

1545. Find Kth Bit in Nth Binary String

Given two positive integers n and k , the binary string S_n is formed as follows:

- $S_1 = "0"$
- $S_i = S_{i-1} + "1" + \text{reverse}(\text{invert}(S_{i-1}))$ for $i > 1$

Where $+$ denotes the concatenation operation, $\text{reverse}(x)$ returns the reversed string x , and $\text{invert}(x)$ inverts all the bits in x (0 changes to 1 and 1 changes to 0).

For example, the first four strings in the above sequence are:

- $S_1 = "0"$
- $S_2 = "011"$
- $S_3 = "0111001"$
- $S_4 = "011100110110001"$

Return *the kth bit in S_n* . It is guaranteed that k is valid for the given n .

Example 1:

Input: $n = 3, k = 1$

Output: `"0"`

Explanation: S_3 is `"0111001"`.
The 1st bit is `"0"`.

Example 2:

Input: $n = 4, k = 11$

Output: `"1"`

Explanation: S_4 is `"011100110110001"`.
The 11th bit is `"1"`.

Constraints:

- $1 \leq n \leq 20$
- $1 \leq k \leq 2^n - 1$

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```
/*
    Brute force
    Time complexity:  $O(2^n)$ 
    Space complexity:  $O(2^n)$ 
*/
class Solution {
public:
    std::string invert(std::string s){
        int n=s.size();
        for(int i=0;i<n;++i){
            s[i]=s[i]=='0'?'1':'0';
        }
        return s;
    }

    std::string reverse(std::string s){
        std::reverse(s.begin(),s.end());
        return s;
    }

    char findKthBit(int n, int k){
        std::string s="0";
        for(int i=2;i<=n;++i){
            s+="1"+reverse(invert(s));
        }
        return s[k-1];
    }
};
```

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```
/*
    Recursion
    Time complexity: O(n)
    Space complexity: O(n)
*/
class Solution{
public:
    char findKthBit(int n, int k) {

        auto solve=[&](int n,int k,auto& self)->char{
            if(n==1) return '0';
            int nb_bits=(1<<n)-1;
            int mid=nb_bits/2+1;
            if(k==mid) return '1';
            if(k<mid) return self(n-1,k,self);
            return self(n-1,nb_bits-k+1,self)=='0'?'1':'0';
        };

        return solve(n,k,solve);
    }
};
```

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```
/*
    Divide and conquer
    Time complexity: O(n)
    Space complexity: O(1)
*/
class Solution {
public:
    char findKthBit(int n, int k){
        int nb_bits=(1<<n)-1;
        int nb_inversions=0;
        while(k>1){
            int mid=nb_bits/2;
            if(k==mid+1) return nb_inversions%2==0?'1':'0';
            if(k>mid){
                k=nb_bits-k+1;
                nb_inversions++;
            }
            nb_bits/=2;
        }
        return nb_inversions%2==0?'0':'1';
    }
};
```

1545. Find Kth Bit in Nth Binary String

```
/*
  Bit manipulation
  Time complexity: O(1)
  Space complexity: O(1)
*/
class Solution {
public:
    char findKthBit(int n, int k) {
        // Find the group where k belong
        int rightmost_set_bit_pos=k&-k;

        // If the group is set => k-th bit is inverted
        // Otherwise => k-th bit remains same as original
        bool is_inverted=((k/rightmost_set_bit_pos)>>1&1)==1;

        // Determine if the original bit (before any inversion) would be 1
        // This is true if k is even (i.e., its least significant bit is 0)
        bool is_original_bit_one=(k&1)==0;

        if (is_inverted){
            // If we're in the inverted part, we need to flip the bit
            return is_original_bit_one?'0':'1';
        }

        // If we're not in the inverted part, return the original bit
        return is_original_bit_one?'1':'0';
    }
};
```