Santa Claus meets Makespan and Matroids: Algorithms and Reductions

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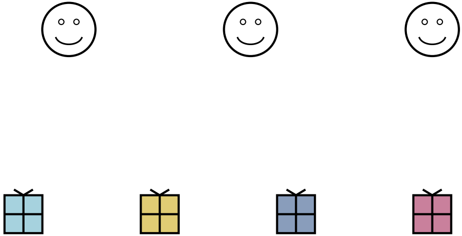
MAPSP 2024

The Santa Claus Problem

m children

n gifts



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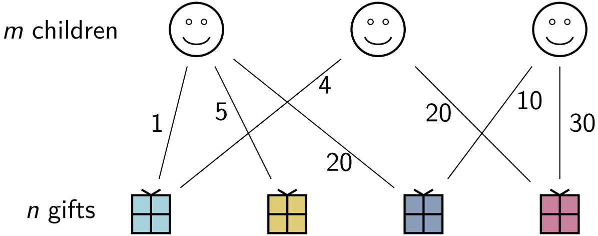

Task: assign gifts; maximize the happiness of the unhappiest child.

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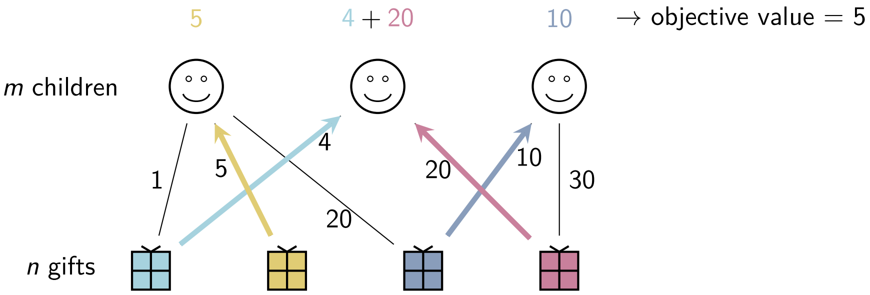

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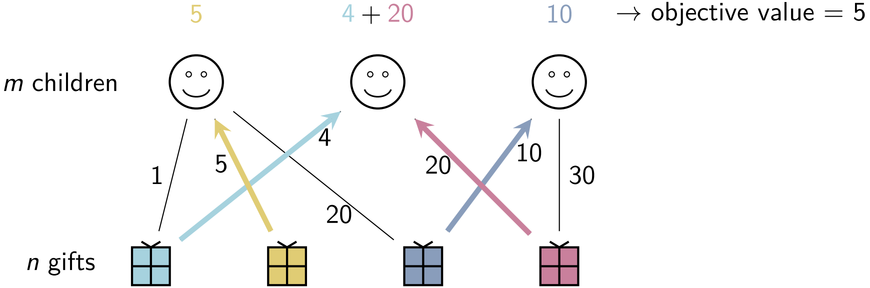

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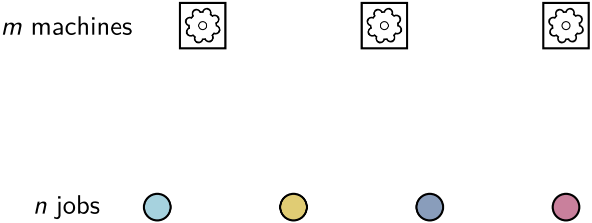
Big open question: Is there a constant factor approximation algorithm?

The Makespan Problem ($R \,||\, C_{\max}$)



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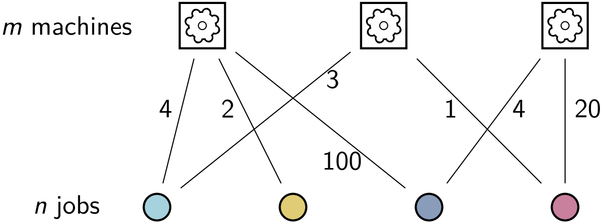

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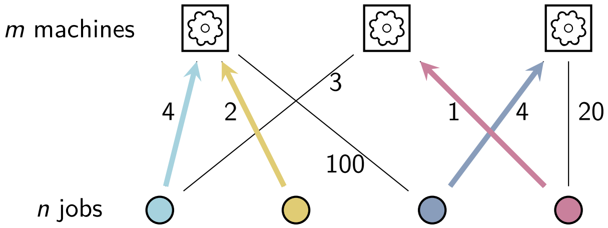

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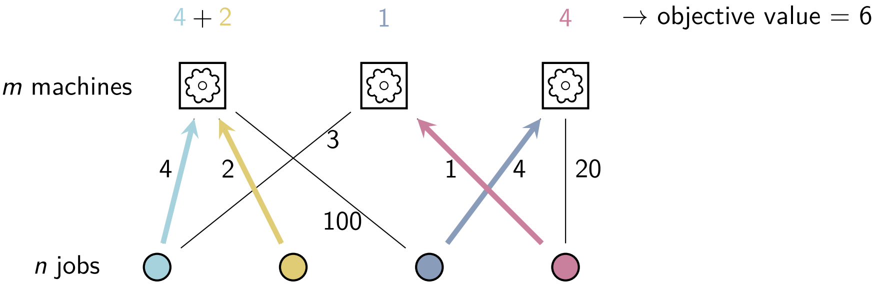

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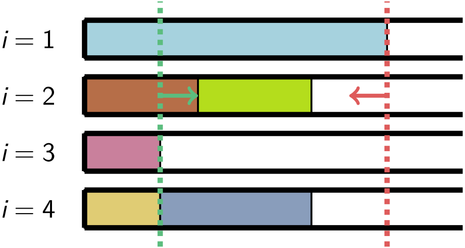
Resource Allocation Problems: Santa Claus and Makespan

Task: assign n gifts / jobs

to m children / machines with (unrelated) values v_{ij} / sizes p_{ij}.



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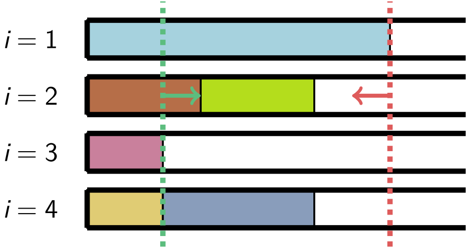

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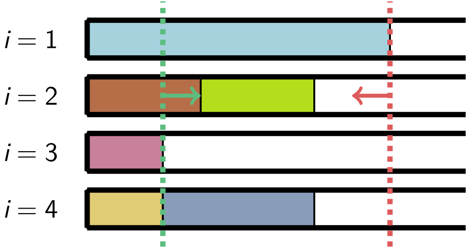

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- 2V-SANTACLAUS / 2S-MAKESPAN: $v_{ij} \in \{0, u, w\}$ / $p_{ij} \in \{\infty, u, w\}$.

Related Work and Similarities

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  [Lenstra, Shmoys, Tardos '90][ST '93]
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configuration LP

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Corollary

For 2S-$\textsc{Makespan}$, there is a

- poly-time $(2 - 1/n^{\epsilon})$-approximation for any $\epsilon > 0$, and a
- quasi-poly-time $(2 - 1/\mathrm{polylog}(n))$-approximation.

Improves in many cases over current SOTA of $2 - 1/m$. [Shchepin and Vakhania '05]

Reduction Framework and Roadmap

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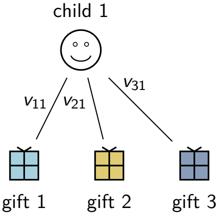
For polynomial reduction: job types, rounding strategies and monotone configurations $\Rightarrow$ loses factor of $1 + \epsilon$.

Reduction to the MAKESPAN Instance I'

$\mathcal{C}_1 \supseteq \{\{1\}, \{2, 3\}\}$



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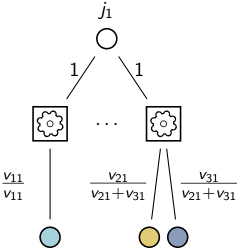

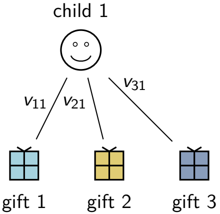

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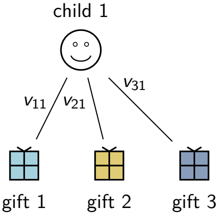

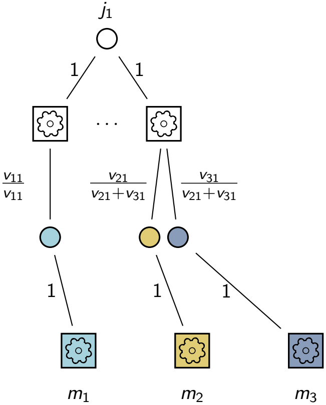

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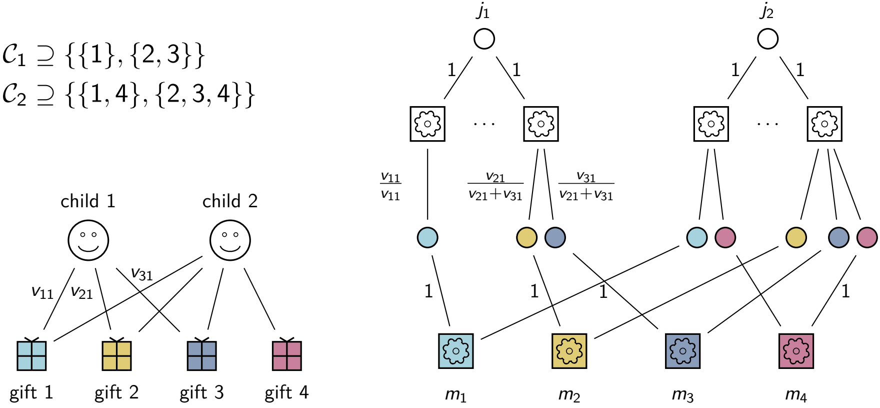

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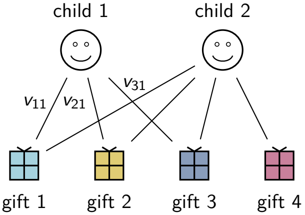

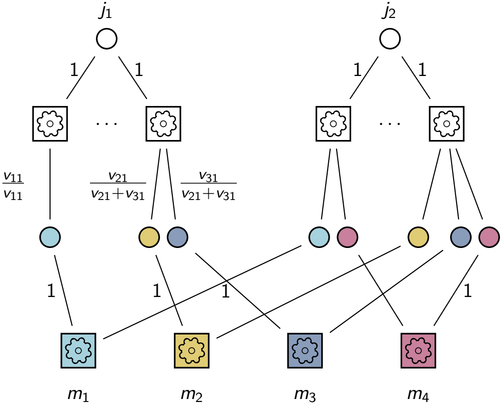

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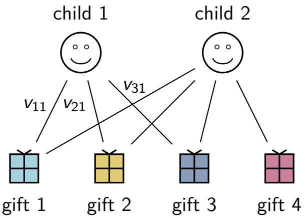

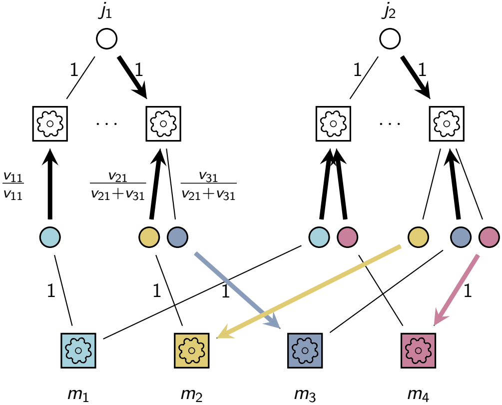

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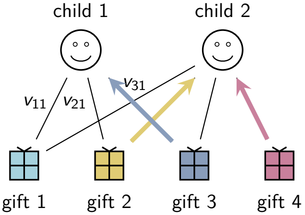

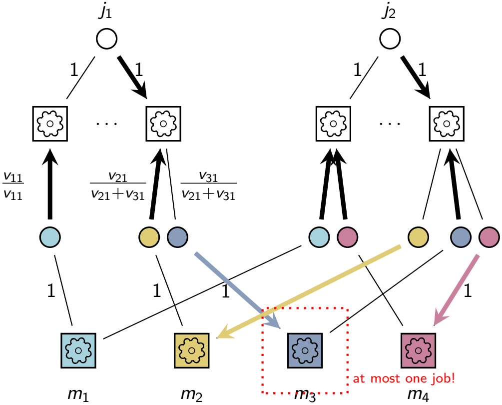

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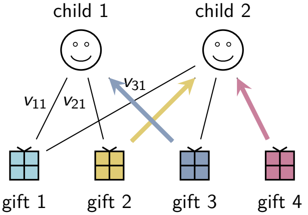

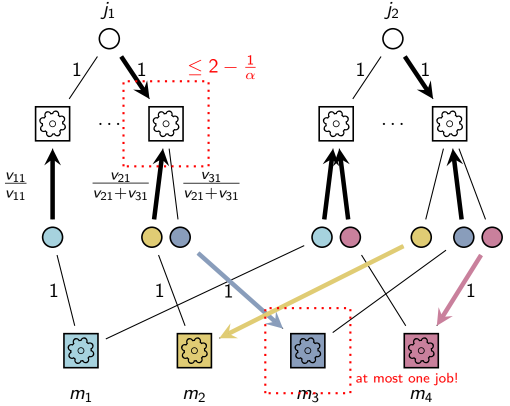

Proof steps:

(i) $\text{OPT}(I) \geq 1 \Longrightarrow \text{OPT}(I') \leq 1$ (omitted)

(ii) Approximate solution for I' has a makespan of at most $2 - \frac{1}{\alpha}$.

(iii) Child 1 receives a total value of $v_{31} \geq \frac{v_{31}}{v_{21}+v_{31}} \geq 2 - (2 - \frac{1}{\alpha}) = \frac{1}{\alpha}$.

Reductions within Special Cases

Theorem 1

better-than-2 for MAKESPAN $\implies$ constant for SANTACLAUS.

Reductions within Special Cases

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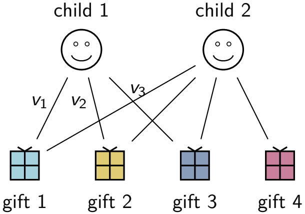
Can we also stay within special instance classes?

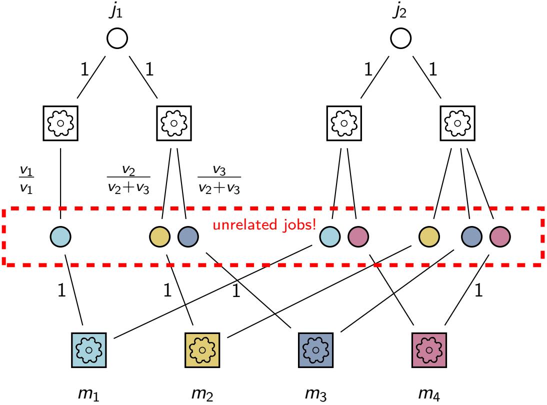
better-than-2 for restricted MAKESPAN

$\stackrel{?????}{\Longrightarrow}$ constant for restricted SANTACLAUS

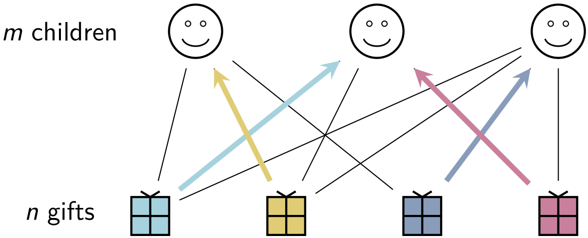
(Recall: restricted SANTACLAUS / MAKESPAN: $v_{ij} \in \{0, v_j\}$ / $p_{ij} \in \{\infty, p_j\}$)

Applying the Reduction to Restricted SantaClaus



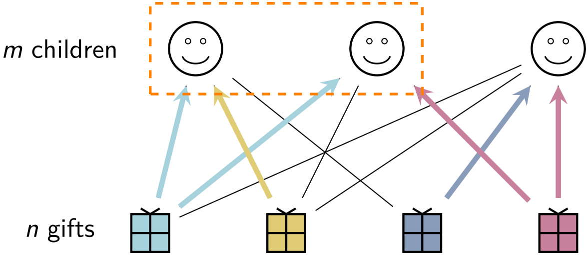


Idea for a Generalization



Idea for a Generalization

blue gift can "copy" to set of children subject to a matroid constraint



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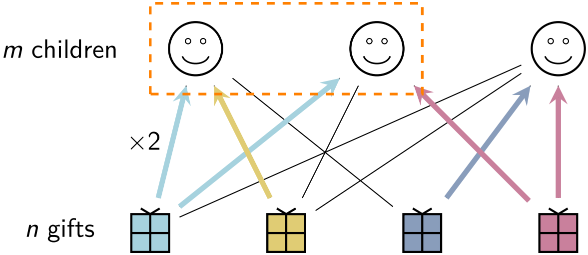

Matroid Generalization

Given: m entities E (children / machines) and n items (gifts / jobs) associated with integer polymatroids $\mathcal{P}_1, \ldots, \mathcal{P}_n$ over E.

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$\mathcal{P}_1$ $\mathcal{P}_2$ $\mathcal{P}_3$ $\mathcal{P}_4$ $\mathcal{P}_5$ $\mathcal{P}_6$

Integer polymatroid = matroid on multisets

Formally,

$$\mathcal{P}_j = \left\{ x \in \mathbb{Z}_{\geq 0}^{E} \ : \ \sum_{i \in S} x(i) \leq f_j(S) \ \forall S \subseteq E \right\}$$

for a submodular, monotone set function $f_j : 2^E \to \mathbb{Z}_{\geq 0}$ with $f_j(\emptyset) = 0$.

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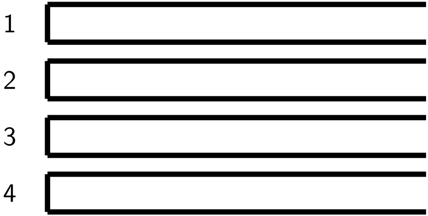
Task for every j: select $x_j \in \mathcal{B}(\mathcal{P}_j)$ and "copy" item j to entity i for $x_j(i)$ times.



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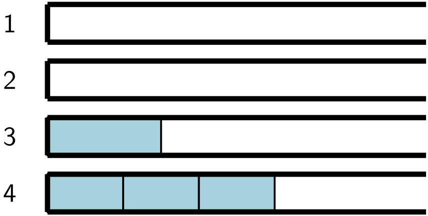

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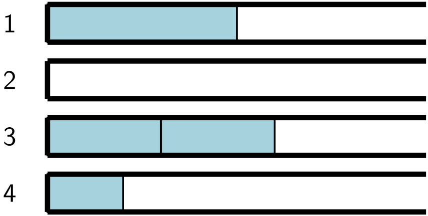

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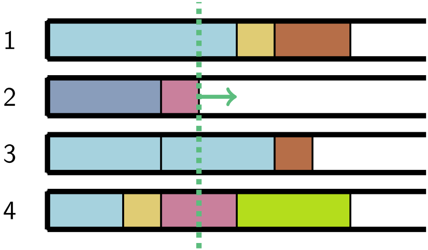

Matroid SantaClaus:

$$\max \min_i \sum_j v_{ij} \cdot x_j(i)$$

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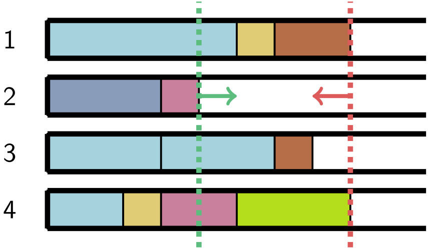

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Matroid Makespan:

$$\min \max_i \sum_j p_{ij} \cdot x_j(i)$$

Our Results

Theorem 3

better-than-2 approximation for restricted matroid 2S-$\textsc{Makespan}$

$$\iff$$

constant approximation for restricted matroid 2V-$\textsc{SantaClaus}$

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Theorem 3

better-than-2 approximation for restricted matroid 2S-MAKESPAN

$\iff$

constant approximation for restricted matroid 2V-SANTACLAUS

Approximation factors: $(2 - 1/\alpha)$-apx. $\iff$ α-apx. for any $\alpha \geq 2$.

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$\Longleftrightarrow$

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Theorem 4

For any $\epsilon > 0$, there is a

- poly-time $(8 + \epsilon)$-apx. for restricted matroid SantaClaus and a
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Corollary

For any $\epsilon > 0$, there exists a $(1.75 + \epsilon)$-apx. for restricted 2S-Makespan.

Improves over best known bound of 1.8945 [Annamalai '19].

Summary and Outlook

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For the 2-value cases, both questions are equivalent, in both the general setting and the restricted matroid setting.

- Reductions within the general restricted (matroid) case?
- Reductions within other special cases, e.g., graph balancing?