1 \to T_1 + T_2 \to T_2 + t : T\| : \|T\| \to \|T\| \to \|T\| \to \|T\| \to \|T\|$$

$$\|T_1 \to T_1 \to T_2 + t : T\| : \|T\| \to \|T\| \to \|T\| \to \|T\| \to \|T\|$$

$$\|T_1 \to T_2 + T_1 \to T_2 + t : T\| : \|T\| \to \|T\| \to \|T\| \to \|T\| \to \|T\|$$

$$\|T_1, T_2 \to T_1 \to T_2 + t : T\| : \|T\| \to \|T\| \to \|T\| \to \|T\| \to \|T\|$$

$$\|T_1, T_2 \to T_1 \to T_2 + t : T\| : \|T\| \to \|T\| \to \|T\| \to \|T\| \to \|T\|$$

$$\|T_1, T_2 \to T_1 \to T_2 + t : T\| : \|T\| \to \|T\| \to \|T\| \to \|T\| \to \|T\| \to \|T\|$$

$$\|T_1, T_2 \to T_1 \to T_2 + t : T\| : \|T\| \to \|T\| \to \|T\| \to \|T\| \to \|T\| \to \|T\| \to \|T\|$$

$$\|T_1, T_2 \to T_1 \to T_2 \to T_1 \to T_2 \to \|T\| \to \|T\|$$

$$\|T_1, T_2 \to T_1 \to T_2 \to T_1 \to T_2 \to \|T\| \to \|T\|$$

図 10 Term Interpretation

$$\sigma_{1} \cup_{\perp}^{+} \sigma_{2} := \begin{cases} \bot & \text{if } \sigma_{1} = \bot \lor \sigma_{2} = \bot \\ \sigma_{1} \cup \sigma_{2} & \text{otherwise} \end{cases}$$

$$\sigma_{1} \cup_{\perp}^{\times} \sigma_{2} := \begin{cases} \bot & \text{if } \sigma_{1} = \bot \land \sigma_{2} = \bot \\ \sigma_{1} & \text{if } \sigma_{2} = \bot \\ \sigma_{2} & \text{if } \sigma_{1} = \bot \\ \sigma_{1} \cup \sigma_{2} & \text{otherwise} \end{cases}$$

$$x \triangleright t \qquad := [x \rightarrow t]$$

$$() \triangleright () \qquad := []$$

$$\text{inl } t \triangleright \text{inl } u \qquad := t \triangleright u$$

$$\text{inl } t \triangleright \text{inr } u \qquad := \bot$$

$$\text{inr } t \triangleright \text{inr } u \qquad := t \triangleright u$$

$$\text{inr } t \triangleright \text{inl } u \qquad := t \triangleright u$$

$$\text{inr } t \triangleright \text{inl } u \qquad := \bot$$

$$t_1 \times t_2 \triangleright u_1 \times u_2 \qquad := (t_1 \triangleright u_1) \cup_{\bot}^{\times} (t_2 \triangleright u_2)$$

$$t_1 \mapsto t_2 \triangleright u_1 \mapsto u_2 \qquad := (t_1 \triangleright u_1) \cup_{\bot}^{\times} (t_2 \triangleright u_2)$$

$$\text{fold }_T t \triangleright \text{fold }_T u \qquad := t \triangleright u$$

$$\text{trace }_T t \triangleright \text{trace }_T u \qquad := t \triangleright u$$

$$t \triangleright u_1 \parallel u_2 \qquad := (t \triangleright u_1) \cup_{\bot}^{+} (t \triangleright u_2)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\bot}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\bot}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\bot}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\bot}^{+} (t_2 \triangleright u)$$

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$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\bot}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\bot}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\bot}^{+} (t_2 \triangleright u)$$

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$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\bot}^{+} (t_2 \triangleright u)$$

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$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\bot}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\bot}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\bot}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\longleftarrow}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\longleftarrow}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\longleftarrow}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\longleftarrow}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\longleftarrow}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\longleftarrow}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_2 \triangleright u \qquad := (t_1 \triangleright u) \cup_{\longleftarrow}^{+} (t_2 \triangleright u)$$

$$t_1 \parallel t_1 \mapsto t_2 \mapsto t_1 \mapsto t_1 \mapsto t_2 \mapsto t_2 \mapsto t_1 \mapsto t_2 \mapsto t_$$

図 12 Constructing Environment

図 13 Consumption of Environment

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\emptyset^{\dagger} \Rightarrow \emptyset
                \emptyset \parallel t \Rightarrow t
                                                                                                                                                                  id^{\dagger} \implies id
(t_1 \parallel t_2) \parallel t_3 \implies t_1 \parallel (t_2 \parallel t_3)
                                                                                                                                                                  ()^{\dagger} \Rightarrow ()
              t_1 \parallel t_2 \implies t_2 \parallel t_1
                                                                                                                                                    (t_1 \parallel t_2)^{\dagger} \implies t_1^{\dagger} \parallel t_2^{\dagger}
                t \parallel t \implies t
                                                                                                                                                    (t_1 \ \mathring{\circ} \ t_2)^{\dagger} \implies t_1^{\dagger} \ \mathring{\circ} \ t_2^{\dagger}
               id \circ t \implies t
                                                                                                                                                    (\operatorname{inl} t)^{\dagger} \implies \operatorname{inl} t^{\dagger}
                 t \circ id \implies t
                                                                                                                                                    (\operatorname{inr} t)^{\dagger} \implies \operatorname{inr} t^{\dagger}
  (t_1 \circ t_2) \circ t_3 \implies t_1 \circ (t_2 \circ t_3)
                                                                                                                                                  (t_1 \times t_2)^{\dagger} \implies t_1^{\dagger} \times t_2^{\dagger}
                 \varnothing : t \implies \varnothing
                                                                                                                                                (t_1 \mapsto t_2)^{\dagger} \implies t_1^{\dagger} \mapsto t_2^{\dagger}
                 t \circ \varnothing \implies \varnothing
                                                                                                                                                (\operatorname{fold}_T t)^{\dagger} \implies \operatorname{fold}_T t^{\dagger}
 (t_1 \parallel t_2) \circ t_3 \implies (t_1 \circ t_3) \parallel (t_2 \circ t_3)
                                                                                                                                              (\operatorname{trace}_T t)^{\dagger} \implies \operatorname{trace}_T t^{\dagger}
 t_1 \circ (t_2 \parallel t_3) \implies (t_1 \circ t_2) \parallel (t_1 \circ t_3)
                                                                                                                                                             (t^{\dagger})^{\dagger} \implies t
                                                                                                                                              inl (t_1 \parallel t_2) \implies inl t_1 \parallel inl t_2
                inl \varnothing \Rightarrow \varnothing
                \operatorname{inr} \varnothing \Rightarrow \varnothing
                                                                                                                                              \operatorname{inr}(t_1 \parallel t_2) \implies \operatorname{inr} t_1 \parallel \operatorname{inr} t_2
               \emptyset \times t \Rightarrow \emptyset
                                                                                                                                           (t_1 \parallel t_2) \times t_3 \implies (t_1 \times t_3) \parallel (t_2 \times t_3)
              t \times \emptyset \implies \emptyset
                                                                                                                                           t_1 \times (t_2 \parallel t_3) \implies (t_1 \times t_2) \parallel (t_1 \times t_3)
             \varnothing\mapsto t\ \Longrightarrow\ \varnothing
                                                                                                                                         (t_1 \parallel t_2) \mapsto t_3 \implies (t_1 \mapsto t_3) \parallel (t_2 \mapsto t_3)
            t\mapsto\varnothing\implies\varnothing
                                                                                                                                         t_1 \mapsto (t_2 \parallel t_3) \implies (t_1 \mapsto t_2) \parallel (t_1 \mapsto t_3)
         fold_T \varnothing \Rightarrow \varnothing
                                                                                                                                       fold_T (t_1 \parallel t_2) \implies fold_T t_1 \parallel fold_T t_2
       \operatorname{trace}_T \varnothing \Rightarrow \varnothing
                                                t_1 \Rightarrow t_1' \implies t_2 \Rightarrow t_2' \implies t_1 \parallel t_2 \Rightarrow t_1' \parallel t_2'
                                                t_1 \Rightarrow t_1' \implies t_2 \Rightarrow t_2' \implies t_1 \circ t_2 \Rightarrow t_1' \circ t_2'
                                                                                         t \implies t' \implies \operatorname{inl} t \implies \operatorname{inl} t'
                                                                                          t \implies t' \implies \inf t \implies \inf t'
                                                t_1 \Rightarrow t_1' \implies t_2 \Rightarrow t_2' \implies t_1 \times t_2 \Rightarrow t_1' \times t_2'
                                                t_1 \; \Rrightarrow \; t_1' \; \Longrightarrow \; t_2 \; \Rrightarrow \; t_2' \; \Longrightarrow \; t_1 \mapsto t_2 \; \Rrightarrow \; t_1' \mapsto t_2'
                                                                                         t \Rightarrow t' \implies \text{fold}_T t \Rightarrow \text{fold}_T t'
                                                                                          t \Rightarrow t' \implies \operatorname{trace}_T t \Rightarrow \operatorname{trace}_T t'
                                               t_1 \Rightarrow t_1' \implies t_2 \Rightarrow t_2' \implies t_1 @ t_2 \Rightarrow t_1' @ t_2'
      (t_1 \mapsto t_2) \otimes t \implies (t_1 \triangleright t) t_2
                        \emptyset @ t \Rightarrow \emptyset
                                                                                                                                       (t_1 \parallel t_2) @ t \implies (t_1 @ t) \parallel (t_2 @ t)
                        t @ \emptyset \Rightarrow \emptyset
                                                                                                                                        t @ (t_1 || t_2) \implies (t @ t_1) || (t @ t_2)
                       id @ t \Rightarrow t
                                                                                                                                           t_1 \, {}_{0}^{\circ} \, t_2 \, @ \, t \implies t_2 \, @ \, (t_1 \, @ \, t)
                                                                                          t @ (inr u) \Rightarrow inr u' \implies (trace_T t) @ u \Rightarrow u'
  t @ (\operatorname{inr} u) \Rightarrow \operatorname{inl} u' \implies (\operatorname{trace}_T t) @ (\operatorname{inl} u') \Rightarrow u'' \implies (\operatorname{trace}_T t) @ u \Rightarrow u''
  t @ (\operatorname{inl} u) \Rightarrow \operatorname{inl} u' \implies (\operatorname{trace}_T t) @ (\operatorname{inl} u') \Rightarrow u'' \implies (\operatorname{trace}_T t) @ u \Rightarrow u''
                                                                                          t \otimes (\text{inl } u) \Rightarrow \text{inr } u' \implies (\text{trace}_T t) \otimes (\text{inl } u) \Rightarrow u'
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図 14 Reduction