$$\frac{}{v : \tau \vdash v : \tau}\,\text{Id}$$

$$\frac{\Gamma \vdash \Delta \qquad \Gamma' \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}\,\text{Mix}$$

$$\frac{\Delta \vdash \Gamma}{\Gamma \vdash \Delta}\,\text{Flip}$$

$$\text{Ex}_\text{L}\,\frac{\Gamma, v' : \tau', v : \tau, \Gamma' \vdash \Delta}{\Gamma, v : \tau, v' : \tau', \Gamma' \vdash \Delta} \qquad \frac{\Gamma \vdash \Delta, v' : \tau', v : \tau, \Delta'}{\Gamma \vdash \Delta, v : \tau, v' : \tau', \Delta'}\,\text{Ex}_\text{R}$$

$$\star_\text{L}\,\frac{\Gamma \vdash v : \tau, \Delta}{\Gamma, \langle\rangle / v : \tau^\star \vdash \Delta} \qquad \frac{\Gamma, v : \tau \vdash \Delta}{\Gamma \vdash \langle\rangle / v : \tau^\star, \Delta}\,\star_\text{R}$$

$$\iota_\text{L}\,\frac{\Gamma \vdash \Delta}{\Gamma, \langle\rangle : \iota \vdash \Delta} \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \langle\rangle : \iota, \Delta}\,\iota_\text{R}$$

$$\otimes_\text{L}\,\frac{\Gamma, v_1 : \tau_1, v_2 : \tau_2 \vdash \Delta}{\Gamma, \langle v_1, v_2\rangle : \tau_1 \otimes \tau_2 \vdash \Delta} \qquad \frac{\Gamma \vdash v_1 : \tau_1, v_2 : \tau_2, \Delta}{\Gamma \vdash \langle v_1, v_2\rangle : \tau_1 \otimes \tau_2, \Delta}\,\otimes_\text{R}$$

$$\oplus_{\text{L}_1}\,\frac{\Gamma, v : \tau_1 \vdash \Delta}{\Gamma, \langle \text{left } v\rangle : \tau_1 \oplus \tau_2 \vdash \Delta} \qquad \frac{\Gamma \vdash v : \tau_1, \Delta}{\Gamma \vdash \langle \text{left } v\rangle : \tau_1 \oplus \tau_2, \Delta}\,\oplus_{\text{R}_1}$$

$$\oplus_{\text{L}_2}\,\frac{\Gamma, v : \tau_2 \vdash \Delta}{\Gamma, \langle \text{right } v\rangle : \tau_1 \oplus \tau_2 \vdash \Delta} \qquad \frac{\Gamma \vdash v : \tau_2, \Delta}{\Gamma \vdash \langle \text{right } v\rangle : \tau_1 \oplus \tau_2, \Delta}\,\oplus_{\text{R}_2}$$

$$\mu_\text{L}\,\frac{\Gamma, v : \tau[\mu x.\tau/x] \vdash \Delta}{\Gamma, \langle v\rangle : \mu x.\tau \vdash \Delta} \qquad \frac{\Gamma \vdash v : \tau[\mu x.\tau/x], \Delta}{\Gamma \vdash \langle v\rangle : \mu x.\tau, \Delta}\,\mu_\text{R}$$

$$!_\text{L}\,\frac{\Xi, v : \tau; \Gamma \vdash \quad ; \Theta}{\Xi; \Gamma, \{v\} : !\tau \vdash \quad ; \Theta} \qquad \frac{\Xi; \quad \vdash \Delta; v : \tau, \Theta}{\Xi; \quad \vdash \{v\} : !\tau, \Delta; \Theta}\,!_\text{R}$$

$$\text{Classical-Ex}_\text{L}\,\frac{\Xi, v' : \tau', v : \tau, \Xi'; \Gamma \vdash \Delta; \Theta}{\Xi, v : \tau, v' : \tau', \Xi'; \Gamma \vdash \Delta; \Theta} \qquad \frac{\Xi; \Gamma \vdash \Delta; \Theta, v' : \tau', v : \tau, \Theta'}{\Xi; \Gamma \vdash \Delta; \Theta, v : \tau, v' : \tau', \Theta'}\,\text{Classical-Ex}_\text{R}$$

$$\text{W}_\text{L}\,\frac{\Xi; \Gamma \vdash \Delta; \Theta}{\Xi, v : \tau; \Gamma \vdash \Delta; \Theta} \qquad \frac{\Xi; \Gamma \vdash \Delta; \Theta}{\Xi; \Gamma \vdash \Delta; v : \tau, \Theta}\,\text{W}_\text{R}$$

$$\text{C}_\text{L}\,\frac{\Xi, v : \tau, v : \tau; \Gamma \vdash \Delta; \Theta}{\Xi, v : \tau; \Gamma \vdash \Delta; \Theta} \qquad \frac{\Xi; \Gamma \vdash \Delta; v : \tau, v : \tau, \Theta}{\Xi; \Gamma \vdash \Delta; v : \tau, \Theta}\,\text{C}_\text{R}$$

$$\Sigma_\text{L}\,\frac{\Xi; \Gamma, \{v_1\} : !\tau_1, v_2 : \tau_2[v_1/x] \vdash \Delta; \Theta}{\Xi; \Gamma, \langle\{v_1\}, v_2\rangle : \Sigma_{x:!\tau_1}\tau_2 \vdash \Delta; \Theta} \qquad \frac{\Xi; \Gamma \vdash \{v_1\} : !\tau_1, v_2 : \tau_2[v_1/x], \Delta; \Theta}{\Xi; \Gamma \vdash \langle\{v_1\}, v_2\rangle : \Sigma_{x:!\tau_1}\tau_2, \Delta; \Theta}\,\Sigma_\text{R}$$

$$\frac{\Xi \vdash \tau : \text{type}}{\Xi \vdash \tau^\star : \text{type}}\,\star_\text{T}$$

$$\frac{}{\vdash \iota : \text{type}}\,\iota_\text{T}$$

$$\frac{\Xi_1 \vdash \tau_1 : \text{type} \qquad \Xi_2 \vdash \tau_2 : \text{type}}{\Xi_1, \Xi_2 \vdash \tau_1 \otimes \tau_2 : \text{type}}\,\otimes_\text{T}$$

$$\frac{\Xi_1 \vdash \tau_1 : \text{type} \qquad \Xi_2 \vdash \tau_2 : \text{type}}{\Xi_1, \Xi_2 \vdash \tau_1 \oplus \tau_2 : \text{type}}\,\oplus_\text{T}$$

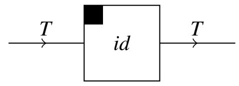
$$\frac{\Xi, x : \text{type} \vdash \tau : \text{type}}{\Xi \vdash \mu x.\tau : \text{type}}\,\mu_\text{T}$$

$$\frac{\Xi \vdash \tau : \text{type}}{\Xi \vdash !\tau : \text{type}}\,!_\text{T}$$

$$\frac{\Xi, x : \tau_1 \vdash \tau_2 : \text{type}}{\Xi \vdash \Sigma_{x:!\tau_1}\tau_2 : \text{type}}\,\Sigma_\text{T}$$

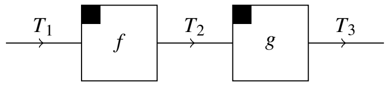
Identity

$$\frac{}{v \quad id \quad v}$$



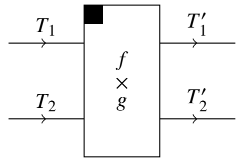
Composition

$$\frac{v_1 \quad f \quad v_2 \qquad v_2 \quad g \quad v_3}{v_1 \quad f;g \quad v_3}$$



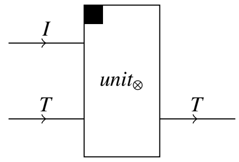
Product

$$\frac{v_1 \quad f \quad v_1' \qquad v_2 \quad g \quad v_2'}{\langle v_1, v_2\rangle \quad f \times g \quad \langle v_1', v_2'\rangle}$$



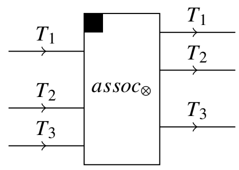
Unitor$_\otimes$

$$\frac{}{\langle\langle\rangle, v\rangle \quad unit_\otimes \quad v}$$



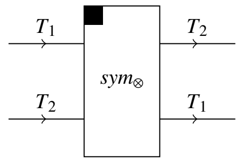
Associator$_\otimes$

$$\frac{}{\langle v_1, \langle v_2, v_3\rangle\rangle \quad assoc_\otimes \quad \langle\langle v_1, v_2\rangle, v_3\rangle}$$



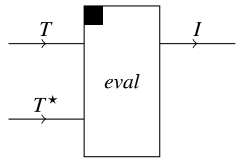
Symmetric$_\otimes$

$$\frac{}{\langle v_1, v_2\rangle \quad sym_\otimes \quad \langle v_2, v_1\rangle}$$



Evaluation$_\otimes$

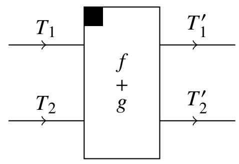
$$\frac{}{\langle v, \langle\rangle / v\rangle \quad eval \quad \langle\rangle}$$



Sum

$$\frac{v \quad f \quad v'}{\langle \text{left } v\rangle \quad f+g \quad \langle \text{left } v'\rangle}$$

$$\frac{v \quad g \quad v'}{\langle \text{right } v\rangle \quad f+g \quad \langle \text{right } v'\rangle}$$

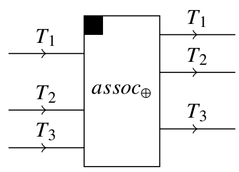


Associator$_\oplus$

$$\frac{}{\langle \text{left } v\rangle \quad assoc_\oplus \quad \langle \text{left } \langle \text{left } v\rangle\rangle}$$

$$\frac{}{\langle \text{right } \langle \text{left } v\rangle\rangle \quad assoc_\oplus \quad \langle \text{left } \langle \text{right } v\rangle\rangle}$$

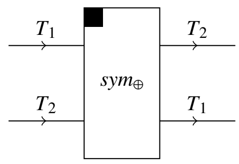
$$\frac{}{\langle \text{right } \langle \text{right } v\rangle\rangle \quad assoc_\oplus \quad \langle \text{right } v\rangle}$$



Symmetric$_\oplus$

$$\frac{}{\langle \text{left } v\rangle \quad sym_\oplus \quad \langle \text{right } v\rangle}$$

$$\frac{}{\langle \text{right } v\rangle \quad sym_\oplus \quad \langle \text{left } v\rangle}$$

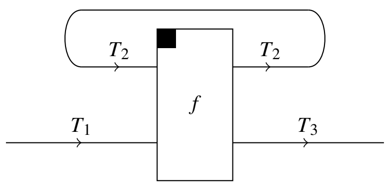


Trace$_\oplus$

$$\frac{\langle \text{right } v_1\rangle \quad f; loop\ f \quad \langle \text{right } v_3\rangle}{v_1 \quad trace\ f \quad v_3}$$

$$\frac{\langle \text{left } v_2\rangle \quad f; loop\ f \quad v}{\langle \text{left } v_2\rangle \quad loop\ f \quad v}$$

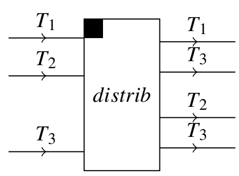
$$\frac{}{\langle \text{right } v_3\rangle \quad loop\ f \quad \langle \text{right } v_3\rangle}$$



Distribution

$$\frac{}{\langle\langle \text{left } v_1\rangle, v_3\rangle \quad distrib \quad \langle \text{left } \langle v_1, v_3\rangle\rangle}$$

$$\frac{}{\langle\langle \text{right } v_2\rangle, v_3\rangle \quad distrib \quad \langle \text{right } \langle v_2, v_3\rangle\rangle}$$



Fold

$$\frac{}{\langle v\rangle \quad fold \quad v}$$