To prove that 3^x appears twice in the sum of distinct powers of three representing n ,let's analyze the given conditions:
$n \geq 3^x$, and, $n - 3^x \geq 3^x$.

Step 1: Understanding the Sum of Distinct Powers of Three

Every positive integer can be uniquely represented as a sum of distinct powers of three (this is essentially the **ternary representation without using the digit** 2 , known as the **balanced ternary representation**). That is,

$$n = \sum_{i=0}^{max_x} a_i 3^i \text{ , where } a_i = \{0, 1\}, max_x = \log_3 x$$

To prove that 3^x is used **twice**, we need to show that the ternary representation of n initially contains 3^x **at least once** and, after subtracting 3^x , the remaining number $n - 3^x$ still contains 3^x , meaning it was counted twice.

Step 2: What Happens When Subtracting 3^x ?

- Since $n \geq 3^x$, we know that 3^x **could** be part of the sum of powers of three for n .
- The second condition, $n - 3^x \geq 3^x$, means that after removing one occurrence of 3^x , the remaining number is still **at least** 3^x . This suggests that there was another occurrence of 3^x in the original representation of n .

Step 3: Contradiction with Uniqueness of Representation

By definition, the **sum of distinct powers of three** cannot contain any power more than once. However, our assumption leads to a contradiction: If 3^x was used twice, then the representation of n was **not** in the form of distinct powers of three, violating the uniqueness of this representation.

Step 4: Conclusion

Since we arrived at a contradiction, our assumption must be false. Therefore, it is **impossible** for 3^x to appear twice in the sum of distinct powers of three. This means that the conditions given,

$n \geq 3^x$, and, $n - 3^x \geq 3^x$.

must imply that n is not correctly expressed as a sum of distinct powers of three. Instead, it suggests that the ternary representation of n (in terms of coefficients a_i) contains a **digit** 2 , specifically at position x . This means n is not a valid sum of distinct powers of three, and it must be adjusted (for example, by carrying over values in a modified ternary system).

Thus, the conditions imply that n is **not** a sum of distinct powers of three, but rather requires a transformation, such as using a balanced ternary system.