$$\frac{}{v : \tau \vdash v : \tau}\ \mathrm{Id}$$

$$\frac{\Gamma \vdash \Delta \qquad \Gamma' \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\ \mathrm{Mix}$$

$$\frac{\Delta \vdash \Gamma}{\Gamma \vdash \Delta}\ \mathrm{Flip}$$

$$\mathrm{Ex_L}\ \frac{\Gamma, v' : \tau', v : \tau, \Gamma' \vdash \Delta}{\Gamma, v : \tau, v' : \tau', \Gamma' \vdash \Delta} \qquad \frac{\Gamma \vdash \Delta, v' : \tau', v : \tau, \Delta'}{\Gamma \vdash \Delta, v : \tau, v' : \tau', \Delta'}\ \mathrm{Ex_R}$$

$$\star_{\mathrm{L}}\ \frac{\Gamma \vdash v : \tau, \Delta}{\Gamma, \langle\rangle/v : \tau^\star \vdash \Delta} \qquad \frac{\Gamma, v : \tau \vdash \Delta}{\Gamma \vdash \langle\rangle/v : \tau^\star, \Delta}\ \star_{\mathrm{R}}$$

$$\iota_{\mathrm{L}}\ \frac{\Gamma \vdash \Delta}{\Gamma, \langle\rangle : \iota \vdash \Delta} \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \langle\rangle : \iota, \Delta}\ \iota_{\mathrm{R}}$$

$$\otimes_{\mathrm{L}}\ \frac{\Gamma, v_1 : \tau_1, v_2 : \tau_2 \vdash \Delta}{\Gamma, \langle v_1, v_2\rangle : \tau_1 \otimes \tau_2 \vdash \Delta} \qquad \frac{\Gamma \vdash v_1 : \tau_1, v_2 : \tau_2, \Delta}{\Gamma \vdash \langle v_1, v_2\rangle : \tau_1 \otimes \tau_2, \Delta}\ \otimes_{\mathrm{R}}$$

$$\oplus_{\mathrm{L}_1}\ \frac{\Gamma, v : \tau_1 \vdash \Delta}{\Gamma, \langle \text{left } v\rangle : \tau_1 \oplus \tau_2 \vdash \Delta} \qquad \frac{\Gamma \vdash v : \tau_1, \Delta}{\Gamma \vdash \langle \text{left } v\rangle : \tau_1 \oplus \tau_2, \Delta}\ \oplus_{\mathrm{R}_1}$$

$$\oplus_{\mathrm{L}_2}\ \frac{\Gamma, v : \tau_2 \vdash \Delta}{\Gamma, \langle \text{right } v\rangle : \tau_1 \oplus \tau_2 \vdash \Delta} \qquad \frac{\Gamma \vdash v : \tau_2, \Delta}{\Gamma \vdash \langle \text{right } v\rangle : \tau_1 \oplus \tau_2, \Delta}\ \oplus_{\mathrm{R}_2}$$

$$\mu_{\mathrm{L}}\ \frac{\Gamma, v : \tau[\mu x.\tau/x] \vdash \Delta}{\Gamma, \langle v\rangle : \mu x.\tau \vdash \Delta} \qquad \frac{\Gamma \vdash v : \tau[\mu x.\tau/x], \Delta}{\Gamma \vdash \langle v\rangle : \mu x.\tau, \Delta}\ \mu_{\mathrm{R}}$$

$$!_{\mathrm{L}}\ \frac{\Xi, v : \tau; \Gamma \vdash \quad ; \Theta}{\Xi; \Gamma, \{v\} : !\tau \vdash \quad ; \Theta} \qquad \frac{\Xi; \quad \vdash \Delta; v : \tau, \Theta}{\Xi; \quad \vdash \{v\} : !\tau, \Delta; \Theta}\ !_{\mathrm{R}}$$

$$\text{Classical-Ex}_{\mathrm{L}}\ \frac{\Xi, v' : \tau', v : \tau, \Xi'; \Gamma \vdash \Delta; \Theta}{\Xi, v : \tau, v' : \tau', \Xi'; \Gamma \vdash \Delta; \Theta} \qquad \frac{\Xi; \Gamma \vdash \Delta; \Theta, v' : \tau', v : \tau, \Theta'}{\Xi; \Gamma \vdash \Delta; \Theta, v : \tau, v' : \tau', \Theta'}\ \text{Classical-Ex}_{\mathrm{R}}$$

$$\mathrm{W_L}\ \frac{\Xi; \Gamma \vdash \Delta; \Theta}{\Xi, v : \tau; \Gamma \vdash \Delta; \Theta} \qquad \frac{\Xi; \Gamma \vdash \Delta; \Theta}{\Xi; \Gamma \vdash \Delta; v : \tau, \Theta}\ \mathrm{W_R}$$

$$\mathrm{C_L}\ \frac{\Xi, v : \tau, v : \tau; \Gamma \vdash \Delta; \Theta}{\Xi, v : \tau; \Gamma \vdash \Delta; \Theta} \qquad \frac{\Xi; \Gamma \vdash \Delta; v : \tau, v : \tau, \Theta}{\Xi; \Gamma \vdash \Delta; v : \tau, \Theta}\ \mathrm{C_R}$$

$$\Sigma_{\mathrm{L}}\ \frac{\Xi; \Gamma, \{v_1\} : !\tau_1, v_2 : \tau_2[v_1/x] \vdash \Delta; \Theta}{\Xi; \Gamma, \langle\{v_1\}, v_2\rangle : \Sigma_{x:!\tau_1}\tau_2 \vdash \Delta; \Theta} \qquad \frac{\Xi; \Gamma \vdash \{v_1\} : !\tau_1, v_2 : \tau_2[v_1/x], \Delta; \Theta}{\Xi; \Gamma \vdash \langle\{v_1\}, v_2\rangle : \Sigma_{x:!\tau_1}\tau_2, \Delta; \Theta}\ \Sigma_{\mathrm{R}}$$

$$\frac{\Xi \vdash \tau : \text{type}}{\Xi \vdash \tau^\star : \text{type}}\ \star_{\mathrm{T}}$$

$$\frac{}{\vdash \iota : \text{type}}\ \iota_{\mathrm{T}}$$

$$\frac{\Xi_1 \vdash \tau_1 : \text{type} \qquad \Xi_2 \vdash \tau_2 : \text{type}}{\Xi_1, \Xi_2 \vdash \tau_1 \otimes \tau_2 : \text{type}}\ \otimes_{\mathrm{T}}$$

$$\frac{\Xi_1 \vdash \tau_1 : \text{type} \qquad \Xi_2 \vdash \tau_2 : \text{type}}{\Xi_1, \Xi_2 \vdash \tau_1 \oplus \tau_2 : \text{type}}\ \oplus_{\mathrm{T}}$$

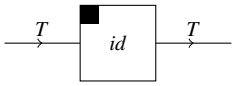
$$\frac{\Xi, x : \text{type} \vdash \tau : \text{type}}{\Xi \vdash \mu x.\tau : \text{type}}\ \mu_{\mathrm{T}}$$

$$\frac{\Xi \vdash \tau : \text{type}}{\Xi \vdash !\tau : \text{type}}\ !_{\mathrm{T}}$$

$$\frac{\Xi, x : \tau_1 \vdash \tau_2 : \text{type}}{\Xi \vdash \Sigma_{x:!\tau_1}\tau_2 : \text{type}}\ \Sigma_{\mathrm{T}}$$

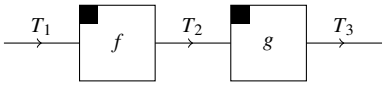
Identity

$$\frac{}{v \quad id \quad v}$$



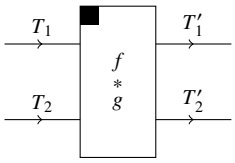
Composition

$$\frac{v_1 \quad f \quad v_2 \qquad v_2 \quad g \quad v_3}{v_1 \quad f;g \quad v_3}$$



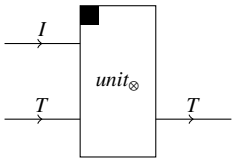
Product

$$\frac{v_1 \quad f \quad v_1' \qquad v_2 \quad g \quad v_2'}{(v_1 * v_2) \quad f * g \quad (v_1' * v_2')}$$



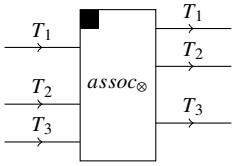
Unitor$_\otimes$

$$\frac{}{((\,) * v) \quad unit_\otimes \quad v}$$



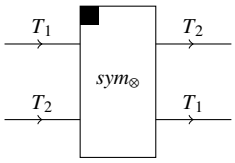
Associator$_\otimes$

$$\frac{}{(v_1 * (v_2 * v_3)) \quad assoc_\otimes \quad ((v_1 * v_2) * v_3)}$$



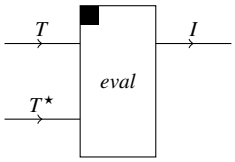
Symmetric$_\otimes$

$$\frac{}{(v_1 * v_2) \quad sym_\otimes \quad (v_2 * v_1)}$$



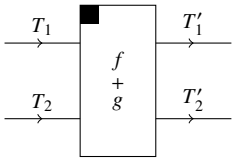
Evaluation$_\otimes$

$$\frac{}{(v * ()/v) \quad eval \quad ()}$$



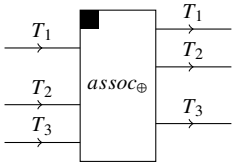
Sum

$$\frac{v_1 \quad f \quad v_1' \qquad v_2 \quad g \quad v_2'}{(v_1 + v_2) \quad f + g \quad (v_1' + v_2')}$$



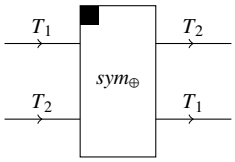
Associator$_\oplus$

$$\frac{}{(v_1 + (v_2 + v_3)) \quad assoc_\otimes \quad ((v_1 + v_2) + v_3)}$$



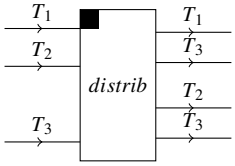
Symmetric$_\oplus$

$$\frac{}{(v_1 + v_2) \quad sym_\oplus \quad (v_1 + v_2)}$$



Distribution

$$\frac{}{((v_1 + v_2) * v_3) \quad distrib \quad ((v_1 * v_3) + (v_2 * v_3))}$$



Fold

$$\frac{}{\langle v \rangle \quad fold \quad v}$$