Theory test on Game Theory

(Innopolis University, Fall semester 2022-23, BS-III)

Description

It is a distance asynchronous individual written test to check that students understand and can apply main definitions, concepts and techniques presented on lectures on weeks 1-52, namely

- Finite Position Games (FPG),
- Knaster-Tarski fix-point theorem,
- backward induction and solutions of FPG's
- games in the extensive and the normal forms
- definition of Pareto optimality and Nash equilibrium.

Test consists of a single task (parameterized by individual birth data). According to the Syllabus, overall cost of the test is 20 points.

Rules

The timeline

- Publication of the Description, Rules, and the task – by Sunday October 2, 2022.
- Consultation on technical issues – during the lecture class (10:50-12:20) on Monday October 3, 2022.
- Individual solutions to be uploaded to Moodle by Sunday October 9, 2022 (prior 24:00 Moscow time zone).
- Publication of grades on Moodle – by Sunday October 16, 2022.
- Test review – to be advertised on Moodle later but in advance.

The main grading criterions

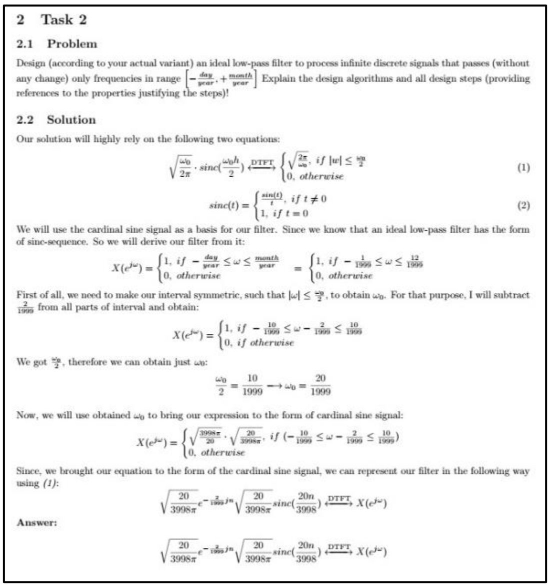
1. human readable but concise
2. well-structuredness (0-5 points for each solution)
3. correct and complete answer (0-5 point for each correct answer)
4. correct and self-complete explained solution (0-15 points)
5. non-crucial computational errors will be treated as tiny mistake (at most 1-point deduction for each task)

Please be aware that the above criterions will be applied as follows: each criterion $i \in [1..4]$ is a prerequisite for all criterions $j \in [(i+1)..5]$; for example, if your paper has no structurally specified answer or conclusion (i.e., "return" operator), then your solution will be ignored.

Comment on grading criterions

- "Human readable but concise" means that your paper should be well-commented but should not excide two 4 pages.

- Well-structuredness is the following "code convention": each task formulation, solution (its parts/sections), and the answer (conclusion) must be explicit in the paper and identified by appropriate headings/keywords (with respect to logical structure).
- Self-completeness means that the paper should be readable independently on any other resource, but lecture notes for weeks 1 and 2.
- An example of a readable, self-complete, and well-structured paper from the Spring-2021 semester BS-III course on Digital Signal Processing is given on the picture to the right.



2 Task 2

2.1 Problem

Design (according to your actual variant) an ideal low-pass filter to process infinite discrete signals that passes (without any change) only frequencies in range $\left[-\frac{day}{year}, +\frac{month}{year}\right]$ Explain the design algorithms and all design steps (providing references to the properties justifying the steps)!

2.2 Solution

Our solution will highly rely on the following two equations:

$$\sqrt{\frac{\omega_0}{2\pi}} \cdot sinc(\frac{\omega_0 h}{2}) \xleftrightarrow{\text{DTFT}} \begin{cases} \sqrt{\frac{2\pi}{\omega_0}},\ if\ |w| \leq \frac{\omega_0}{2} \\ 0,\ otherwise \end{cases} \tag{1}$$

$$sinc(t) = \begin{cases} \frac{sin(t)}{t},\ if\ t \neq 0 \\ 1,\ if\ t = 0 \end{cases} \tag{2}$$

We will use the cardinal sine signal as a basis for our filter. Since we know that an ideal low-pass filter has the form of sinc-sequence. So we will derive our filter from it:

$$X(e^{j\omega}) = \begin{cases} 1,\ if\ -\frac{day}{year} \leq \omega \leq \frac{month}{year} \\ 0,\ otherwise \end{cases} = \begin{cases} 1,\ if\ -\frac{1}{1999} \leq \omega \leq \frac{12}{1999} \\ 0,\ otherwise \end{cases}$$

First of all, we need to make our interval symmetric, such that $|\omega| \leq \frac{\omega_0}{2}$, to obtain ω_0. For that purpose, I will subtract $\frac{2}{1999}$ from all parts of interval and obtain:

$$X(e^{j\omega}) = \begin{cases} 1,\ if\ -\frac{10}{1999} \leq \omega - \frac{2}{1999} \leq \frac{10}{1999} \\ 0,\ if\ otherwise \end{cases}$$

We got $\frac{\omega_0}{2}$, therefore we can obtain just ω_0:

$$\frac{\omega_0}{2} = \frac{10}{1999} \longrightarrow \omega_0 = \frac{20}{1999}$$

Now, we will use obtained ω_0 to bring our expression to the form of cardinal sine signal:

$$X(e^{j\omega}) = \begin{cases} \sqrt{\frac{3998\pi}{20}} \cdot \sqrt{\frac{20}{3998\pi}},\ if\ (-\frac{10}{1999} \leq \omega - \frac{2}{1999} \leq \frac{10}{1999}) \\ 0,\ otherwise \end{cases}$$

Since, we brought our equation to the form of the cardinal sine signal, we can represent our filter in the following way using *(1)*:

$$\sqrt{\frac{20}{3998\pi}} e^{-\frac{2}{1999}jn} \sqrt{\frac{20}{3998\pi}} sinc(\frac{20n}{3998}) \xleftrightarrow{\text{DTFT}} X(e^{j\omega})$$

Answer:

$$\sqrt{\frac{20}{3998\pi}} e^{-\frac{2}{1999}jn} \sqrt{\frac{20}{3998\pi}} sinc(\frac{20n}{3998}) \xleftrightarrow{\text{DTFT}} X(e^{j\omega})$$

<u>Submission rules and formats</u>

- Students must upload individual solutions in two files: the source file (in one of 5 formats – OpenDocument .odt, Word 2007 document .docx, Word document .doc, Markdown with extension .txt, or application/x-tex .tex with any plain class like article) and the result of PDF-conversion of the source file (i.e., pdf file).
- Each submitted file should be named by student first name and surname (for example NikolayShilov.docx and NikolayShilov.pdf)
- On the top of the front page of each submission should start with student first name and surname followed by birthdate (in the format $day.month.year$, for example 24.04.1961).
- Submissions with scanned or photo images of hand-written solutions will be discarded without consideration!

<u>Late Submission Policy</u>

According to the Syllabus, "Students who do not take/submit on time any mid-term assignment without legal excuse (e.g., documented medical) may/can do it not later than one week after the scheduled date with 30% deduction from the grade for this assignment."

Task (20 points)

Consider the following game between Eloise and Abelard.

- Positions of the game are integers in $[1..(day + month + year)]$, players move in turns Eloise-Abelard-etc.
- Moves (for both players) are $(+n)$ where n is the maximal integer in $[1..(year)]$ such that the next position is in the admissible range $[1..(day + month + year)]$); for example, if $day.month.year$ is 24.04.1961, then a player can move from position 12 to the next position 1973 (i.e., $n = 1961$), but from position 1234 to 1989 (i.e., $n = 755$).
- A player wins and the play stops as soon as the player moves to the final position $(day + month + year)$.

Answer the following questions (in any order that is better for you but using the original question' numbers).

1. Starting with a definition, represent the game as a finite position game (FPG).
2. Give a (general) definition for a winning strategy for a player in FPG.
3. Describe in the set-theoretic terms backward induction foe Eloise and compute (using the description) all initial game positions where Eloise has a winning strategy. (Explain your answer.)
4. List all initial game positions where Abelard has a winning strategy. (Explain your answer.)

Let Abelard select the initial position in the range $[1..(day + month + year)]$ and then pass turn to Eloise for the first move.

5. Starting with definition, draw (sketch) the game in the extensive form (using $+1$ for individual win and -1 for individual loss).
6. Starting with definition, list all plays of the game in the extensive form that are Pareto optimal. (Explain your answer.)
7. Starting with definition, list all plays of the game in the extensive form that are Nash equilibria. (Explain your answer.)
8. Starting with definition, represent the game in the normal form.