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Explanation of the Game:

Eloise and Abelard play the following game in numbers:

- Positions are integers from 1 to $05 \ + \ 11 \ + \ 2001 \ = \ 2017$, players move in turns.
- Moves for both players are $(+ \ n)$ where n is $min(2001, \ 2017 - p)$ where $p$is the current position.
- A player wins and the play stops as soon as the player moves to the final position (2017)

Solution:

1. In general, a finite game of two players A and B is a tuble G = (PA, PB , MA, MB , FA, FB), where:
    a. PA and PB, are disjoint finite sets of positions for A and B,
    b. $MA \subseteq PA$ × ($PA \cup PB$) and $MB \subseteq PB$ × ($PA \cup PB$) are admissible moves of A and B,
    c. $FA \subseteq$ ($PA \cup PB$) and $FB \subseteq$ ($PA \cup PB$) are disjoint final positions (where A and B won already respectively).

    Thus, in our Game depending on fixed initial position we can say:

    $$PEloise \ = \{initial \ + \ (n), \ initial + (n \ * \ 3), \ ....., \ min(intial \ + (n \ * \ k), \ 2017)\}, where \ initial \ = \ The \ start \ point \ ,$$
    $$k \in [\ integer \ of \ ((2017 - initial) \ / \ n \ ), \ integer \ of \ ((2017 - initial) \ / \ n \ ) - 1], where \ k \ is \ odd.$$

    $$PAlbert \ = \ P \ / \ PEloise \ , \ where \ P \ = \ \{initial + n, \ ......, \ min(initial + \ (n \ * \ w), \ 2017)\}, \ w \in N$$
    $MEloise \ \subseteq PEloise \times PAlbert$ and $MAlbert \ \subseteq PAlbert \times PEloise$

    $$FEloise \ \subseteq PEloise \ and \ FAlbert \subseteq PAlbert$$

    But, we already know that on fixed initial position:

    $$MEloise \ = \{[\ PEloise^{(i)} \ , \ PAlbert^{(i)}]\}, \ where \ i \ \in N.$$

    And we can represent it as:

    $$MEloise \ = \{[\ PEloise^{(i)} \ , \ min(PEloise^{(i)} + n, \ 2017)]\}, \ where \ i \ \in N.$$

    Same way we can represent:

    $$MAlbert \ = \{[\ min(PEloise^{(i)} + n, \ 2017) \ , \ min(PEloise^{(i)} + 2n, \ 2017) \ ]\}, \ where \ i \ \in \ N$$

    If $FEloise \ = \{2017\} \ then \ FAlbert \ = \ \{\emptyset\}$ and vise versa.
2. In general, a player A has a winning strategy against set of players lets call them B in a position p iff
    - either p is a final position.
    - or A has a move $(p, \ q) \in MA$ that leads to a position q such that:
        - q is not a final position
        - any move $(q, \ r) \in Mi$ from player $i \ where \ i \in B$ leads to a position r where A has a winning strategy in that position.

3. game G = (PA, PB , MA, MB , FA, FB) can be solved on behalf of Eloise on base of the following observation: the set WA where Eloise has a winning strategy is the least set of positions that satisfies the following equation:

$$X = FA \cup \{p \in PA\backslash F : \exists q((p,\ q) \in MA \,\&\, q \in X)\} \cup \{p \in PB \backslash F : \forall q((p,\ q) \in MB \ then\ q \in X)\}\ where\ F = FA \cup FB$$

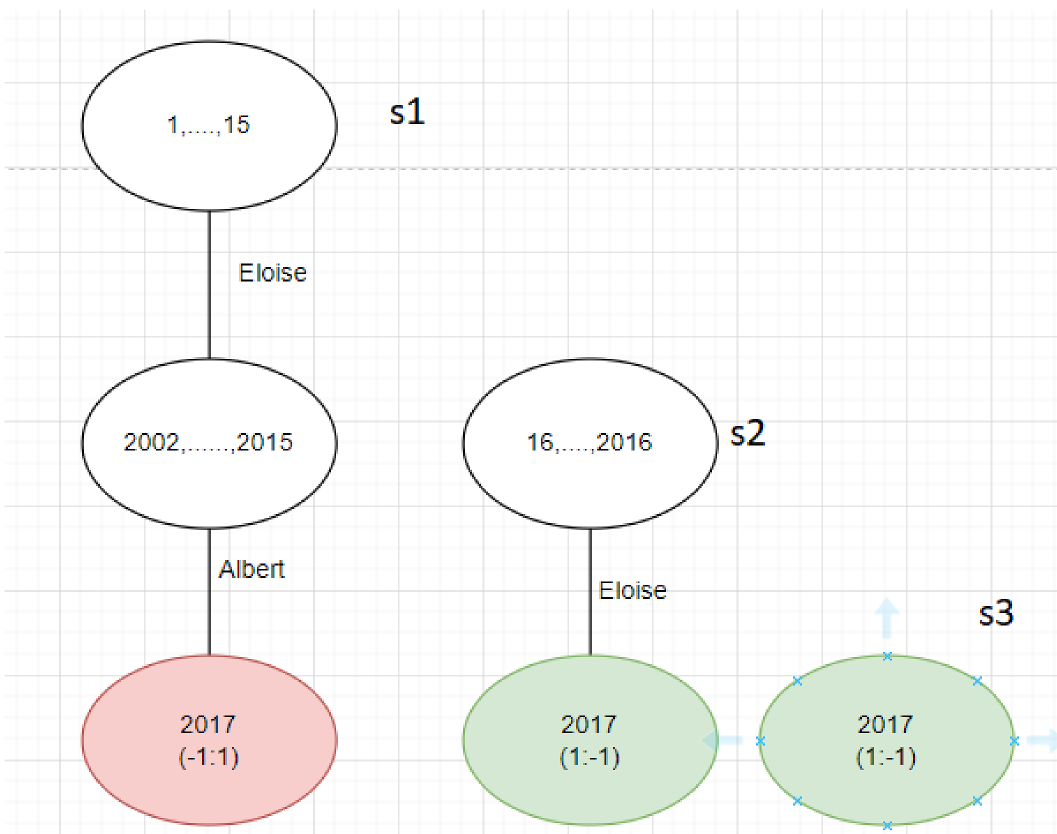
The computation:

$$XEloise = \{2017\} \cup \{2017 - 2001,......,2016\} \cup \{2017 - 2001,......,2016\}\ where\ F = 2017$$

$$XEloise = \{16,\ 17,\ 18,\ ....,\ 2017\}$$

Which mean X is a set of winning positions for Eloise which is: final position adding to it all the winning positions that come from the winning positions of X.

4. $XAlbert = P/Xeloise\ where\ P = \{1,\ 2,\ 3,\ 4,\ .....,\ 2017\}$ ={1,....,15}
As the set if moves $n \in [2001]$ then there is only one way to move and all these initial position are already winning position for Albert, however any move from this position to position + n will lead to winning position for the other component and as Eloise start the first move, Albert will win.
5. We can imagine the situation of Albert choosing the initial position as he is making a move to this position then the turn will go to Eloise, in the skitch below:
S1 refers to the situation where Albert choosed the initial point from the range [1,...,15].
S2 refers to the situation where Albert choosed the initial point from the range [16,...,2016].
S3 refers to the situation where Albert choosed [2017] as initial point.
Extensive form:



NOTATION:

Let G be a play in a game in the normal form of players A, B, etc.
Let π be the payoff function that maps every play $S = (sA, sB, \dots)$ of the game (where sA, sB, … are strategies of the players A, B, etc.) into vector of the payoffs in this play $\pi(S) = (\pi A(S) : \pi B(S) : \dots)$.
For any play $S = (sA, sB, \dots)$ and any strategy sX' of a player X in A, B, etc., let $SX{:}sX'$ be result of X playing sX' instead of sX in S (while all other players do not change their strategies).

6- In general, a play $S = (sA, sB, \dots)$ is Pareto optimal, if there is no any play S' such that $\pi(S') \geq \pi(S)$ holds componentwise and $\pi X(S') > \pi X(X)$ for some player X of the game.

From the definition: s1=(-1,1) and s2=(1,-1) and s3=(1,-1) are All Pareto optimal

Because:

πalbert (-1,1)=1>πalbert(AI,BJ)

πeloise (1,-1)=1>πeloise(AI,BJ)

Which mean s1 and s2 ans s3 cannot be better off without making another player worse.

7-Nash equilibria: A play $S = (sA, sB, \dots)$ is acceptable for a player X in A, B, etc., if $\pi X(S) \geq \pi X(SX{:}sX')$ for any legal strategy sX' of the player X.

From the definition: s1=(-1,1) is Nash equilibria because it let (πalbert (-1,1)=1) ≥ (πalbert(1,-1)=-1), and πeloise (-1,1)=-1 is the only and biggest result of moves that eloise can make.

Which mean albert will not like to change the result, holding all other decisions constant

8- As the moves of Eloise is depend on the initial position that Albert choose, we can write the normal form as:

Albert Eloise	Initial 1 +2017-Position	Initial 2	initial3
Eloise move is always depend on Albert moves	-1	1	1

-1 mean Albert win and Eloise lose
1 mean Eloise win and Albert lose

Or we can make the normal form in this way:

Albert Eloise	Initial1 +2017-Position	Initial2	Initial3
+2001	(-1,1)	Not a play	Not a play
+2017-Postion	Not a play	(1,-1)	Not a play

Position	Not a play	Not a play	(1,-1)

(-1,1) means Eloise win and get 1 and Albert lose and get -1
(1,-1) means Albert win and get -1 and Eloise lose and get 1