

Type Formation rules

$$\frac{}{\Gamma \vdash I} \quad \frac{\Gamma \vdash T_1 \quad \Gamma \vdash T_2}{\Gamma \vdash T_1 \oplus T_2} \quad \frac{\Gamma \vdash T_1 \quad \Gamma \vdash T_2}{\Gamma \vdash T_1 \otimes T_2} \quad \frac{\Gamma \vdash T_1 \quad \Gamma \vdash T_2}{\Gamma \vdash T_1 \wp T_2} \quad \frac{\Gamma \vdash T_1 \quad \Gamma \vdash T_2}{\Gamma \vdash T_1 \multimap T_2} \quad \frac{}{\Gamma, X \vdash X} \quad \frac{\Gamma, X \vdash T}{\Gamma \vdash \mu X. T}$$

Typing rules for Values

$$\frac{}{x : T \vdash x : T} \text{ Variable}$$

$$I_L \frac{\Gamma \vdash \Delta}{\Gamma, () : I \vdash \Delta} \quad I_R \frac{\Gamma \vdash \Delta}{\Gamma \vdash () : I, \Delta}$$

$$\oplus_{L_l} \frac{\Gamma, v_1 : T_1 \vdash \Delta}{\Gamma, \text{inl } v_1 : T_1 \oplus T_2 \vdash \Delta} \quad \oplus_{R_l} \frac{\Gamma \vdash v_1 : T_1, \Delta}{\Gamma \vdash \text{inl } v_1 : T_1 \oplus T_2, \Delta}$$

$$\oplus_{L_r} \frac{\Gamma, v_2 : T_2 \vdash \Delta}{\Gamma, \text{inr } v_2 : T_1 \oplus T_2 \vdash \Delta} \quad \oplus_{R_r} \frac{\Gamma \vdash v_2 : T_2, \Delta}{\Gamma \vdash \text{inr } v_2 : T_1 \oplus T_2, \Delta}$$

$$\otimes_L \frac{\Gamma, v_1 : T_1, v_2 : T_2 \vdash \Delta}{\Gamma, v_1 \times v_2 : T_1 \otimes T_2 \vdash \Delta} \quad \otimes_R \frac{\Gamma_1 \vdash v_1 : T_1, \Delta_1 \quad \Gamma_2 \vdash v_2 : T_2, \Delta_2}{\Gamma_1; \Gamma_2 \vdash v_1 \times v_2 : T_1 \otimes T_2, \Delta_1; \Delta_2}$$

$$\wp_L \frac{\Gamma_1, v_1 : T_1 \vdash \Delta_1 \quad \Gamma_2, v_2 : T_2 \vdash \Delta_2}{\Gamma_1; \Gamma_2, v_1 \parallel v_2 : T_1 \wp T_2 \vdash \Delta_1; \Delta_2} \quad \wp_R \frac{\Gamma \vdash v_1 : T_1, v_2 : T_2, \Delta}{\Gamma \vdash v_1 \parallel v_2 : T_1 \wp T_2, \Delta}$$

$$\multimap_L \frac{\Gamma_1 \vdash v_1 : T_1, \Delta_1 \quad \Gamma_2, v_2 : T_2 \vdash \Delta_2}{\Gamma_1; \Gamma_2, v_1 \mapsto v_2 : T_1 \multimap T_2 \vdash \Delta_1; \Delta_2} \quad \multimap_R \frac{\Gamma, v_1 : T_1 \vdash v_2 : T_2, \Delta}{\Gamma \vdash v_1 \mapsto v_2 : T_1 \multimap T_2, \Delta}$$

$$\mu_L \frac{\Gamma, v : T[\mu X. T/X] \vdash \Delta}{\Gamma, \text{fold } v : \mu X. T \vdash \Delta} \quad \mu_R \frac{\Gamma \vdash v : T[\mu X. T/X], \Delta}{\Gamma \vdash \text{fold } v : \mu X. T, \Delta}$$

Typing rules for Terms

$$\text{Application} \frac{\vdash v_2 : T_1 \multimap T_2 \quad \vdash v_1 : T_1}{\vdash v_2 v_1 : T_2} \quad \text{Composition} \frac{\vdash v_1 : T_1 \multimap T_2 \quad \vdash v_2 : T_2 \multimap T_3}{\vdash v_1 \circ v_2 : T_1 \multimap T_3}$$

$$\text{Transpose} \frac{\vdash v : T_2 \multimap T_1}{\vdash v^\dagger : T_1 \multimap T_2} \quad \text{Measurement} \frac{\vdash v : T_1 \wp T_2}{\vdash \text{measure } v : T_1 \oplus T_2}$$

Substitution

$$Y[S/X] = \begin{cases} S & \text{if } Y = X \\ Y & \text{otherwise} \end{cases}$$

$$I[S/X] = I$$

$$T_1 \oplus T_2[S/X] = T_1[S/X] \oplus T_2[S/X]$$

$$T_1 \otimes T_2[S/X] = T_1[S/X] \otimes T_2[S/X]$$

$$T_1 \wp T_2[S/X] = T_1[S/X] \wp T_2[S/X]$$

$$T_1 \multimap T_2[S/X] = T_1[S/X] \multimap T_2[S/X]$$

$$\mu Y. T[S/X] = \mu Y. (T[S/X])$$