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1 Task1
1.1 Problem

Consider a game of two players (Alice and Bob) with the following payoff matrix

$$\begin{bmatrix} \text{day} & 24 & \text{ye} & 19 \\ \text{month} & 4 & \text{ar} & 61 \end{bmatrix}$$

Rows of the matrix correspond to strategies $A1$and $A2$of Alice, columns – to strategies $B1$,$B2$,$B3$, $B4$ of Bob. Firstly, characterize the game using terms and concepts introduced in the lecture notes. Then solve the game using mixed strategies.

2.2 Solution

Our game is a zero sum game, represented in the matrix as follows:

$$\begin{bmatrix} 5 & 24 & 20 & 19 \\ 11 & 4 & 01 & 61 \end{bmatrix}$$

Let *G* be a fixed game in the normal form (of players Alice, Bob), *X* be a player (in *A*lice, *B*ob), and $\{X1,\ldots Xn\}$ be the set of the player *X* pure strategies in the game *G*.
A mixed strategy of the player *X* is a probability distribution on the set of its pure strategies $\{X1,\ldots Xn\}$.
In our task,
"A mixed strategy of Alice" is a probability distribution on the set of its pure strategies $\{A1, A2\}$.
"A mixed strategy of Bob" is a probability distribution on the set of its pure strategies $\{B1, B2, B3, B4\}$.

Any mixed strategy of Alice can be represented as a vector $(a1, a2)$, where $1 = \sum_{1\le i\le 2} ai$ and $0\le a1\le 1,\ 0\le a2\le 1$ are probabilities that Alice plays strategies $A1, A2$.

Any mixed strategy of Bob can be represented as a vector $(b1, b2)$, where $1 = \sum_{1\le i\le 4} bi$ and $0\le b1\le 1,\ 0\le b2\le 1$ are probabilities that Alice plays strategies $B1, B2,\ B3,\ B4$.
The mixed extension *Mix*(G) is the following game (of the same players) in the normal form.
Individual strategies are mixed individual strategies.
For any play S=(($a1$,am),($b1$,...,$b4$)) the payoff $\pi^{mix}(S)$ is:

$$(\pi^{mix}_{Alice}(S):\pi^{mix}_{Bob}(S)) \text{ where}$$

$$\pi^{mix}_{Alice}(S) = \sum_{1\le i\le 2,\ 1\le j\le 4} (\pi_{Alice}(i, j) \times (a_i \times b_j))$$

$$\pi^{mix}_{Bob}(S) = \sum_{1\le i\le 2,\ 1\le j\le 4} (\pi_{Bob}(i, j) \times (a_i \times b_j))$$

Lets find nash equilibrium using maxmin and minmax

Find min for rows, max for columns		Bob				
		B1	**B2**	**B3**	**B4**	**min**
Alice	**A1**	5	24	20	19	5
	A2	11	4	01	61	1
	max	11	24	20	61	

The maxmin of rows is: 5 and the minmax of columns is:8

Maxmin not equal to minmax which mean Saddle points and nash equilibrium don't exist in pure strategies

Let us show if the fourth column is dominated in mixed strategies by two first

$$19 > 5\alpha + 24(1 - \alpha) \rightarrow -5 > -9\alpha \rightarrow \alpha < \frac{5}{9}$$

$$61 > 11\alpha + 4(1 - \alpha) \rightarrow 57 > 10\alpha \rightarrow \alpha < 5.7$$

$$\alpha \in]-\infty, \frac{5}{9}[$$

Hence, the fourth column B4 might be eliminated $\begin{bmatrix} 5 & 24 & 20 \\ 11 & 4 & 01 \end{bmatrix}$

Let us show if the second column is dominated in mixed strategies by first and third

$$24 > 5b + 20(1 - b) \Rightarrow 4 >- 15B -> b >- \frac{4}{5}$$

$$4 > 11b + 1(1 - b) \Rightarrow 3 > 10B -> b < \frac{3}{10}$$

$$b \in] - \frac{4}{15}, \frac{3}{10}[$$

Hence, the second column B2 might be eliminated $\begin{bmatrix} 5 & 20 \\ 11 & 1 \end{bmatrix}$

Lets find nash equilibrium using maxmin and minmax after elimination (Just to make sure I know there is not)

Find min for rows,max for columns		Bob		
		B1	**B3**	**min**
Alice	**A1**	5	20	5
	A2	11	1	1
	max	11	20	

The maxmin of rows is: 5 and the minmax of columns is:11

Maxmin not equal to minmax which mean Saddle points and nash equilibrium don't exist in pure strategies

Let's use formulas from slides 26-47 of GT3fall22 to compute the common dimension D and the game value v.

$$D = G_{11} + G_{22} - G_{12} - G_{21} = 5 + 1 - 20 - 11 = -25$$

$$a1 = \frac{G_{22} - G_{21}}{D} = \frac{1-11}{-25} = \frac{10}{25}$$

$$a2 = \frac{G_{11} - G_{12}}{D} = \frac{5-20}{-25} = \frac{15}{25}$$

$$b1 = \frac{G_{22} - G_{12}}{D} = \frac{1-20}{-25} = \frac{19}{25}$$

$$b2 = \frac{G_{11} - G_{21}}{D} = \frac{5-11}{-25} = \frac{16}{25}$$

$$v = \frac{G_{11}G_{22} - G_{12}G_{21}}{D} = \frac{5*1-20*11}{-25} = \frac{215}{25} = 8.6$$

Answer:

$$a1 = \frac{10}{25}\ a2 = \frac{15}{25}\ b1 = \frac{19}{25}\ b2 = \frac{16}{25}\ v = 8.6$$

2 Task2

2.1 Problem

Consider a game of two players (Alice and Bob) with the following payoff matrix

$$\begin{bmatrix} \text{day:}24 & \text{ye:}19 \\ \text{month:}4 & \text{ar:}61 \end{bmatrix}$$

Rows of the matrix correspond to the strategies $A1$ and $A2$of Alice; columns correspond to the strategies $B1$ and $B2$of Bob.

Firstly, characterize the game using terms and concepts introduced in the lecture notes. Then solve the game using mixed strategies.

2.2 Solution

The game represented in the matrix:

$$\begin{bmatrix} 5:24 & 20:19 \\ 11:4 & 01:61 \end{bmatrix}$$

Let G be a game in the normal form with players Alice and Bob with the payoff function π that maps every play$S = (S_{Alice}, S_{Bob})$ of the game (where S_{Alice}, S_{Bob} are individual strategies of the players A, B, etc.) into vector of the payoffs in this play $\pi(S)$=(πAlice(S):πBob(S)).

$$S *= ((a1 *, a2 *), (b1 *, b2 *))$$

1-Strategy S * is a mixed equilibrium.

2- for Alice and Bob following holds:

$$-\pi_x^{mix}(x_i, S_{-x}^*) = \pi_x^{mix}(S *) \text{ for any } i \epsilon [1... n_x] \text{ such that } x_i^* > 0$$

$$-\pi_x^{mix}(x_i, S_{-x}^*) \le \pi_x^{mix}(S *) \text{ for any } i \epsilon [1... n_x] \text{ such that } x_i^* = 0$$

Let us compute the expected payoff of Alice playing its pure strategies A1 and A2 separately against a mixed strategy $(q, (1 - q))$ of the player B:

$$-\pi_{Alice}^{mix}(A_1, (1, (1 - q)) = 5q + 20(1 - q) = -25q + 20$$

$$-\pi^{mix}_{Alice}(A_2, (1, (1 - q)) = 11q + 1(1 - q) = 10q + 1$$

$$\pi^{mix}_{Alice}(A_1, (1, (1 - q)) = \pi^{mix}_{Alice}(A_2, (1, (1 - q)) \rightarrow - 25 + 20q = 10q + 1 \rightarrow q = \frac{19}{35}$$

Let us compute the expected payoff of Alice playing its pure strategies B1 and B2 separately against a mixed strategy $(p, (1 - p))$ of the player B:

$$-\pi^{mix}_{Bob}(B_1, (1, (1 - p)) = 24p + 4(1 - p) = 20p + 4$$

$$-\pi^{mix}_{Bob}(B_2, (1, (1 - p)) = 19p + 61(1 - p) =- 42p + 61$$

$$\pi^{mix}_{Bob}(B_1, (1, (1 - p)) = \pi^{mix}_{Bob}(B_2, (1, (1 - p)) \rightarrow 20p + 4 =- 42p + 61 \rightarrow p = \frac{57}{62}$$

Answer:

$((\frac{19}{35}, \frac{16}{35})(\frac{57}{62}, \frac{5}{62}))$ is the unique purely mixed equilibrium in the game.

3 Task3

3.1 Problem

Consider the problem. Rational Agents in the Marketplace. What are individual agents' beliefs, desires, and intentions in the model of the problem? Let agents A and B compete for a salesman, and the matrix of their game flip-or-bid game be

A/B	**bid**	**flip**
bid	-ye:-ar	0:-month
flip	-day:0	-day:-month

where $LA =- day\ and\ LB =- month$are individual (negative) losses in case of flip,
$FA =- ye\ and\ FB =- ar$are individual (also negative) fins for simultaneous bidding.
– characterize and solve the flip-or-bid game.

3.2 Solution

There are m≥1 customers and n≥1 salesmen at the marketplace, Every salesman has a indivisible piece of cake and every buyer wants to buy exactly one All salesmen offer their cakes to buyers by individual prices $\{P_{s,b}\epsilon N:\ 1 \le b \le m\ and\ 1 \le s \le n\}$.

All buyers are rational agents that can communicate, negotiate, make concessions, and flip individually and swap their salesmen pairwise (and only pairwise) in a peer to peer manner.
All flips and swaps must be rational for both participating agents. Since time is money, all agents fine themselves a fixed individual amount $\{F_b\epsilon Z:\ 1 \le b \le m\ and\ F_b < 0\}$every time negotiations fail.

We assume that the total number of customers m is common knowledge, and that every customer $b\in[1..m]$ knows its individual sorted price list$Pb = \{Ps, b\in\boldsymbol{N}: 1\le s\le n\}$
(i.e., the cheapest the first, the most expensive the last), as well as its individual fine Fb
We interpret every individual price list Pb as buyer's rationale for flipping/swapping traders, and individual fine Fbas its rationale for concessions in negotiations.We also assume a bounded rationality as follows: a buyer never mind to remember data of other customers and always is looking for the most rational local action (i.e., just one step ahead).

Buyer's beliefs represent its ideas/opinion about itself, otherBuyers, and the network; theses ideas/opinions may be incorrect, incomplete, and inconsistent. **Which for example their believes could be buy with the minimum price always**

Buyer's desires represent its long term aims, obligations and purposes (that may be controversial). **get best price for highest quality or value at the end.**

An agent intentionsons are used for shor-term planning (related to its of course). **Keep having cash and not get rid of all the money.**

Our Game can be represent as following:

A/B	**bid**	**flip**
bid	-20:-01	0:-11
flip	-5:0	-5:-11

where $LA =- 5$ and $LB =- 11$ are individual (negative) losses in case of flip,
$FA =- 20$ and $FB =- 01$ are individual (also negative) fins for simultaneous bidding.

Nash equilibrium is a game theory concept that illustrates the result of a particular game. Most fundamentally, Nash equilibrium is the result of a collection of decisions in which no player would like to change the result, holding all other decisions constant (because a player can only change his or her behavior)

From the definition of Nash equilibruim (-5:0) is the only Nash equilibruim pure strategy, and because of that there is no need to find Nash equilibruim in mixed strategies.

A play $(flib, bid)$is is acceptable for A because:

$$\pi_A(flib, bid) = -5 > -20 = \pi_A(bid, bid)$$

A play$(flib, bid)$) is is acceptable for B because:

$$\pi_B(flib, bid) = 0 > -11 = \pi_B(flib, flib)$$

Hence the play $(flib, bid)$ is acceptable for both players, it is a Nash equilibrium of the game, $flib$ is an equilibrium strategy for A, and bid is an equilibrium strategy for B.

Answer:

$(flib, bid)$ is Nash equilibrium in pure strategy with payoff (-5,0)

Note: Some matrices can be modified and updated by extension called **MathType,** it is not image every thing made by hand.

Dear professor Nikolay Shilov, I worked so hard for this course and I worked for it more than any other course in the semester. I hoped for A from the first lecture but the first theoretical test disappointed me, so I hoped for B and I am still hoping for B. Please do not disappointed me.

Kind Regard,

Mahmoud Mousatat