Final theoretical examination on Game Theory

(Innopolis University, Fall semester 2022/23, BS-III)

Description

It is a distance asynchronous individual written test to check that students understand and can apply main definitions, concepts and techniques presented on lectures (topics 3 and 4 presented on weeks 7-11 and 12-14), namely

- Mixed vs. pure strategies
- Minimax von Neumann Theorem for zero-sum games
- Nash Equilibrium in Mixed Strategies bimatrix games
- Solving bimatrix games in mixed strategies
- Believe-Desire-Intension agent model and multiagent algorithms

The examination comprises 3 tasks with individual variants of data (based on birth data). According to the Syllabus, overall cost of the test is 30 points.

Rules

The timeline

- Publication of the Description, Rules, and the Tasks – on Sunday December 4, 2022.
- Consultation on technical issues – TBD but prior to the submission date.
- Individual solutions to be uploaded to Moodle by Sunday December 18, 2022 (prior 24:00 Moscow time zone).
- Publication of grades on Moodle – by Wednesday December 21, 2022.
- Review sessions (for class and individual) – on Thursday December 22, 2022.

The main grading criterions

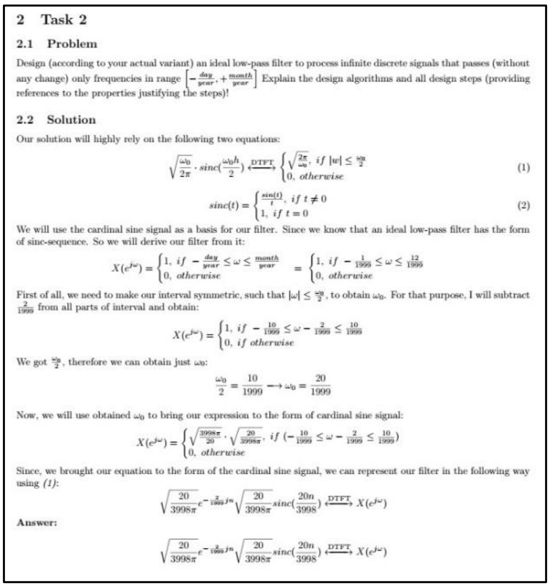
1. human readable but concise
2. well-structuredness (0-5 points for each task)
3. correct and complete answer (0-5 point for each correct answer)
4. correct and self-complete explained solutions (0-5 points for each solution)
5. non-crucial computational errors will be treated as tiny mistake (at most 1-point deduction for each task)

Please be aware that the above criterions will be applied as follows: each criterion $i \in [1..4]$ is a prerequisite for all criterions $j \in [(i+1)..5]$; for example, if your paper has no structurally specified answer or conclusion (i.e., “return” operator), then your solution will be ignored.

Comment on grading criterions

- “Human readable but concise” means that your paper should be well-commented but recommended not excide two 5 pages.

- Well-structuredness is the following “code convention”: each task formulation, solution (its parts/sections), and the answer (conclusion) must be explicit in the paper and identified by appropriate headings/keywords (with respect to logical structure).
- Self-completeness means that the paper should be readable independently on any other resource, but lecture notes for weeks 1 and 2.
- An example of a readable, self-complete, and well-structured paper from the Spring-2021 semester BS-III course on Digital Signal Processing is given on the picture to the right.



2 Task 2

2.1 Problem

Design (according to your actual variant) an ideal low-pass filter to process infinite discrete signals that passes (without any change) only frequencies in range $\left[-\frac{day}{year}, +\frac{month}{year}\right]$ Explain the design algorithms and all design steps (providing references to the properties justifying the steps)!

2.2 Solution

Our solution will highly rely on the following two equations:

$$\sqrt{\frac{\omega_0}{2\pi}} \cdot sinc(\frac{\omega_0 h}{2}) \xleftrightarrow{DTFT} \begin{cases} \sqrt{\frac{2\pi}{\omega_0}}, \ if \ |w| \leq \frac{\omega_0}{2} \\ 0, \ otherwise \end{cases} \quad (1)$$

$$sinc(t) = \begin{cases} \frac{sin(t)}{t}, \ if \ t \neq 0 \\ 1, \ if \ t = 0 \end{cases} \quad (2)$$

We will use the cardinal sine signal as a basis for our filter. Since we know that an ideal low-pass filter has the form of sinc-sequence. So we will derive our filter from it:

$$X(e^{j\omega}) = \begin{cases} 1, \ if \ -\frac{day}{year} \leq \omega \leq \frac{month}{year} \\ 0, \ otherwise \end{cases} = \begin{cases} 1, \ if \ -\frac{1}{1999} \leq \omega \leq \frac{12}{1999} \\ 0, \ otherwise \end{cases}$$

First of all, we need to make our interval symmetric, such that $|\omega| \leq \frac{\omega_0}{2}$, to obtain ω_0. For that purpose, I will subtract $\frac{2}{1999}$ from all parts of interval and obtain:

$$X(e^{j\omega}) = \begin{cases} 1, \ if \ -\frac{10}{1999} \leq \omega - \frac{2}{1999} \leq \frac{10}{1999} \\ 0, \ if \ otherwise \end{cases}$$

We got $\frac{\omega_0}{2}$, therefore we can obtain just ω_0:

$$\frac{\omega_0}{2} = \frac{10}{1999} \longrightarrow \omega_0 = \frac{20}{1999}$$

Now, we will use obtained ω_0 to bring our expression to the form of cardinal sine signal:

$$X(e^{j\omega}) = \begin{cases} \sqrt{\frac{3998\pi}{20}} \cdot \sqrt{\frac{20}{3998\pi}}, \ if \ (-\frac{10}{1999} \leq \omega - \frac{2}{1999} \leq \frac{10}{1999}) \\ 0, \ otherwise \end{cases}$$

Since, we brought our equation to the form of the cardinal sine signal, we can represent our filter in the following way using *(1)*:

$$\sqrt{\frac{20}{3998\pi}} e^{-\frac{2}{1999}jn} \sqrt{\frac{20}{3998\pi}} sinc(\frac{20n}{3998}) \xleftrightarrow{DTFT} X(e^{j\omega})$$

Answer:

$$\sqrt{\frac{20}{3998\pi}} e^{-\frac{2}{1999}jn} \sqrt{\frac{20}{3998\pi}} sinc(\frac{20n}{3998}) \xleftrightarrow{DTFT} X(e^{j\omega})$$

- Please be aware that the above example is just an illustrative one, while actual structural headings may be different. An example of recommended structure/outlines of a solution can be found in lecture notes on the topics (available on Moodle).

Submission rules and formats

- Students must upload individual solutions in two files: the source file (in one of 5 formats – OpenDocument .odt, Word 2007 document .docx, Word document .doc, Markdown with extension .txt, or application/x-tex .tex with any plain class like article) and the result of PDF-conversion of the source file (i.e., pdf file).
- Each submitted file should be named by student first name and surname (for example NikolayShilov.docx and NikolayShilov.pdf)
- On the top of the front page of each submission should start with student first name and surname.
- Submissions with scanned or photo images of hand-written solutions will be discarded without consideration!

Regulation regarding compulsory of the examination

- Students who already have accumulated number of points that they consider sufficient for them may not to attempt the final exam.

Late Submission Policy

According to the Syllabus, “Late submission policy is not applicable to the final examination.”

Notational convention

Let day and $month$ be the day and the month of your birth, ye and ar are numbers presented by the two leftmost and rightmost two digits of your birth year (i.e., like 19 and 61 in the last column are the leftmost and the rightmost two digits of 1961).

Task 1 (10 points)

Consider a game of two players (Alice and Bob) with the following payoff matrix $\begin{bmatrix} day & 24 & ye & 19 \\ month & 4 & ar & 61 \end{bmatrix}$. Rows of the matrix corresponds to strategies $A1$ and $A2$ of Alice, columns – to strategies $B1$, $B2$, $B3$, $B4$ of Bob.

Firstly, characterize the game using terms and concepts introduced in the lecture notes. Then solve the game in mixed strategies.

Task 2 (10 points)

Consider a game of two players (Alice and Bob) with the following payoff matrix $\begin{bmatrix} day: 24 & ye: 19 \\ month: 4 & ar: 61 \end{bmatrix}$. Rows of the matrix corresponds to strategies $A1$ and $A2$ of Alice, columns – to strategies $B1$ and $B2$ of Bob.

Firstly, characterize the game using terms and concepts introduced in the lecture notes. Then solve the game in mixed strategies.

Task 3 (10 points)

Consider problem Rational Agents at the Marketplace (from lecture notes on topic 4). What are individual agents' beliefs, desires, and intentions in the model of the problem? Let agents A and B compete for a salesman, and the matrix of their game flip-or-bid game be $\begin{matrix} A\backslash B & bid & flip \\ bid & -ye: -ar & 0: -month \\ flip & -day: 0 & -day: -month \end{matrix}$ where $L_A = -day$ and $L_B = -month$ are individual (negative) losses in case of flip, $F_A = -ye$ and $F_B = -ar$ are individual (also negative) fins for simultaneous bidding. – Characterize and solve the *flip-or-bid game*.