Formation rules for Types

$$\frac{}{X \vdash X}\ var \qquad \frac{}{\Gamma \vdash I}\ I \qquad \frac{\Gamma \vdash T_1 \qquad \Gamma \vdash T_2}{\Gamma \vdash T_1 \otimes T_2}\ \otimes \qquad \frac{\Gamma \vdash T_1 \qquad \Gamma \vdash T_2}{\Gamma \vdash T_1 \oplus T_2}\ \oplus \qquad \frac{\Gamma \vdash T}{\Gamma \vdash T^\star}\ \star \qquad \frac{X \vdash T}{\Gamma \vdash \mu\, X.T}\ \mu$$

Formation rules for Values

$$\frac{}{x : X \vdash x : X}\ var \qquad \frac{}{\Gamma \vdash * : I}\ I \qquad \frac{\Gamma \vdash v : T}{\Gamma \vdash v^\star : T^\star}\ \star \qquad \frac{x : X \vdash v : T}{\Gamma \vdash \mu\, x.v : \mu\, X.T}\ \mu$$

$$\frac{\Gamma \vdash v_1 : T_1 \qquad \Gamma \vdash v_2 : T_2}{\Gamma \vdash v_1 \times v_2 : T_1 \otimes T_2}\ \otimes$$

$$\frac{\Gamma \vdash v_1 : T_1}{\Gamma \vdash inl\ v_1 : T_1 \oplus T_2}\ \oplus_l \qquad \frac{\Gamma \vdash v_1 : T_1 \qquad \Gamma \vdash v_2 : T_2}{\Gamma \vdash v_1 + v_2 : T_1 \oplus T_2}\ \oplus \qquad \frac{\Gamma \vdash v_2 : T_2}{\Gamma \vdash inr\ v_2 : T_1 \oplus T_2}\ \oplus_r$$

Formation rules for Functions

$$\frac{}{\vdash id : T \leftrightarrow T}\ id \qquad \frac{\vdash f_1 : T_1 \leftrightarrow T_2 \qquad \vdash f_2 : T_2 \leftrightarrow T_3}{\vdash f_1 \fatsemi f_2 : T_1 \leftrightarrow T_3}\ \fatsemi$$

$$\frac{\vdash f_1 : T_1 \leftrightarrow T_3 \qquad \vdash f_2 : T_2 \leftrightarrow T_4}{\vdash f_1 \times f_2 : T_1 \otimes T_2 \leftrightarrow T_3 \otimes T_4}\ \otimes \qquad \frac{\vdash f_1 : T_1 \leftrightarrow T_3 \qquad \vdash f_2 : T_2 \leftrightarrow T_4}{\vdash f_1 + f_2 : T_1 \oplus T_2 \leftrightarrow T_3 \oplus T_4}\ \oplus$$

$$\frac{\vdash f : T_1 \leftrightarrow T_2}{\vdash f^\star : T_2^\star \leftrightarrow T_1^\star}\ \star \qquad \frac{\vdash f : T_1 \leftrightarrow T_2}{\vdash f^\dagger : T_2 \leftrightarrow T_1}\ \dagger$$

$$\frac{}{\vdash assoc_\otimes : T_1 \otimes (T_2 \otimes T_3) \leftrightarrow (T_1 \otimes T_2) \otimes T_3}\ assoc_\otimes \qquad \frac{}{\vdash assoc_\oplus : T_1 \oplus (T_2 \oplus T_3) \leftrightarrow (T_1 \oplus T_2) \oplus T_3}\ assoc_\oplus$$

$$\frac{}{\vdash sym_\otimes : T_1 \otimes T_2 \leftrightarrow T_2 \otimes T_1}\ sym_\otimes \qquad \frac{}{\vdash sym_\oplus : T_1 \oplus T_2 \leftrightarrow T_2 \oplus T_1}\ sym_\oplus$$

$$\frac{}{\vdash unit_\otimes : I \otimes T \leftrightarrow T}\ unit_\otimes \qquad \frac{}{\vdash distrib : (T_1 \oplus T_2) \otimes T_3 \leftrightarrow (T_1 \otimes T_3) \oplus (T_2 \otimes T_3)}\ distrib$$

$$\frac{}{\vdash eval : T \otimes T^\star \leftrightarrow I}\ eval \qquad \frac{}{\vdash unfold : \mu\, X.T \leftrightarrow T[\mu\, X.T/X]}\ unfold$$

Transformation rules

$$\frac{}{\vdash id : v \leftrightarrow v}\ id \qquad \frac{\vdash f_1 : v_1 \leftrightarrow v_2 \qquad \vdash f_2 : v_2 \leftrightarrow v_3}{\vdash f_1 \fatsemi f_2 : v_1 \leftrightarrow v_3}\ \fatsemi$$

$$\frac{\vdash f_1 : v_1 \leftrightarrow v_3 \qquad \vdash f_2 : v_2 \leftrightarrow v_4}{\vdash f_1 \otimes f_2 : v_1 \times v_2 \leftrightarrow v_3 \times v_4}\ \otimes \qquad \frac{\vdash f_1 : v_1 \leftrightarrow v_3 \qquad \vdash f_2 : v_2 \leftrightarrow v_4}{\vdash f_1 + f_2 : v_1 + v_2 \leftrightarrow v_3 + v_4}\ \oplus$$

$$\frac{\vdash f_1 : v_1 \leftrightarrow v_3}{\vdash f_1 + f_2 : inl\ v_1 \leftrightarrow inl\ v_3}\ \oplus_l \qquad \frac{\vdash f_2 : v_2 \leftrightarrow v_4}{\vdash f_1 + f_2 : inr\ v_2 \leftrightarrow inr\ v_4}\ \oplus_r$$

$$\frac{\vdash f : v_1 \leftrightarrow v_2}{\vdash f^\star : v_2^\star \leftrightarrow v_1^\star}\ \star \qquad \frac{\vdash f : v_1 \leftrightarrow v_2}{\vdash f^\dagger : v_2 \leftrightarrow v_1}\ \dagger$$

$$\frac{}{\vdash assoc_\otimes : v_1 \times (v_2 \times v_3) \leftrightarrow (v_1 \times v_2) \times v_3}\ assoc_\otimes \qquad \frac{}{\vdash assoc_\oplus : v_1 + (v_2 + v_3) \leftrightarrow (v_1 + v_2) + v_3}\ assoc_\oplus$$

$$\frac{}{\vdash assoc_\oplus : inl\ v_1 \leftrightarrow inl\ (inl\ v_1)}\ assoc_{\oplus_1} \qquad \frac{}{\vdash assoc_\oplus : inr\ (v_2 + v_3) \leftrightarrow (inr\ v_2) + v_3}\ assoc_{\oplus_{23}}$$

$$\frac{}{\vdash assoc_\oplus : inr\ (inl\ v_2) \leftrightarrow inl\ (inr\ v_2)}\ assoc_{\oplus_2} \qquad \frac{}{\vdash assoc_\oplus : v_1 + (inr\ v_3) \leftrightarrow (inr\ v_1) + v_3}\ assoc_{\oplus_{13}}$$

$$\frac{}{\vdash assoc_\oplus : inr\ (inr\ v_3) \leftrightarrow inr\ v_3}\ assoc_{\oplus_3} \qquad \frac{}{\vdash assoc_\oplus : v_1 + (inl\ v_2) \leftrightarrow inl\ (v_1 + v_2)}\ assoc_{\oplus_{12}}$$

$$\frac{}{\vdash sym_\otimes : v_1 \times v_2 \leftrightarrow v_2 \times v_1}\ sym_\otimes \qquad \frac{}{\vdash sym_\oplus : v_1 + v_2 \leftrightarrow v_2 + v_1}\ sym_\oplus$$

$$\frac{}{\vdash sym_\oplus : inl\ v_1 \leftrightarrow inr\ v_1}\ sym_{\oplus_l} \qquad \frac{}{\vdash sym_\oplus : inr\ v_2 \leftrightarrow inl\ v_2}\ sym_{\oplus_r}$$

$$\frac{}{\vdash unit_\otimes : unit \times v \leftrightarrow v}\ unit_\otimes \qquad \frac{}{\vdash distrib : (v_1 + v_2) \times v_3 \leftrightarrow (v_1 \times v_3) + (v_2 \times v_3)}\ distrib$$

$$\frac{}{\vdash distrib : (inl\ v_1) \times v_3 \leftrightarrow inl\ (v_1 \times v_3)}\ distrib_l \qquad \frac{}{\vdash distrib : (inr\ v_2) \times v_3 \leftrightarrow inr\ (v_2 \times v_3)}\ distrib_r$$

$$\frac{}{\vdash eval : v \otimes v^\star \leftrightarrow unit}\ eval \qquad \frac{}{\vdash unfold : \mu\, x.v \leftrightarrow v[\mu\, x.v/x]}\ unfold$$

Substitution rules

$$
\begin{aligned}
X[S/X] &= S \\
I[S/X] &= I \\
T_1 \otimes T_2[S/X] &= T_1[S/X] \otimes T_2[S/X] \\
T_1 \oplus T_2[S/X] &= T_1[S/X] \oplus T_2[S/X] \\
T^\star[S/X] &= T[S/X]^\star \\
\mu\, Y.T[S/X] &= \mu\, Y.(T[S/X])
\end{aligned}
$$

$$
\begin{aligned}
x[u/x] &= u \\
unit[u/x] &= unit \\
v_1 \times v_2[u/x] &= v_1[u/x] \times v_2[u/x] \\
v_1 + v_2[u/x] &= v_1[u/x] + v_2[u/x] \\
inl\ v_1[u/x] &= inl\ (v_1[u/x]) \\
inr\ v_2[u/x] &= inr\ (v_2[u/x]) \\
v^\star[u/x] &= v[u/x] \\
\mu\, y.v[u/x] &= \mu\, y.(v[u/x])
\end{aligned}
$$