858. Mirror Reflection

This approach is not my idea. It is used by many guys.

https://leetcode.com/problems/mirror-reflection/discuss/2376355/Python3-oror-4-lines-geometry-w-explanation-oror-TM%3A-9281

https://leetcode.com/problems/mirror-reflection/discuss/2377070/Pseudocode-Explain-Why-Odd-and-Even-Matter

https://www.google.com/search?q=mirror+reflection+leetcode&source=lmns&bih=891&biw=1678&hl=en&sa=X&ved=2ahUKEwi62v79prT5AhUBhxoKHexgC3IQ_AUoAHoECAEQAA

The only thing that I did, that I have added more details to understand the solutions of many guys.

Let’s take this sample:

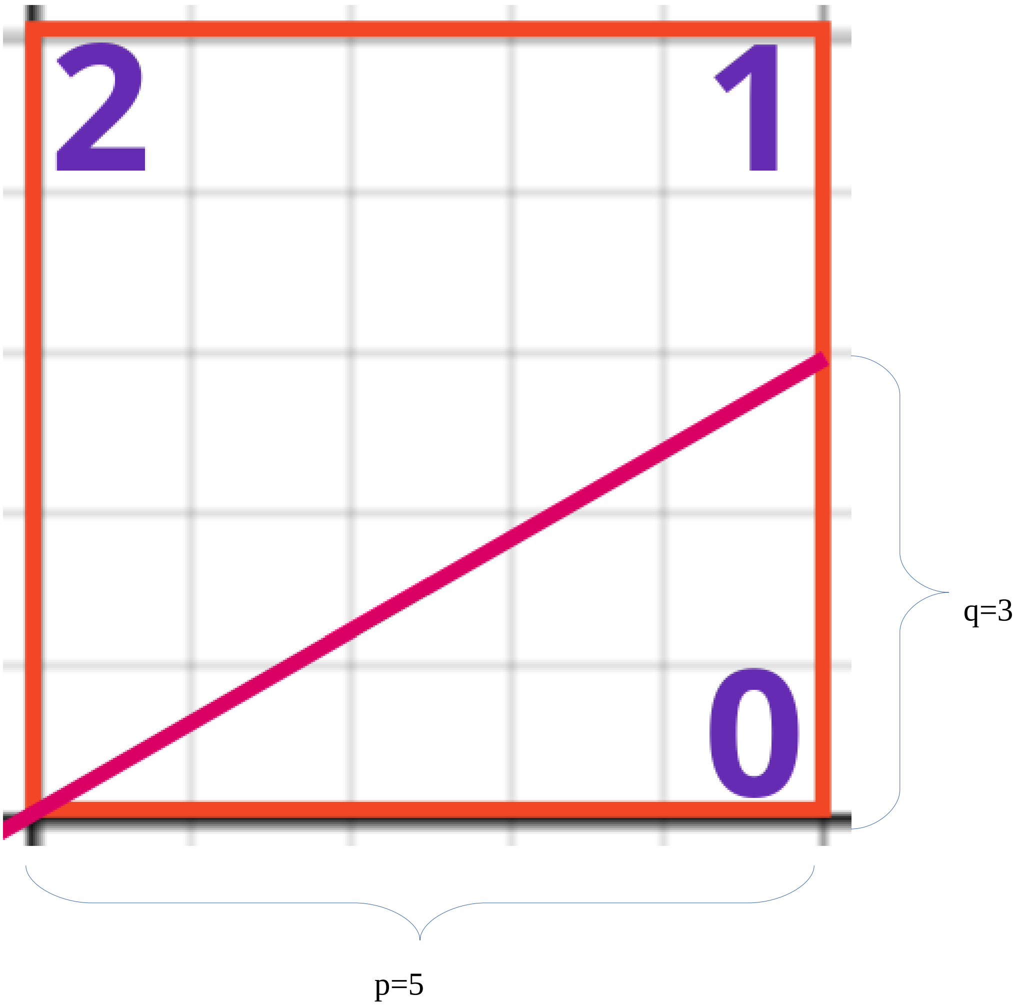
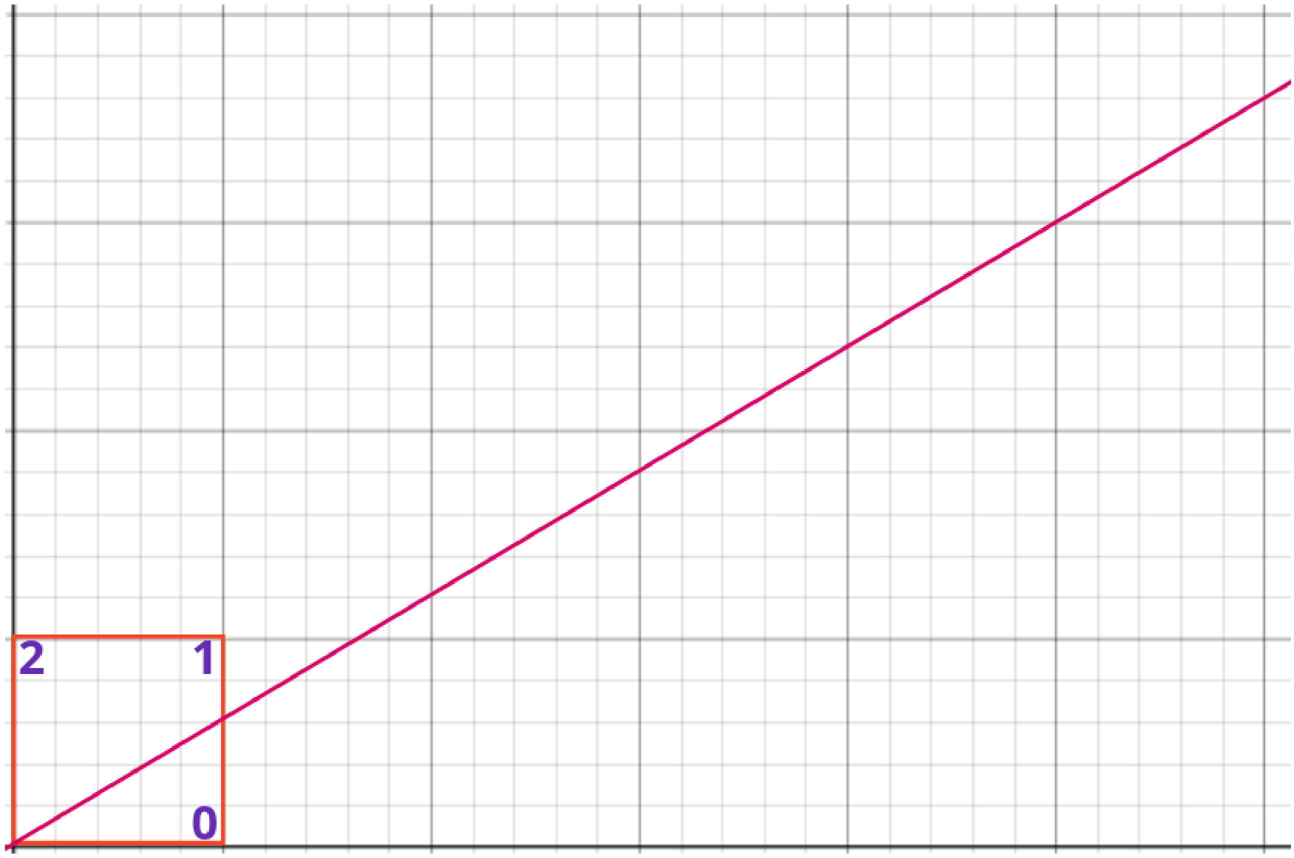
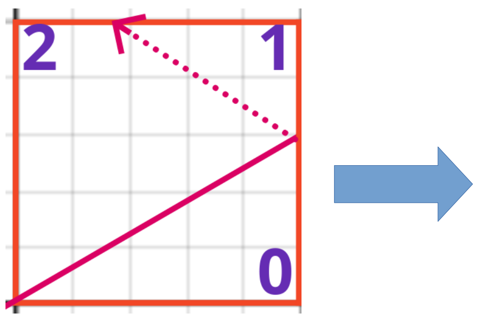


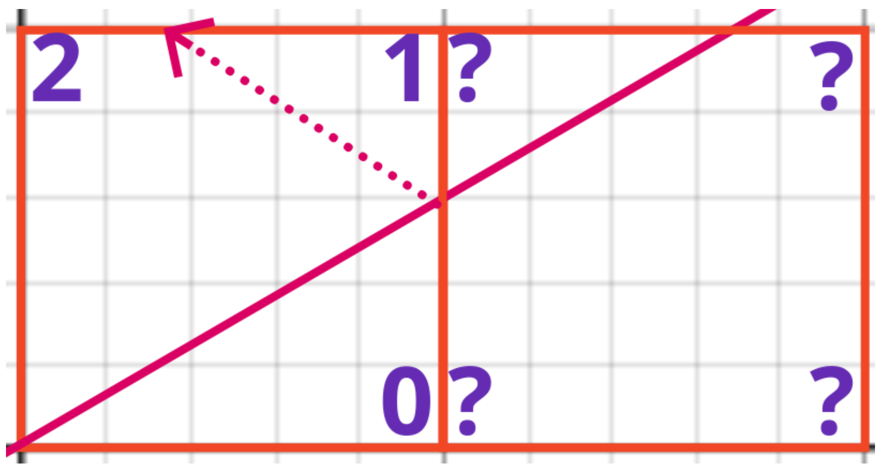
Image now that the LASER is not reflected and pass throw many rooms stacked horizontally and vertically in a manner which simulate the reflection in the original room. (English please!!!!)



LASER not reflected



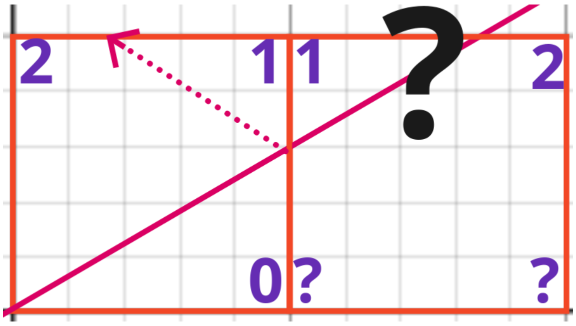
LASER reflected



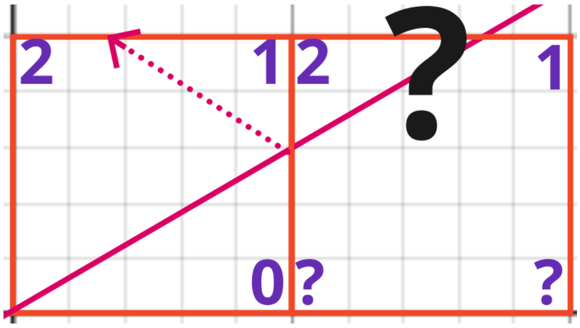
LASER inside the first room

what is the labels of the corners?

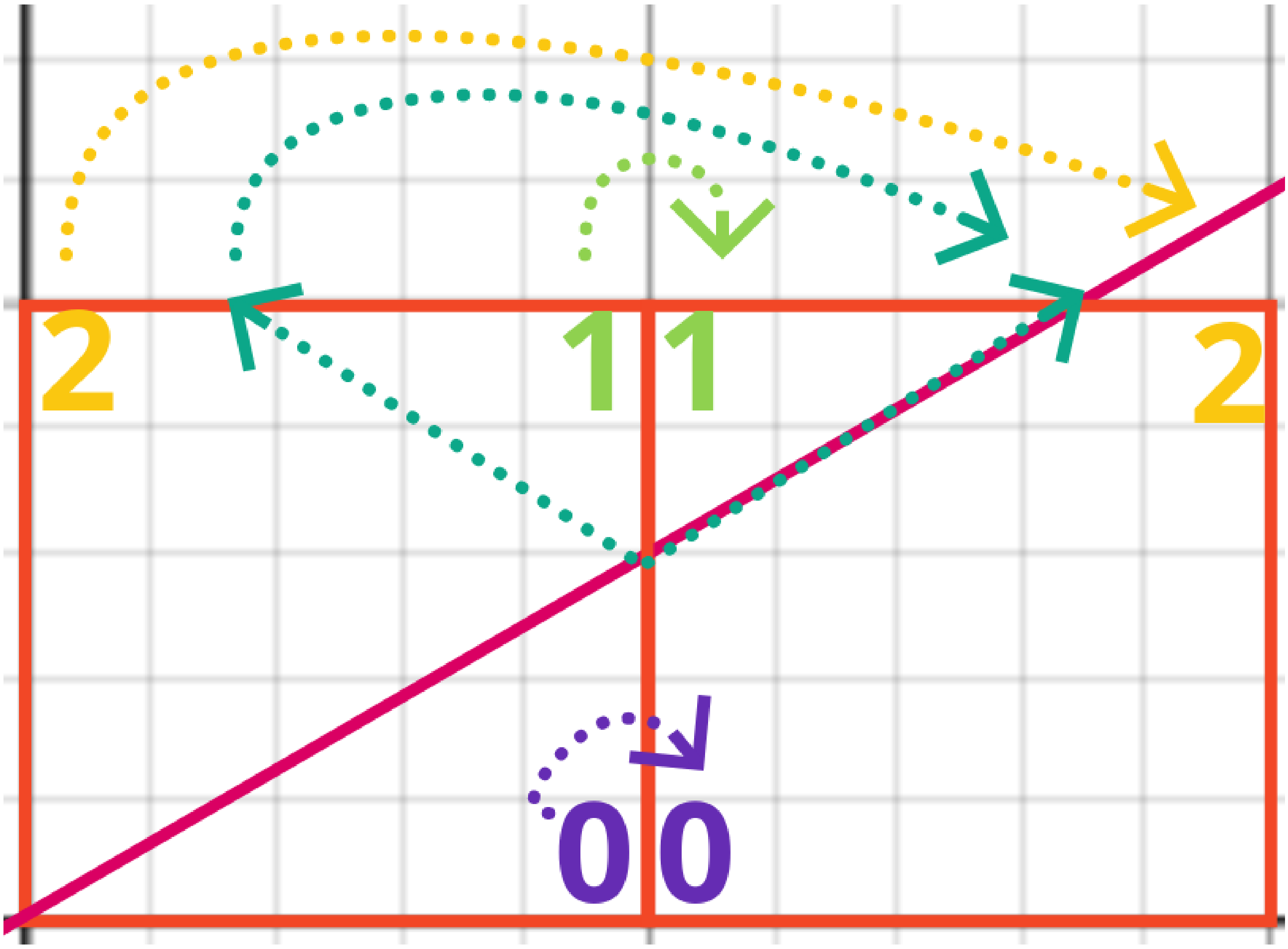
In the original room the LASER is reflected on wall [1,2]. So, the upper upper wall is labeled [1,2] too.



OR

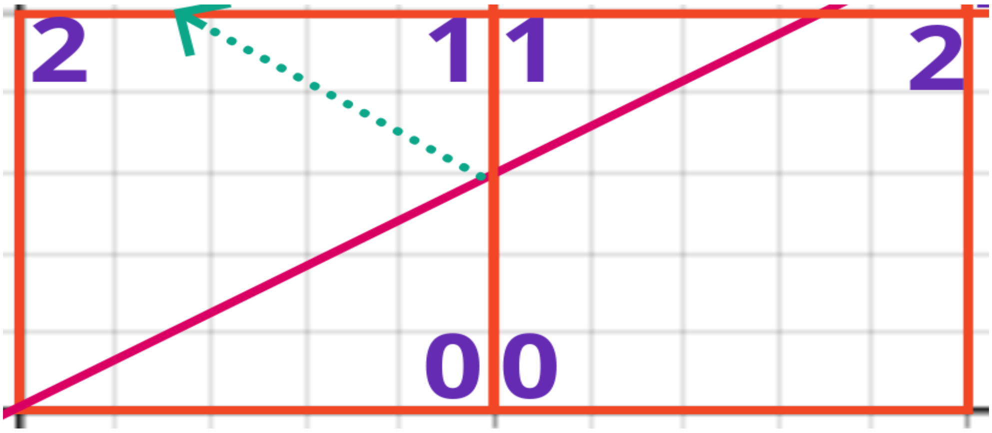


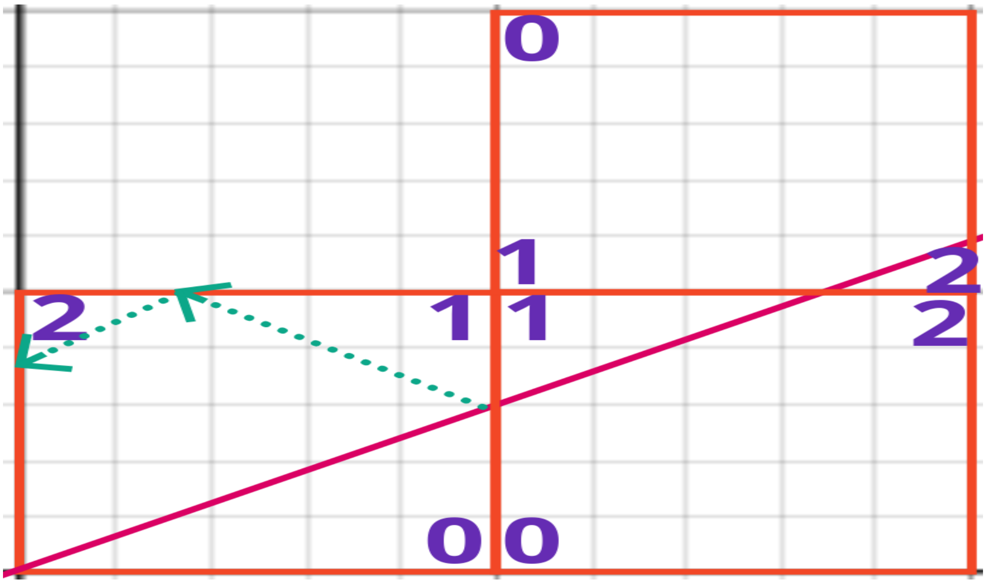
As I said, the rooms simulate the reflection of the light in the original room, the reflected light in the original room must coincide with the extended light in the other rooms.

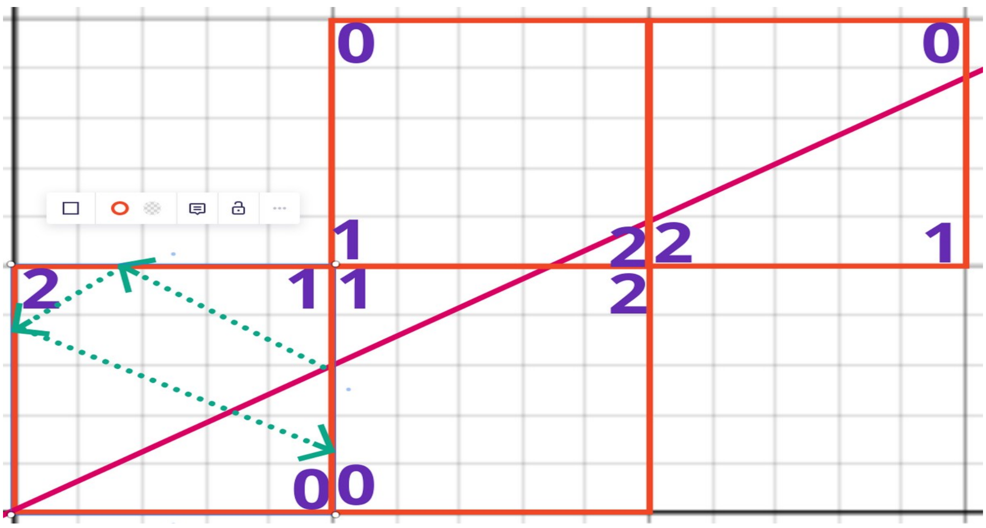


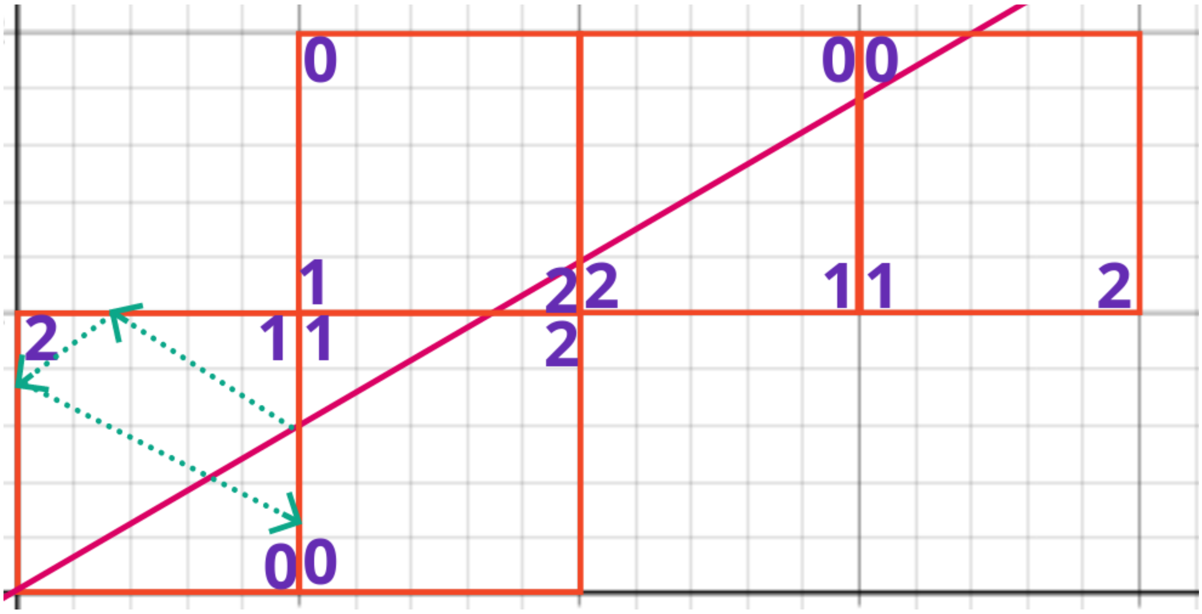
We need to flip horizontally to the right the room in order the lights coincide

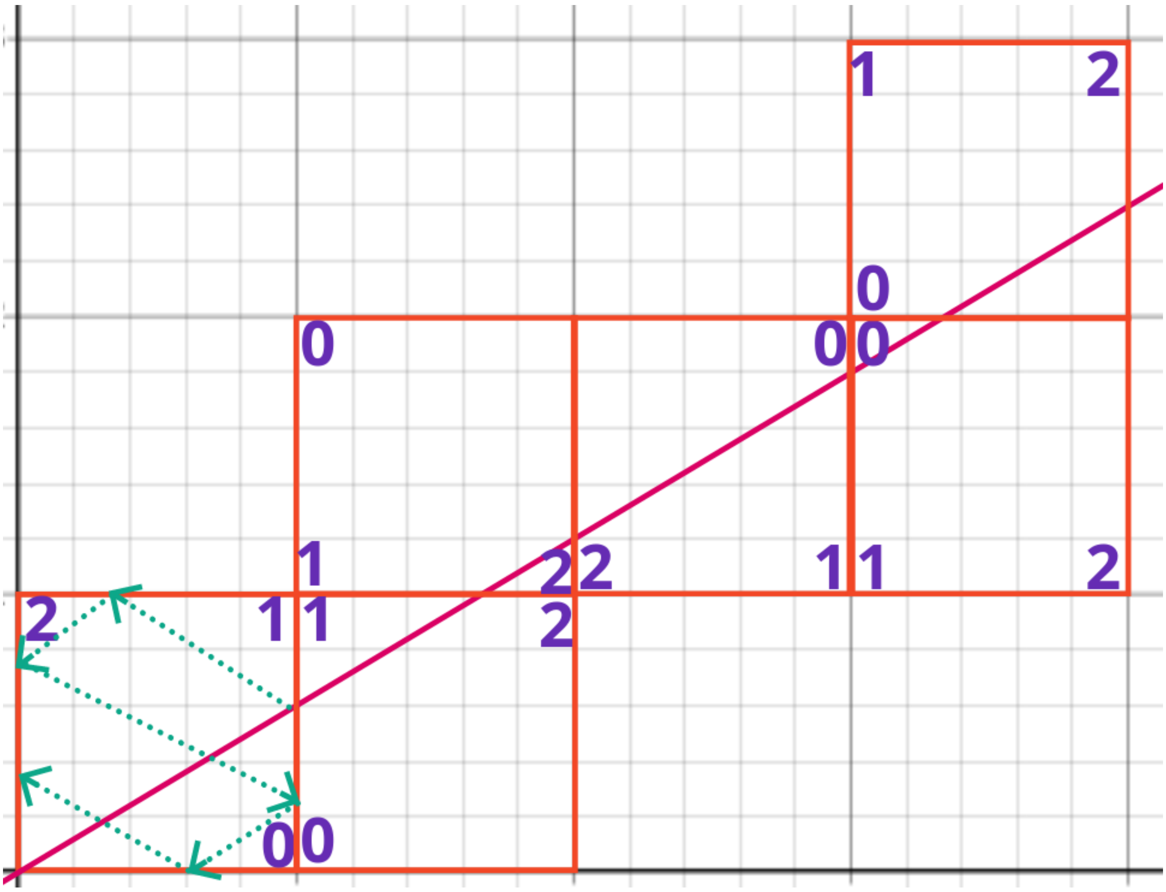
We continue flipping the rooms horizontally to the right or vertically to the top until the extended light reach a corner.








0
0 0
1
2 2
1 1
2
2
1 1
2
0 0


1
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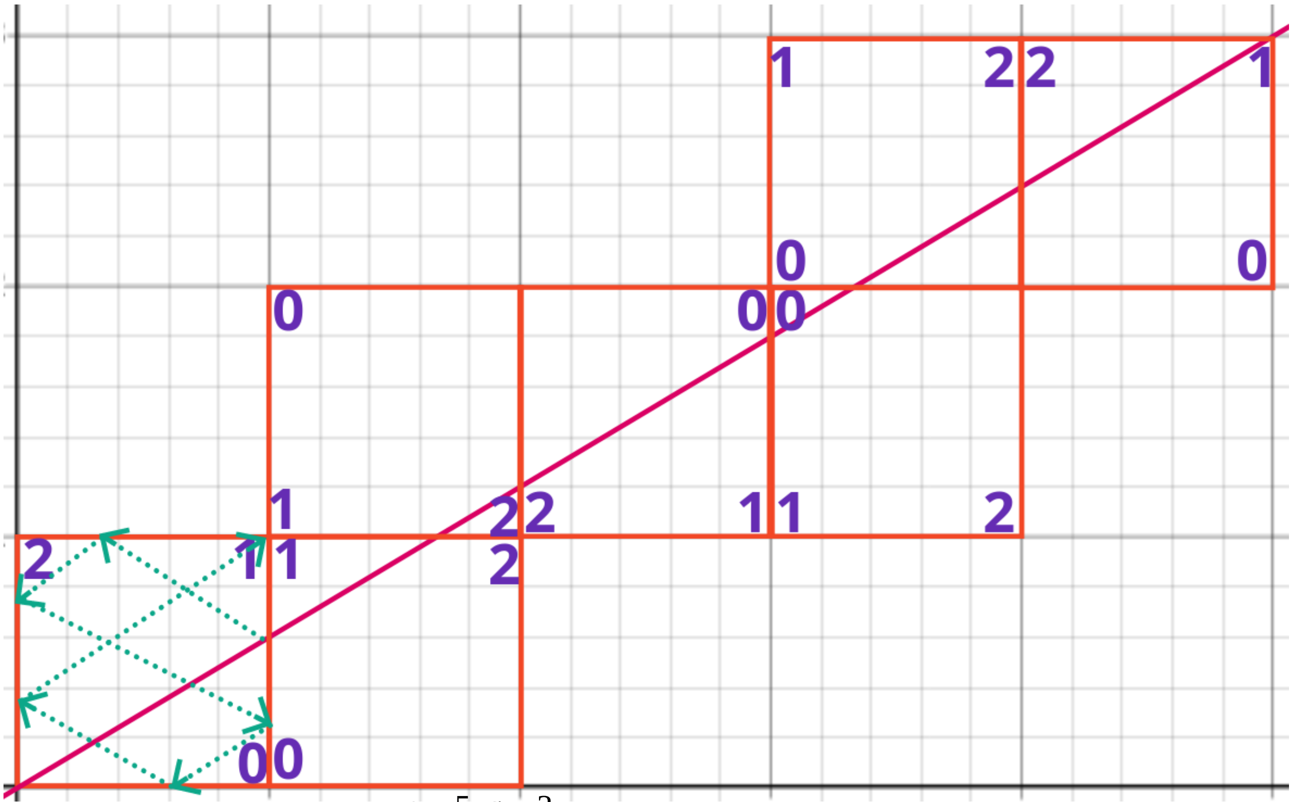
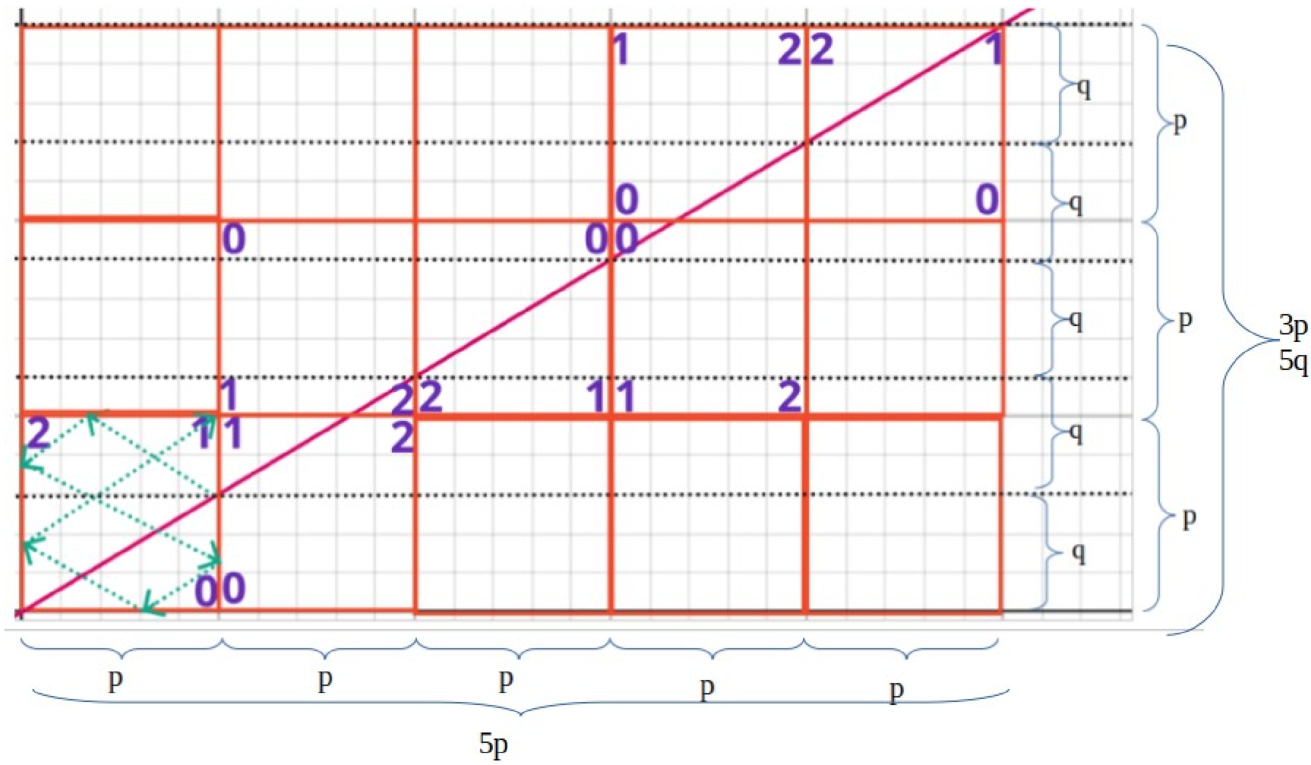
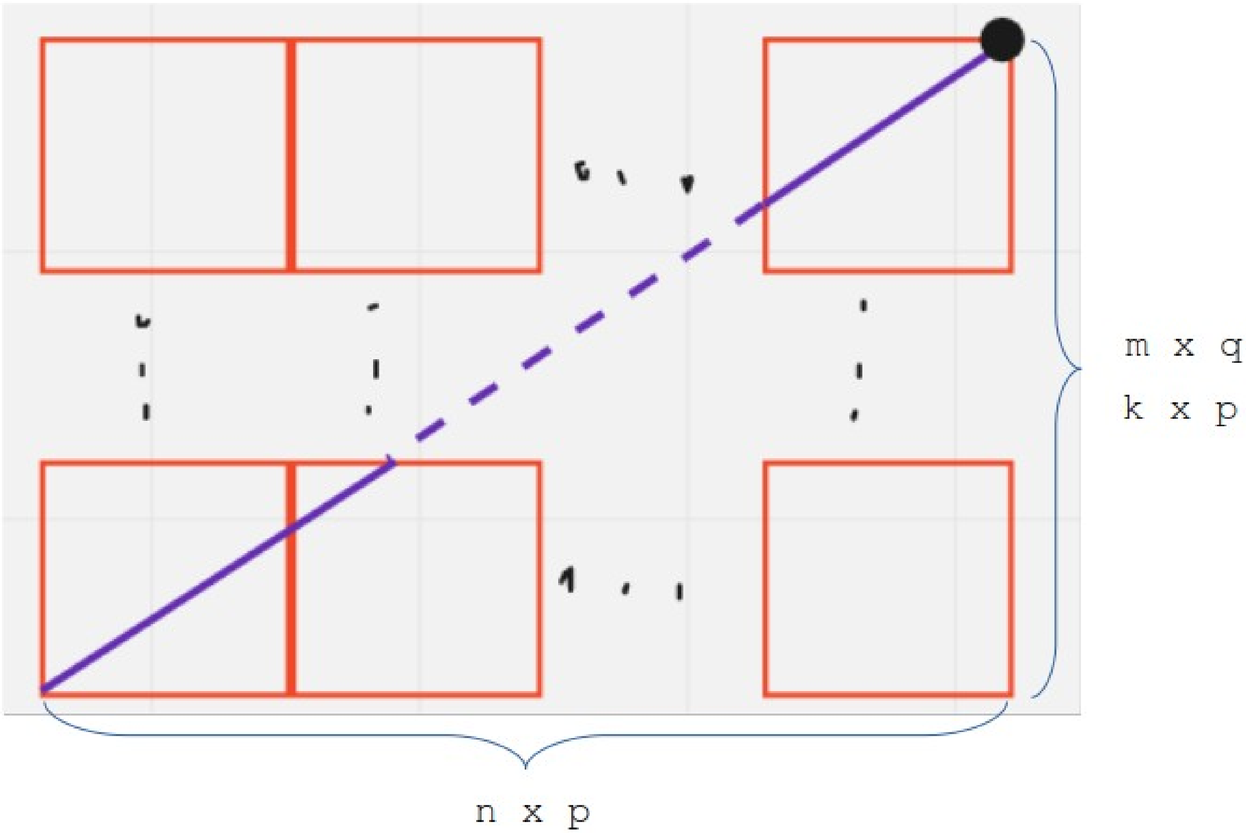


p = 5, q = 3
The light hit corner number 1



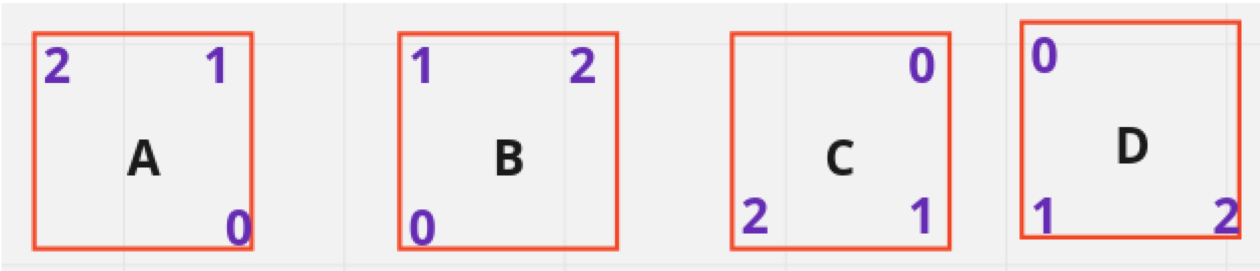
In general case, we have:



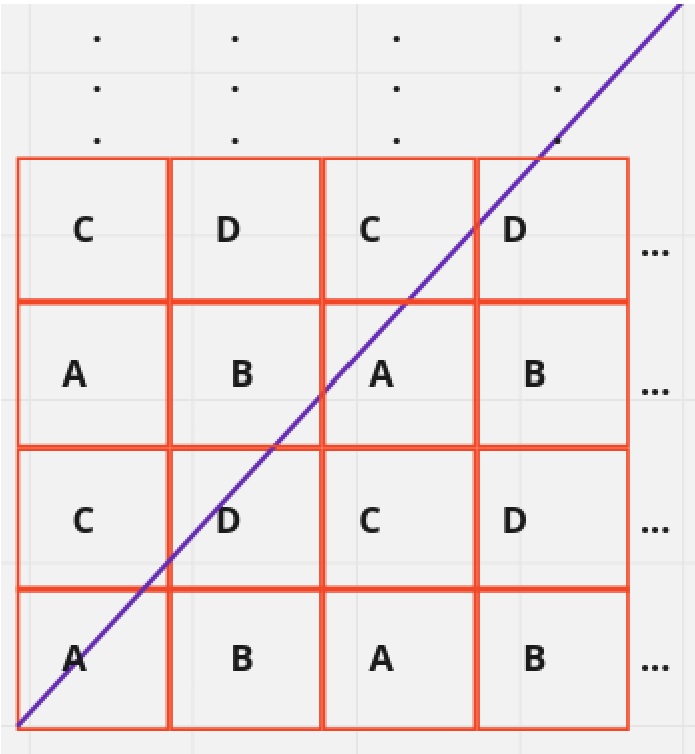
where:

- n : the number of rooms stacked horizontally,
- k : the number of rooms stacked vertically,
- m : the parts of length q

If we label the rooms like in the illustration below:



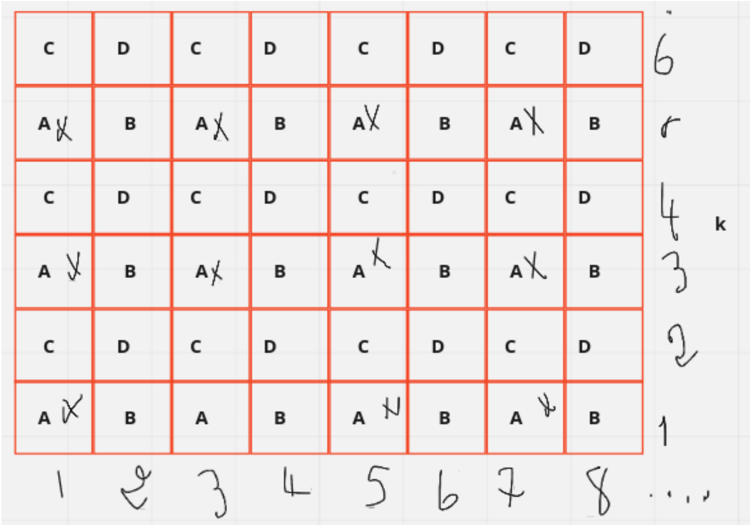
we obtain:



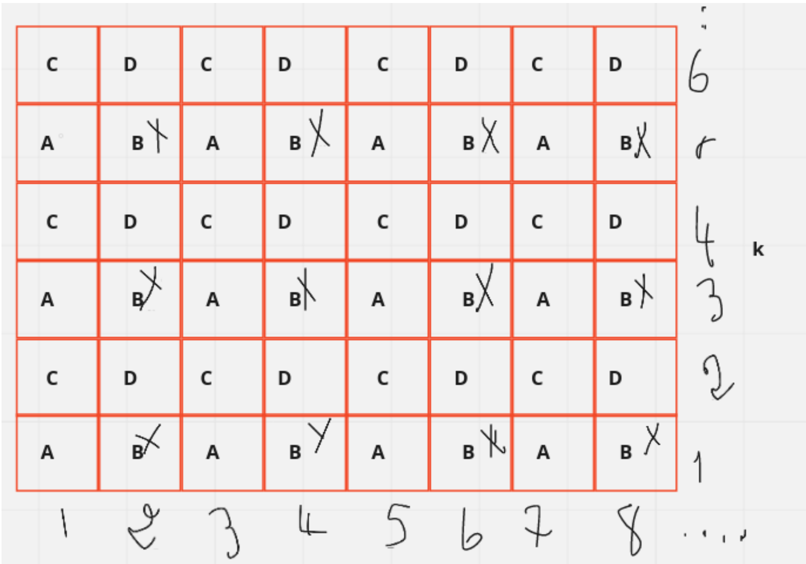
The end room is where the light hit a corner, so it is A or B or C or D .

we have four cases:

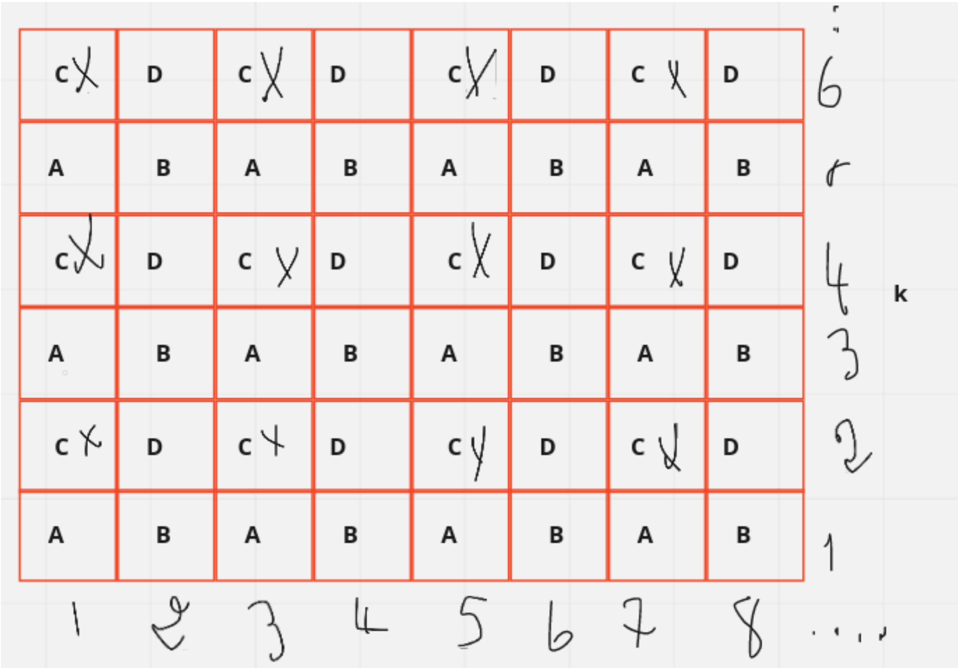
if n is odd and k is odd, the light end at room A , which is corner 1



if n is even and k is odd, the light ends in room B which is corner 2



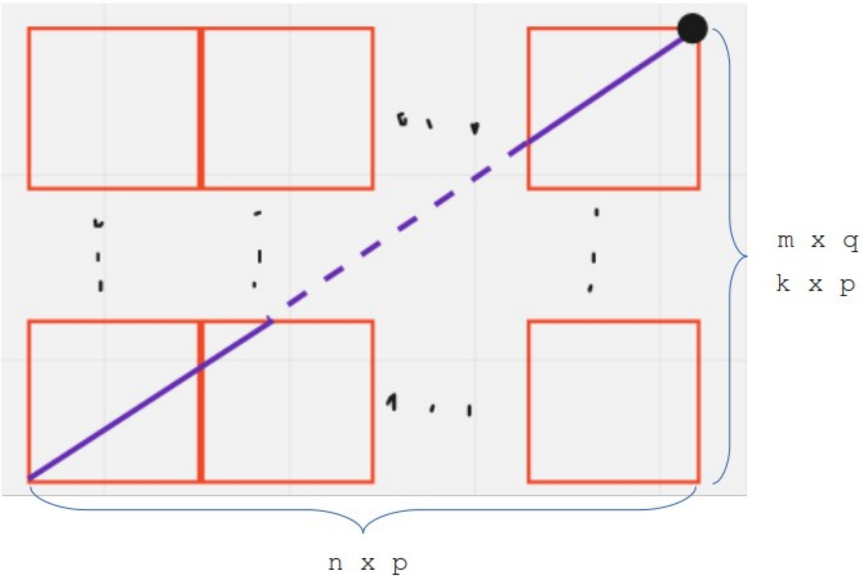
if n is odd and k is even, the light ends in room C which is corner 0 :



if n is even and k is even, the light ends in room D which has no corner.

The ultimate question now, how to find n and k ?

As we have seen, the height of the vertically stacked rooms is $m \times q = k \times p$, and the width of the horizontally stacked room is $n \times p$

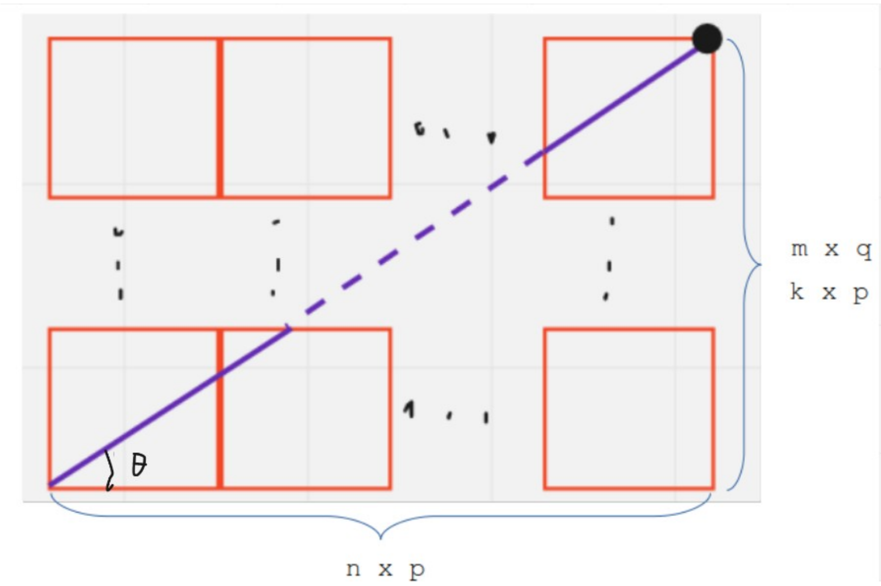


Let say that $\begin{cases} x_1 = n \times p \\ x_2 = m \times q \end{cases}$ => $\frac{x_1}{x_2} = \frac{n \times p}{m \times q}$ => $\frac{x_1 \times m}{x_2 \times n} = \frac{p}{q}$

We know if $z = LCM(p, q)$ => $\begin{cases} \frac{z}{p} = w_1 \\ \frac{z}{q} = w_2 \end{cases}$ => $\frac{\frac{z}{q}}{\frac{z}{p}} = \frac{w_2}{w_1}$ => $\frac{p}{q} = \frac{w_2}{w_1} = \frac{x_1 \times m}{x_2 \times n}$

we have $\frac{x_1 \times m}{x_2 \times n} = \frac{p}{q}$ => $\begin{cases} \frac{\frac{x_1 \times m}{x_1}}{\frac{x_2 \times n}{x_1}} = \frac{p}{q} \\ \frac{\frac{x_1 \times m}{x_2}}{\frac{x_2 \times n}{x_2}} = \frac{p}{q} \end{cases}$ => $\begin{cases} \frac{m}{\frac{x_2 \times n}{x_1}} = \frac{p}{q} \\ \frac{\frac{x_1 \times m}{x_2}}{n} = \frac{p}{q} \end{cases}$

$$\begin{cases} \frac{z}{p}=\frac{x_2\times n}{x_1} \\ \frac{z}{q}=m \\ \frac{z}{p}=n \\ \frac{z}{q}=\frac{x_1\times m}{x_2} \end{cases} \Rightarrow \begin{cases} \frac{z}{p}=\frac{m\times q\times n}{n\times p} \\ \frac{z}{q}=m \\ \frac{z}{p}=n \\ \frac{z}{q}=\frac{n\times p\times m}{m\times q} \end{cases} \Rightarrow \begin{cases} \frac{z}{p}=\frac{m\times q}{p} \\ \frac{z}{q}=m \\ \frac{z}{p}=n \\ \frac{z}{q}=\frac{n\times p}{q} \end{cases} \Rightarrow \begin{cases} z=m\times q \\ z=m\times q \\ z=n\times p \\ z=n\times p \end{cases}$$

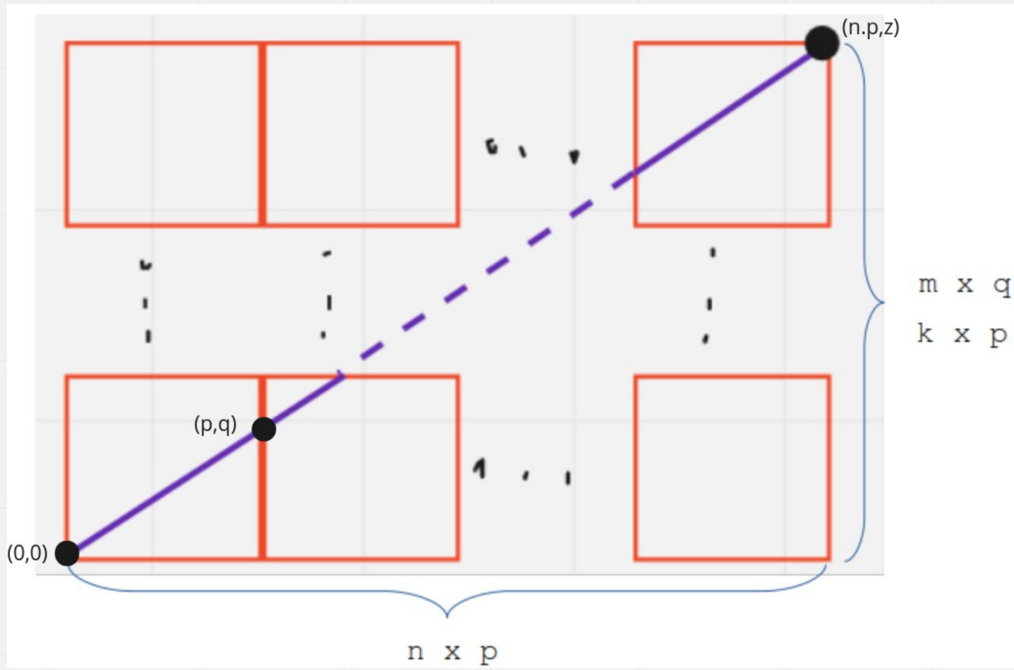


$\tan\theta=\frac{m\times q}{n\times p}$, we know $0<\theta<44$, so $0<\tan\theta<1$, that leads to $m.q<n.p$

We can conclude that $z=LCM(p,q)=m.q$

We have $k.p=z$ => $\boxed{k=\frac{z}{p}}$

To find n , we know about the xy-coordinates of three points on the extended line :



we can compute the slope a of that line:

$$\begin{cases} 0=a\times 0+b \\ q=a\times p+b \end{cases} \quad => \quad \begin{cases} b=0 \\ a=\frac{q}{p} \end{cases}$$

we have the point $(n.p,z)$: $z=a.n.p$ => $z=\frac{q.n.p}{p}$ => $\boxed{n=\frac{z}{q}}$

As we have seen earlier:

$$F(p,q)=\begin{cases} 0, if\ n\, mod\, 2\neq 0 \text{ and } k\, mod\, 2=0 \\ 1, if\ n\, mod\, 2\neq 0 \text{ and } k\, mod\, 2\neq 0 \\ 2, if\ n\, mod\, 2=0 \text{ and } k\, mod\, 2\neq 0 \\ -1, otherwise \end{cases}$$

we can reduce it to: $F(p,q)=\begin{cases} 2, if\ n\, mod\, 2=0 \\ k\, mod\, 2, otherwise \end{cases}$, which is equal to:

$$F(p,q)=\begin{cases} 2, if\ \frac{z}{q} mod\, 2=0 \\ \frac{z}{p} mod\, 2, otherwise \end{cases} \quad \text{with} \quad z=lcm(p,q)$$

C++

```
class Solution {
    public:
        int gcd(int a, int b){
            return a % b == 0 ? b : gcd(b,a % b);
        }

        int lcm (int a, int b){
            return (a*b)/gcd(a,b);
        }

        int mirrorReflection(int p, int q) {
            int z = lcm(p,q);

            if (z / q % 2 == 0) return 2;

            return z / p % 2;
        }
};
```